METHODS OF PROTEIN SEPARATION

;. PHYSICS MADE-EASY

NG TYPICAL UNIVERSITY QUESTIONS WITH
ANSWERS SYSTEMATICALLY ARRANGED
WITH
UNIVERSITY PAPERS UP TO DATE
AND 163 ILLUSTRATIONS

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etc., etc.

1940
ATMA RAM & SONS
EDUCATIONAL PUBLISHERS & BOOKSELLERS
LAHORE

Published by
Brij Lal Pury of Messrs. Atma Ram & Sons
Booksellers, Publishers & Printers
Anarkali, Lahore,

Printed by
Ram Lal Pur
at
The University Tuto
Lal Chand Street, A
Lahore

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PHYSICS MADE-EASY

FOR B.A. & B.Sc. STUDENTS

PART I

PROPERTIES OF MATTER

Q. 1. Explain fully what is meant by the dimensions of physical quantities? State and explain, giving illustrations, the uses of dimensional equations.

Ans. Any physical quantity measures some physical property, and its value is expressed by a unit and the number of times the quantity contains this unit. The numerical value is *inversely* proportional to the magnitude of the unit selected. Thus a length of 1 mile may also be expressed by 1760 yards, or 1760×3 feet. Most of the physical quantities can be derived from three units. These are taken to be mass M, length L, and time T, and are called *fundamental units.*

The **dimensions** *of a physical quantity show how its nature and the magnitude of its unit depend on the magnitudes of the fundamental units selected.* Area is obtained by multiplying length by length, and its dimensions are given by $L \times L$, or L^2, or $M^0L^2T^0$, as the value of area does not depend on the units of mass and time selected. A unit area of 1 square yard is 3^2 times the unit area of 1 square foot, as 1 yard is equal to 3 feet. The units of mass, length, and time are denoted by [M], [L], and [T] respectively. The velocity of a moving body is obtained by dividing the distance travelled by the time taken. If a body, moving with a velocity of v units, travels distance s units in t units of time, then

$$S[L] = v[V] \times t[T]$$

$$v[V] = \frac{s[L]}{t\,[T]}$$

When s and t are both equal to 1, v is also equal to 1.

$\therefore \quad V = \frac{[L]}{[T]}$, or the dimensions of velocity are given by $\frac{L}{T}$, or

$M^0L^1T^{-1}$. This indicates that the magnitude of a unit of velocity is *directly* proportional to the unit of length selected and *inversely* proportional to the unit of time selected. Thus a velocity of 1 mile per hour is equal to 1760 yards per hour, $\frac{1}{60}$ mile per minute, or $\frac{1760}{60}$ yards per minute. Similarly, as the acceleration of a body is its rate of change of velocity, its dimensions are $\dfrac{L^1T^{-1}}{T}$, or L^1T^{-2}, that is, the magnitude of a unit of acceleration is directly proportional to the unit of length and inversely proportional to the *square* of the unit of time used. *When the magnitude of the unit of a physical quantity is proportional to the nth power of a fundamental unit, it is said to be of n dimensions in that fundamental unit.* The above expressions, which indicate the relation between the derived units and the fundamental units, are called **dimensional equations** and are written :

$$[\text{Area}] = [M^0L^2T^0], \quad [\text{Velocity}] = [M^0L^1T^{-1}],$$
$$[\text{Acceleration}] = [M^0L^1T^{-2}]$$

Uses of Dimensional Equations. (1) Knowing the dimensional equation of a physical quantity, we can find its numerical value when changing from one system of units to another. The value of a physical quantity is equal to the product of its numerical value and the value of its unit, and is the *same on all systems of units*. If its dimensional equation is $[M^aL^bT^c]$, and its numerical value is N_1 when the fundamental units are M_1, L_1, T_1, then its value N_2 on another system of units M_2, L_2, T_2 is given by

$$N_2 \times [M_2{}^aL_2{}^bT_2{}^c] = N_1 \times [M_1{}^a \times L_1{}^b \times T_1{}^c]$$

or

$$N_2 = N_1 \times \left(\frac{M_1}{M_2}\right)^a \times \left(\frac{L_1}{L_2}\right)^b \times \left(\frac{T_1}{T_2}\right)^c$$

The dimensions of force (= mass × acceleration) are $M^1L^1T^{-2}$, and its units on the foot-pound-second and centimetre-gram-second systems are called poundal and dyne respectively. If a force of N_1 poundals is equal to N_2 dynes, the relation is given by

$$N_2 = N_1 \times \left(\frac{\text{pound}}{\text{gram}}\right)^1 \times \left(\frac{\text{foot}}{\text{cm.}}\right)^1 \times \left(\frac{\text{second}}{\text{second}}\right)^{-2}$$
$$= N_1 \times (453{\cdot}6) \times (12 \times 2{\cdot}54)$$

(2) Two or more physical quantities of the same nature only can be added up, and the resultant is also of the same nature. As it is not possible to compare two physical quantities of different kinds, *the dimensions of the two sides and of all the terms of an equation must be the same.* Applying this criterion, we can check any equation. For example, we can test the equation for the time period of a compound pendulum,

$$t=2\pi\sqrt{\frac{l^2+k^2}{lg}}=2\pi\sqrt{\frac{l}{g}+\frac{k^2}{lg}}$$

Dimensions of t are $[T^1]$

$$,, \quad ,, \quad \frac{l}{g} \text{ are } \frac{[L^1]}{[L^1T^{-2}]}=[T^2]$$

$$,, \quad ,, \quad \frac{k^2}{lg} \text{ are } \frac{[L^2]}{[L^1L^1T^{-2}]}=[T^2]$$

$$\therefore \quad ,, \quad ,, \quad \sqrt{\frac{l}{g}+\frac{k^2}{lg}} \text{ are } [\sqrt{T^2}]=[T^1].$$

Thus the two terms on the right hand side have the same dimensions, and the dimensions of the two sides of the equation are also equal.

(3) In many cases the form of expression for a physical quantity can be found if we know the factors on which it depends. This is due to the reason already explained in (2). For example, the velocity V of sound in a gas depends on its pressure P and density D. Let $V=KP^aD^b$, where K is a numerical quantity which has no dimensions.

Dimensions of $V=[M^0L^1T^{-1}]$.

$$,, \quad ,, \quad P=\left[\frac{\text{Force}}{\text{Area}}\right]=\left[\frac{M^1L^1T^{-2}}{L^2}\right]=[M^1L^{-1}T^{-2}]$$

$$,, \quad ,, \quad P^a=[M^aL^{-a}T^{-2a}]$$

$$,, \quad ,, \quad D=\left[\frac{\text{Mass}}{\text{Volume}}\right]=\left[\frac{M^1}{L^3}\right]=[M^1L^{-3}]$$

$$\therefore \quad ,, \quad ,, \quad D^b=[M^bL^{-3b}]$$

$$\text{and } ,, \quad ,, \quad P^aD^b=[M^{a+b}L^{-a-3b}T^{-2a}].$$

The dimensions of the two sides of the above equation must be the same, that is, 0 in mass, 1 in length, and -1 in time.

$$\therefore \quad -2a=-1, \text{ or } a=\tfrac{1}{2}$$

and
$$a+b=0, \text{ or } b=-a=-\tfrac{1}{2}$$

$$\therefore \quad V=KP^{\frac{1}{2}}D^{-\frac{1}{2}}=K\sqrt{\frac{P}{D}}.$$

Q. 2. Deduce the dimensions of (a) the coefficient of viscosity, and (b) the constant of gravitation, G.

Obtain a formula for the time of swing of a simple pendulum from a knowledge of the dimensions of the physical quantities involved. (*P.U. 1936*)

Ans. The force F required to maintain the relative velocity V of an area A of a layer of a fluid, of coefficient of viscosity n, with respect to a parallel layer at a distance x from it is given by the relation

$$F=\frac{nAV}{x}, \text{ or } n=\frac{Fx}{AV}$$

$$\therefore \quad [n]=\frac{[F]\times[x]}{[A]\times[V]}=\frac{[M^1L^1T^{-2}]\times[L^1]}{[L^2]\times[L^1T^{-1}]}$$
$$=[M^1L^{-1}T^{-1}]$$

Thus the dimensions of the coefficient of viscosity are 1 in mass, -1 in length, and -1 in time.

According to the law of gravitation, any particle of mass M_1 attracts any other particle of mass M_2 with a force F which is proportional to the product of their masses and inversely proportional to the square of their distance d apart.

$$\therefore \quad F=\alpha\frac{M_1M_2}{d^2}$$

$$\text{or} \quad F=\frac{GM_1M_2}{d^2},$$

where G is the constant of gravitation.

$$\therefore \quad G=\frac{Fd^2}{M_1M_2}$$

or
$$[G]=\frac{[F]\times[d^2]}{[M_1]\times[M_2]}=\frac{[M^1L^1T^{-2}]\times[L^2]}{[M^1]\times[M^1]}$$
$$=[M^{-1}L^3T^{-2}]$$

Therefore the dimensions of G are -1 in mass, 3 in length, and -2 in time.

The period of vibration t of a simple pendulum may depend on its mass M, length l, accelaration due to gravity g, and the angle of swing θ. Out of these θ alone has no dimensions.

$$\text{Let} \quad t = \mathrm{K}\mathrm{M}^a l^b g^c \theta^d$$

$$\therefore \quad [t] = [\mathrm{M}^a] \times [\mathrm{L}^b] \times [\mathrm{L}^c\mathrm{T}^{-2c}]$$

or

$$[\mathrm{T}] = [\mathrm{M}^a \mathrm{L}^{b+c} \mathrm{T}^{-2c}]$$

As the dimensions of the two sides must be the same,

$$\therefore \quad -2c = 1, \quad \text{or} \quad c = -\tfrac{1}{2}$$

$$b + c = 0, \quad \text{or} \quad b = -c = \tfrac{1}{2}$$

and

$$a = 0$$

$$\therefore \quad t = \mathrm{K} l^{\frac{1}{2}} g^{-\frac{1}{2}} = \mathrm{K}\sqrt{\frac{l}{g}}.$$

Q. 3. What is simple harmonic motion? Derive expressions for its characteristic properties,

Two simple harmonic motions of the same period, but of different amplitudes and phases, act in the *same* direction on a particle. Prove that the resultant motion of the particle is simple harmonic.

Ans. Let a particle P (Fig. 1) move with *uniform* speed v around a circle of radius a, and let it make a second particle Q move along any diameter such that the line joining the two is *perpendicular* to that diameter. When P describes the upper half of the circle anticlockwise, Q moves from A to B. Then Q moves back from B to A as P goes along the lower half of the circle. This oscillatory motion of Q about c is called **simple harmonic motion.**

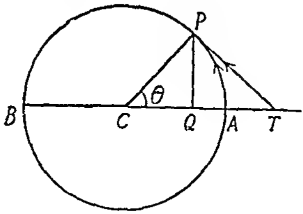


Fig. 1.

The motion of Q may be due to that of P, or it may take place under the action of some force acting along its path, when the circle of reference is imaginary. Further, the path of Q may be straight or curved,

Properties. (1) The oscillatory motion of Q takes place about C, and the radius of the circle of reference, or the **maximum** displacement of Q on either side of C, is called its **amplitude.**

(2) The **displacement** x of Q is measured from C along its path, and at any instant is given by

$$x = a \cos \theta,$$
$$= a \cos wt.$$

where w is the uniform angular velocity of P, t is the time in which it has moved from A to its present position, and θ is the angle described by radius CP in this time.

(3) The **velocity** of Q is equal to the *component* of the velocity of P along the diameter AB. The velocity at P is along the *tangent* TP at that point, and its component along AC is equal to $v \sin \theta$.

$$\text{Velocity of Q} = v \sin \theta$$
$$= wa \sin \theta$$
$$= wa \frac{\sqrt{a^2 - x^2}}{a}$$
$$= w\sqrt{a^2 - x^2}$$

Thus the velocity of Q is different at different points on its path; the *greater* the displacement x, the *smaller* the velocity. It is zero at A, and increases to its maximum value at C. After that it decreases, and becomes zero again at B.

(4) The acceleration of P is equal to aw^2, and is at all points directed towards the centre of the circle. Its *component* along the path of Q gives the acceleration of Q.

$$\text{Acceleration of Q} = w^2 a \cos \theta$$
$$= w^2 c\,Q$$
$$= w^2 x$$
$$= Kx,$$

where K is equal to w^2. Therefore the **acceleration** *of Q is proportional to its displacement x from* C *and is always directed towards it*, the constant of propotionality being equal to w^2. It is greatest at the extreme positions of Q, and from either side reduces to zero at C.

5. When P goes once round the circle, Q completes one vibration. Therefore the **time period** T of Q is given by

$$T=\frac{2\pi}{w}$$
$$=\frac{2\pi}{\sqrt{K}}$$

The **frequency** of vibration n is equal to the number of vibrations executed by Q in a *unit time*, and, therefore, is given by $\frac{1}{T}$.

Thus the motion of a particle is simple harmonic if its acceleration is always directed towards a fixed point in its path and is proportional to its displacement from that point.

Resultant Motion. Let a_1 and a_2 be the amplitudes of the two simple harmonic motions, x_1 and x_2 the corresponding separate displacements at any instant, and e the phase angle by which the second motion is *ahead* of the first. As the two motions have the same time periods, the phase difference between them remains the *same* throughout. The resultant displacement x is equal to the algebraic sum of the individual displacements, because the two displacements are along the *same* line.

$$x_1=a_1\cos\theta$$
$$x_2=a_2\cos(\theta+e)$$
$$x=x_1+x_2$$
$$=a_1\cos\theta+a_2\cos(\theta+e)$$
$$=a_1\cos\theta+a_2\cos\theta.\cos e-a_2\sin\theta.\sin e$$
$$=\cos\theta(a_1+a_2\cos e)-\sin\theta.\,a_2\sin e$$

Let $a\cos e'$ be equal to $a_1+a_2\cos e$, and $a\sin e'$ equal to $a_2\sin e$, so that

$$\tan e'=\frac{a_2\sin e}{a_1+a_2\cos e}$$
$$a^2=(a_1+a_2\cos e)^2+(a_2\sin e)^2$$
$$=a_1^2+2a_1a_2\cos e+a_2^2\cos^2 e+a_2^2\sin^2 e$$
$$=a_1^2+2a_1a_2\cos e+a_2^2$$

$$\therefore\ x = \cos\theta . a \cos e' - \sin\theta .\ a \sin e'$$
$$= a\,(\cos\theta . \cos e' - \sin\theta .\ \sin e')$$
$$= a \cos(\theta + e')$$

Thus the resultant motion of the particle is simple harmonic along the same line. Its amptitude is a, and is ahead of the first motion by phase angle e'.

Q. 4. Find the acceleration of a particle which is travelling round the circumference of a circle of radius R with uniform speed v.

Assuming that the moon describes a circular orbit of radius $3{\cdot}84 \times 10^8$ metres in $27{\cdot}3$ days, and that the outer satellite of Mars describes a circular orbit of radius $2{\cdot}35 \times 10^7$ metres in $1{\cdot}26$ day, find the ratio of the mass of Mars to the mass of the earth.

(*Camb. University*)

Ans. Let a particle move with constant angular velocity w in a circle of radius R (Fig. 2). Its linear velocity $v = (Rw)$ is at every point along the tangent there. It remains fixed in magnitude, but changes constantly in direction. Therefore it must be under the action of some inward force which produces acceleration in it, and the velocity is changed in the *direction of this force*. As the magnitude of velocity remains the same, the acceleration at any point must be *perpendicular* to the direction of velocity there, *i.e.*, along the radius; otherwise the acceleration would have component *along* the tangent there, and the magnitude of velocity would change.

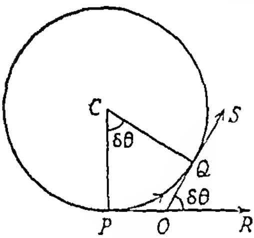


Fig. 2.

Consider two points P and Q *very close* together, so that $\angle PCQ = \delta\theta$ radian is very small, and the particle moves from P to Q in a very short time δt. Draw PR and OQS tangents at P and Q respectively to represent the velocities there. Angle SOR between the tangents is equal to the angle between the corresponding radii. When θ is indefinitely small, its cosine is equal to 1, and its sine is equal to θ (*radian*).

Component of velocity at Q parallel to POR $= v \cos\delta\theta = v$

" " " " " $\perp$ " " $= v \sin\delta\theta = v.\delta\theta$

Therefore, when Q is *indefinitely close to* P, there is no change of velocity along POR, but the particle is given velocity $v\delta\theta$ along PC in time δt.

$$\therefore \text{ Acceleration } at \text{ P} = \frac{V\delta\theta}{\delta t} = Vw = Rw^2$$

$$= \frac{V^2}{R} \text{ along PC.}$$

The magnitude of this *centripetal* acceleration remains constant, but its direction *changes* constantly, and at every point is towards the *centre* of the circle. It is for this reason that Q is taken indefinitely close to P to find the acceleration there.

Problem. If a planet of mass M be at a distance R from its satellite of mass m, then the force exerted by either on the other is equal to $\frac{GMm}{R^2}$, where G is the gravitation constant. This is the force which, acting on the satellite, makes it move in a circle of radius R, with the planet at its centre. Therefore the centripetal acceleration of the satellite is equal to $\frac{GMm}{mR^2} = \frac{GM}{R^2}$. It is also equal to Rw^2, where w is the angular velocity with which the satellite describes its circular orbit.

$$\therefore \quad \frac{GM}{R^2} = Rw^2$$

or

$$M = \frac{R^3 w^2}{G}.$$

Let M_1, w_1, and R_1 be the mass of earth, angular velocity of moon, and radius of moon's orbit respectively, and M_2, w_2, and R_2 be the corresponding terms for Mars and its satellite.

$$\therefore \; M_1 = \frac{R_1^3 w_1^2}{G}$$

$$M_2 = \frac{R_2^3 w_2^2}{G}$$

Dividing, we get

$$\frac{M_2}{M_1} = \frac{R_2^3 w_2^2}{R_1^3 w_1^2} = \left(\frac{R_2}{R_1}\right)^3 \times \left(\frac{w_2}{w_1}\right)^2$$

Radius of moon's orbit $= 3{\cdot}84 \times 10^8$ metres

Angular velocity of moon $= \dfrac{2\pi}{27{\cdot}3}$ radians per day.

Radius of the orbit of Mar's satellite $= 2{\cdot}35 \times 10^7$ metres.

Angular velocity ,, ,, ,, $= \dfrac{2\pi}{1{\cdot}26}$ radians per day.

$$\therefore \quad \frac{\text{Mass of Mars } M_2}{\text{Mass of Earth } M_1} = \left(\frac{2{\cdot}35 \times 10^7}{3{\cdot}84 \times 10^8}\right)^3 \times \left(\frac{2\pi}{1{\cdot}26} \times \frac{27{\cdot}3}{2\pi}\right)^2$$
$$= {\cdot}1076$$

Q. 5. How does the value of g change at different places on the surface of the earth due to its rotation?

The mass of a railway train is 100 tons. What will be its weight when (*a*) stationary, (*b*) travelling due east, (*c*) travelling due west, along the equator at 60 miles per hour? Radius of the earth is 4000 miles.

(P. U. 1933)

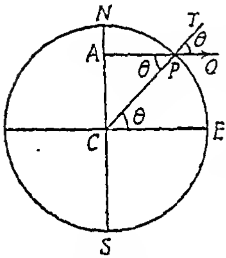


Fig. 3.

Ans. Fig. 3 is a section of the earth, assumed to be a perfect sphere of radius R, through its polar diameter NS, about which it rotates with angular velocity w, and CE is its equatorial radius. All the points on the earth revolve in circles with the same angular velocity, but different speeds, and the radius of the circle described by a point is equal to its distance from the *axis of rotation* NS.

A particle P in the latitude θ describes a circle of radius PA equal to R cos θ, and its centrifugal acceleration R cos θ w^2 is in the direction APQ *away* from A, its centre of circular motion. The acceleration g due to the force of gravitation is the same at all points, as the earth is assumed to be a perfect sphere, and is directed towards its *centre* C. The component of the centrifugal acceleration in the direction CPT is R cos θ w^2. cos θ, and, therefore, the apparent acceleration at P towards C is equal to $g -$ R $\cos^2\theta . w^2$.

The centrifugal acceleration is *greatest* for points on the equator, as for them the radius of their circular motion is R. It goes on decreasing as we go away from the equator, and

becomes zero at the poles. Therefore the apparent acceleration of a body towards the centre of the earth is the least on the equator, and goes on increasing as we approach the poles.

Problem. The earth revolves on its axis from *west to east*, and the angular velocity of the train with respect to this axis of rotation is increased or decreased according as the train moves towards east or west. When the train moves from west to east, its centrifugal acceleration increases, and its apparent acceleration towards the centre of the earth decreases. Therefore its apparent weight *decreases*. On the other hand, when train moves from east to west, *opposite* to the direction of rotation of the earth, its centrifugal acceleration is decreased, and its apparent acceleration towards the centre of the earth is increased. In this case its apparent weight *increases*.

When the train is stationary, its apparent weight is 100 tons wt., or 100×2240 lbs. wt.

Radius of the earth $= 4000$ miles.

$= 4000 \times 1760 \times 3$ ft.

Velocity of a point on the equator $= \dfrac{2\pi \times 4000 \times 1760 \times 3}{24 \times 60 \times 60}$

$= 1536$ ft. per sec.

Centrifugal acceleration ,, ,, $= \dfrac{1536 \times 1536}{4000 \times 1760 \times 3}$

$= {\cdot}1117$ ft./sec^2

Velocity of train with respect to earth's surface

$= 60$ miles per hour.

$= 88$ ft. per sec.

(*a*) Train moving from west to east.

Resultant velocity of the train $= 1536 + 88$

$= 1624$ ft. per sec.

Centrifugal acceleration ,, $= \dfrac{1624 \times 1624}{4000 \times 1760 \times 3}$

$= {\cdot}1249$ ft./sec.^{2}

Increase in centrifugal acceleration ($=$ *Decrease* in downward acceleration) $= {\cdot}1249 - {\cdot}1117$

$= {\cdot}0132$ ft. per sec. per sec.

Decrease in apparent weight $= 100 \times 2240 \times {}^{\cdot}0132$ poundals

$$= \frac{100 \times 2240 \times {}^{\cdot}0132}{32 \times 2240}$$

$$= {}^{\cdot}0412 \text{ ton wt.}$$

$\therefore$ Apparent weight of the train $= 100 - {}^{\cdot}0412$

$= 99{}^{\cdot}9588$ tons wt.

(*b*) Train going west.

Resultant velocity of the train $= 1536 - 88 = 1448$ ft./sec.

Centrifugal acceleration „ „ $= \dfrac{1448 \times 1448}{4000 \times 1760 \times 3}$

$= {}^{\cdot}09928$ ft./sec^2

Decrease in centrifugal acceleration (= *Increase* in downward acceleration) $= {}^{\cdot}1117 - {}^{\cdot}09928$

$= {}^{\cdot}01242$ ft./sec.^{2}

Increaee in apparent weight $= 100 \times 2240 \times {}^{\cdot}01242$ poundals

$$= \frac{100 \times 2240 \times {}^{\cdot}01242}{32 \times 2240}$$

$= {}^{\cdot}0388$ tons wt.

Apparent weight of the train $= 100 + {}^{\cdot}0388$

$= 100{}^{\cdot}0388$ tons wt.

Q. 6. What is moment of inertia of a body? State the units in which it is generally measured.

Find the moment of inertia of a thin uniform circular plate of mass M and radius R_1, with a concentric hole of radius R_2, about an axis passing normally through the centre. (*P. U. 1935*)

Ans. **Moment of Inertia.** When a body rotates about an axis, the effect of its rotation is determined not only by its mass M and angular velocity w, but it also depends on the position of the axis and the distribution of the mass about that axis. The linear velocily of a particle is equal to the product of angular velocity and its distance from the axis, and is different for different particles, though their angular velocity is the same. Consider particles of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis, and having linear velocities $v_1, v_2, \ldots\ldots\ldots\ldots\ldots$

Kinetic energy of first particle $=\frac{1}{2}m_1v_1^2=\frac{1}{2}m_1(r_1w)^2$
$=\frac{1}{2}m_1r_1^2w^2$

" " " second " $=\frac{1}{2}m_2v_2^2=\frac{1}{2}m_2r_2^2w^2$

....

Total " " " rotating body $=\frac{1}{2}m_1r_1^2w^2+\frac{1}{2}m_2r_2^2w^2+\ ..$
$=\Sigma\frac{1}{2}mr^2w^2=\frac{1}{2}(\Sigma mr^2)w^2$
$=\frac{1}{2}MK^2w^2$, $[MK^2=\Sigma mr^2$
$=\frac{1}{2}Iw^2$.

Here K^2 is the *mean square* distance (not square of mean distance) of the particles of the body from the axis of rotation, and I $(=MK^2)$ is called the *moment of inertia* of the body about this axis. It has a definite value for a body about a given axis, and is equal to twice its kinetic energy of rotation when its angular velocity is 1 unit, $[w=1,\ K.E.=\frac{1}{2}I,$ or $I=2\times K.E.]$

Generally mass is measured in pounds, and distances are given in feet, therefore moment of inertia is usually measured in lb. ft.^{2} units.

Moment of Inertia of Disc. Fig. 4 is a section of the disc by a plane perpendicular to its thickness t.

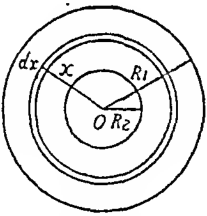


Fig. 4.

Face area of the disc $=\pi R_1^2-\pi R_2^2$
Volume " " $=\pi(R_1^2-R_2^2)t$

Mass per unit volume $=\dfrac{M}{\pi(R_1^2-R_2^2)t}$

Consider a coaxial disc of inner radius x and outer radius $x+dx$. All its particles are at the *same* distance from the axis passing normally through O.

Face area of this ring $=\pi(x+dx)^2-\pi x^2=2\pi xdx$

Volume " " " $=2\pi xdx\times t$

Mass " " " $=2\pi xdx\times t\times\dfrac{M}{\pi(R_1^2-R_2^2)t}$

$$=\frac{2Mxdx}{(R_1^2-R_2^2)}$$

M. I. of this ring about the axis

$$=\frac{2Mx\,dx\times x^2}{(R_1^2-R_2^2)}$$

$\therefore$ M. I. of the whole disc about the axis

$$=\int_{R_2}^{R_1}\frac{2M}{(R_1^2-R_2^2)}\,x^3\,dx$$

$$=\frac{2M}{(R_1^2-R_2^2)}\left[\frac{x^4}{4}\right]_{R_2}^{R_1}$$

$$=\frac{2M}{(R_1^1-R_2^2)}\left(\frac{R_1^4-R_2^4}{4}\right)$$

$$=M\left(\frac{R_1^2+R_2^2}{2}\right)$$

Q. 7. State and prove the principles of perpendicular and parallel axes as applied to moments of inertia.

Find the moment of inertia of a cylinder about an axis perpendicular to its length and passing through its centre of gravity.

Ans. **Principle of perpendicular axis.** *The sum of the moments of inertia of a plane lamina about any two perpendicular axes in its plane is equal to its moment of inertia about an axis perpendicular to its plane and passing through the point of intersection of the first two axes.*

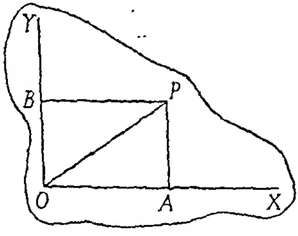


Fig. 5.

Let OX and OY (Fig. 5) be the two perpendicular axes in the plane of the lamina, and let the axis of Z be perpendicular to this plane and pass through O. The moments of inertia of a particle P, of mass m, about these axes are given by $m\times PA^2$, $m\times PB^2$, and $m\times PO^2$ respectively. Then for the whole lamina,

M. I. about x-axis $=I_x=\Sigma m\times PA^2$

" " y-axis $=I_y=\Sigma m\times PB^2$

$$\text{M. I. about } z\text{-axis} = I_z = \Sigma m \times PO^2 = \Sigma m\ (PA^2 + PB^2)$$
$$= \Sigma m \times PA^2 + \Sigma m \times PB^2$$
$$= I_x + I_y.$$

Principle of parallel axes. *The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of gravity and the product of its mass and the square of the distance between the two axes.*

Let any axis and a parallel axis through the centre of gravity of the body cut the plane of paper in A and G respectively, and P be a particle of mass m (Fig. 6). Draw PB perpendicular on AG, then

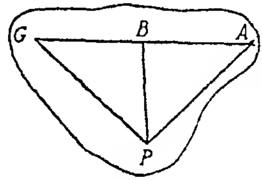


Fig. 6

$$PA^2 = PG^2 + GA^2 - 2GB \times GA.$$

M. I. of P about axis through $G = m \times PG^2$

" " " " $A = m \times PA^2$

" of body " " $A = I_A = \Sigma m \times PA^2$

$$= \Sigma m\ (PG^2 + GA^2 - 2GB \times GA)$$
$$= \Sigma m \times PG^2 + \Sigma m \times GA^2 - \Sigma m \times 2GB \times GA.$$

Now, $m \times GB$ is the moment of the weight of the particle P about the axis through G. As any body *balances* about an axis passing through its centre of gravity, the algebraic sum of such moments for all the particles of the body, *i.e.*, $\Sigma m \times GB$, is zero. Further, $\Sigma m \times GA^2 = GA^2 \times \Sigma m = M \times GA^2$, where M is the mass of the body, and $\Sigma m \times PG^2 = I_G$ is the moment of inertia of the whole body about the axis through G.

$$\therefore\ I_A = I_G + M \times GA^2$$

Moment of inertia of a cylinder. In Q. 6, the moment of inertia of a disc, about an axis perpendicular to its faces and passing through its centre of gravity, has been found. If the disc is not hollow, $R_2 = 0$, and its moment of inertia about the axis is $M\dfrac{R^2}{2}$, where R is its radius. Therefore, according to the principle of perpendicular axes, the sum of the moments of inertia of an *indefinitely thin* disc about any

two perpendicular diameters is equal to $M\frac{R^2}{2}$, or its moment of inertia about any diameter is $M\frac{R^2}{4}$, as, by symmetry, it is the same about all its diameters.

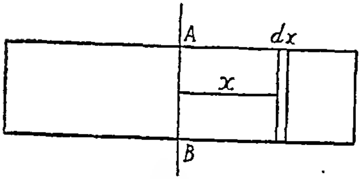


Fig. 7.

Let AB (Fig. 7) be the axis passing through the centre of gravity of a cylinder of length l, radius R, and mass M, and consider a very thin disc of thickness dx, at a distance x from this axis.

$$\text{Volume of the cylinder} = \pi R^2 \times l$$

$$\text{Mass per unit volume} = \frac{M}{\pi R^2 l}$$

$$\text{Volume of the disc} = \pi R^2 \times dx$$

$$\text{Mass ,, ,,} = \frac{M}{\pi R^2 l} \times \pi R^2 dx = \frac{M}{l} dx$$

$$\text{M. I. of the disc about its any diameter} = \frac{M dx}{l} \times \frac{R^2}{4}$$

,, ,, ,, axis AB (By principle of parallel axis)

$$= \frac{M dx}{l} \times \frac{R^2}{4} + \frac{M dx}{l} \times x^2$$

$$= \frac{M}{l}\left(\frac{R^2}{4} + x^2\right) dx$$

$$\therefore \text{M. I. of the whole cylinder about AB} = 2\int_0^{l/2} \frac{M}{l}\left(\frac{R^2}{4} + x^2\right) dx$$

$$= 2\frac{M}{l}\left[\frac{R^2 x}{4} + \frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{2M}{l}\left[\frac{R^2 l}{4 \times 2} + \frac{1}{3}\left(\frac{l}{2}\right)^3\right]$$

$$= M\left(\frac{R^2}{4} + \frac{l^2}{12}\right)$$

Q. 8. Derive the formula for determining the moment of inertia of a sphere about a diameter.

A flywheel weighs 10 tons, and the whole of the weight may be considered to be concentrated at a distance of 3 feet from the axis. What is the amount of energy stored in the flywheel when rotating at a speed of 100 revolutions per minute ? *(P. U. 1934)*

Ans. **M. I. of sphere.** Fig. 8 shows a section of a sphere of mass M and radius R through its centre C. If its moment of inertia is to be found about the diameter AB, consider an indefinitely thin disc of thickness dx and radius y formed by two planes *perpendicular* to AB. The moment of inertia of this disc about AB is equal to the product of its mass and

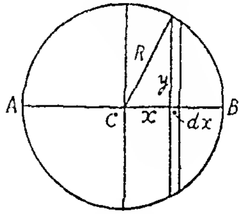


Fig. 8.

$\frac{y^2}{2}$, as AB passes through its centre and is perpendicular to its faces.

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

$$\text{Mass per unit volume} = \frac{M}{\frac{4}{3}\pi R^3}$$

$$\text{Volume of the disc} = \pi y^2 \times dx$$

$$\text{Mass} \quad " \quad " \quad = \frac{3M}{4\pi R^3} \times \pi y^2 dx = \frac{3My^2dx}{4R^3}$$

$$\therefore \text{M. I. of this disc about AB} = \frac{3My^2dx}{4R^3} \times \frac{y^2}{2}$$

$$= \frac{3My^4dx}{8R^3} = \frac{3M(R^2-x^2)^2dx}{8R^3}$$

$$[\because R^2 = x^2 + y^2$$

The sphere may be considered to consist of such discs whose distance x from C changes from 0 to R for *each* hemisphere.

$$\therefore \text{M. I. of sphere about AB} = 2\int_0^R \frac{3M(R^2-x^2)^2}{8R^3}dx$$

$$= \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2x^2 + x^4)dx$$

$$= \frac{3M}{4R^3}\left[R^4x - 2\frac{R^2x^3}{3} + \frac{x^5}{5}\right]_0^R$$

$$= \frac{3M}{4R^3}\left[R^5 - 2\frac{R^5}{3} + \frac{R^5}{5}\right]$$

$$= \frac{3M}{4R^3} \times \frac{8R^5}{15} = \frac{2}{5}MR^2$$

When a body of mass M rotates, its kinetic energy of rotation is equal to $\frac{1}{2}MK^2w^2$, where K and w are its radius of gyration and angular velocity respectively about the axis of rotation.

Mass of flywheel $= 10 \times 2240$ lbs.

Angular velocity $= \frac{100 \times 2\pi}{60}$ *radians* per second.

Radius of gyration $= 3$ ft.

$\therefore$ K.E. of rotation $= \frac{1}{2} \times 22400 \times 3^2 \times \left(\frac{100 \times 2 \times 3{\cdot}142}{60}\right)^2$

$= 1106 \times 10^4$ *foot-poundals*.

Q. 9. Derive an expression for the acceleration of a body rolling-freely down an inclined plane.

A solid ball and a hollow cylinder of the same mass roll freely down an inclined plane. Which will reach the bottom first? (*P. U. Subsidiary, 1938*)

Ans. Let a body of mass M and radius R roll freely, that is, *without slipping*, down a plane inclined at angle θ to the horizontal (Fig. 9). When it has moved a distance S from the *start*, let the linear velocity of its axis of rotation be v, and w be its angular velocity about this axis. In one rotation the axis of rotation moves through a distance $2\pi R$,

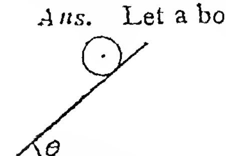


Fig. 9.

while a point on the rim turns through 2π radians in the same time, therefore at any instant v is equal to $R\omega$.

If I is the moment of inertia and K the radius of gyration of the body about the axis of rotation, its kinetic energy consists of two parts; $\frac{1}{2}I\omega^2$ or $\frac{1}{2}MK^2\omega^2$ is due to the rotatory motion, and $\frac{1}{2}Mv^2$ is due to its motion of translation. In moving a distance S down the inclined plane, the body comes down *vertically* through a distance S sin θ, and its potential energy is decreased by $MgS \sin \theta$. As there is no slipping, no energy is dissipated, and, therefore, the gain of kinetic energy is equal to the loss of potential energy.

$$MgS \sin \theta = \tfrac{1}{2}MK^2\omega^2 + \tfrac{1}{2}Mv^2$$

$$= \tfrac{1}{2}MK^2\frac{v^2}{R^2} + \tfrac{1}{2}Mv^2 = \frac{M}{2}\left(\frac{K^2}{R^2}+1\right)v^2$$

Differentiating both sides with respect to time t, we get

$$Mg \sin \theta.\frac{dS}{dt} = \frac{M}{2}\left(\frac{K^2}{R^2}+1\right)v\, 2.\frac{dv}{dt}.$$

Here $\dfrac{dv}{dt}$ and $\dfrac{dS}{dt}$ respectively denote the acceleration a and linear velocity v of the body.

$$\therefore \quad g \sin \theta.\, v = M\left(\frac{K^2}{R^2}+1\right)va.$$

or $$\text{Acceleration } a = \frac{g \sin \theta}{\left(\dfrac{K^2}{R^2}+1\right)}$$

This shows that for a given angle θ of the inclined plane, the acceleration is *inversely* proportional to $\left(\dfrac{K^2}{R^2}+1\right)$. The *greater* the value of K^2 *as compared* with R^2, the *smaller* is the acceleration, and vice versa. It is independent of the mass of the body.

It is proved in Q. 8 that the moment of inertia of a solid sphere, of radius R and mass M, about a diameter is equal to $\frac{2}{5}MR^2$, or $K^2 = \frac{2}{5}R^2$. In the case of a very thin hollow cylinder, all its particles are at the *same* distance from its axis, and hence its radius of gyration K about this axis is equal to its

radius R. As the value of $\frac{K^2}{R^2}$ for a hollow cylinder is *greater* than that for a solid sphere, the acceleration of the former down an inclined plane is smaller than that of the latter. Therefore the *solid sphere*, moving with greater acceleration, will reach the bottom of the inclined plane first.

Q. 10. Describe a conical pendulum, and obtain an expression for the period of its motion. Show how the period of revolution of a conical pendulum may be made independent of the exact value of the radius of its circular path. *(B. U. 1928)*

Ans. **Conical Pendulum.** It consists of a *small* heavy bob of mass M suspended by a *very light* inextensible string. The bob is displaced to one side and given circular motion in a horizontal plane. It traces out a circular path repeatedly under the action of a centripetal force, while the string describes a cone. The centre C (Fig. 10) of the horizontal circle is vertically below the point of suspension A. The forces acting on the bob are the tension T of the string, acting along BA, and the force of gravity Mg. When the bob rotates with angular velocity w in a horizontal circle of radius R, the vertical component of T balances the downward force of gravity on the bob, and its horizontal component provides the centripetal force MRw^2 for the circular motion. The length l of the pendulum is equal to the distance from the point of suspension A to the centre of gravity of the bob B.

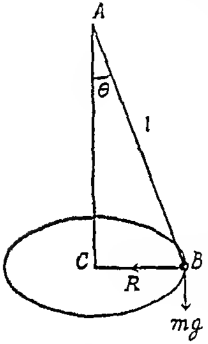


Fig. 10.

If at any instant the string make angle θ with the vertical, and the bob moves with angular velocity w in a horizontal circle of radius R, then

$$R = l \sin\theta \quad \ldots \quad \ldots(1)$$

$$T\cos\theta = Mg \quad \ldots \quad \ldots(2)$$

$$T\sin\theta = MRw^2$$

$$= Ml\sin\theta . w^2 \qquad \text{[from (1)}$$

or

$$T = Mlw^2 \quad \ldots(3)$$

Dividing (2) by (3), we get

$$\cos\theta = \frac{g}{l w^2}$$

or

$$w^2 = \frac{g}{l\cos\theta}$$

This proves that at any place, and for a given value of l and θ, the angular velocity w is *fixed*, and the *pendulum has a definite time period t*.

$$\therefore\ t = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{\dfrac{g}{l\cos\theta}}}$$

$$= 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

When R is *very small as compared with l*, θ is very small: $\cos\theta$ is practically equal to 1, and, therefore, t is independent of θ and R. Under this condition the above result becomes

$$t = 2\pi\sqrt{\frac{l}{g}}$$

Q. 11. Derive the formula for the period of a compound pendulum and prove that the centres of oscillation and suspension are interchangeable. Indicate how this principle is utilised in an accurate determination of gravity. (*P. U. 1938*)

Ans. **Compound Pendulum.** Any rigid body which can *freely* oscillate about a *horizontal* axis, passing through *it*, is called a compound pendulum. Fig. 11 is its *vertical* section through its centre of gravity G; the horizontal axis of suspension cuts it at the centre of suspension S, and at a distance l_1 *from* G. In the normal position of the pendulum, G is vertically below S. When it is displaced to one side and let free, it moves backward under the force of gravity, and begins to oscillate about its normal position,

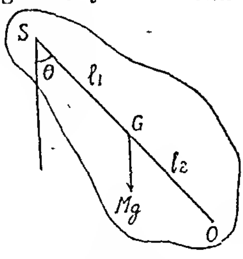


Fig. 11.

In any position where line SG is inclined at θ to the vertical, the only force Mg, acting at G, exerts a restoring moment $Mgl_1 \sin\theta$ about the axis of suspension and produces angular acceleration α about this axis. The linear acceleration of a particle is equal to the product of its angular acceleration and its distance from the axis of suspension, and is *different* for different particles, though their angular acceleration is the *same*.

Consider particles of masses m_1, m_2,....at distances r_1, r_2,...... and having linear accelerations a_1, a_2,......

Force acting on the first particle $= m_1 \times a_1 = m_1 r_1 \alpha$

Moment of this force about the axis of suspension

$$= m_1 r_1 \alpha \times r_1 = m_1 r_1^2 \alpha$$

Force acting on the second particle $= m_2 \times a_2 = m_2 r_2 \alpha$

Its moment about the axis of suspension

$$= m_2 r_2 \alpha \times r_2 = m_2 r_2^2 \alpha.$$

$\therefore$ Sum of moments for all particles $= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \ldots\ldots$

$$= \Sigma m r^2 \alpha = (\Sigma m r^2)\alpha = I\alpha$$

Here I is the moment of inertia of the pendulum about the axis of *suspension*, and, according to the principle of parallel axes, is equal to $M(K^2 + l_1^2)$, where K is its radius of gyration about a *parallel axis through* G.

$$\therefore \quad Mgl_1 \sin\theta = I\alpha = M(K^2 + l_1^2)\,\alpha$$

When θ is *very small*, $\sin\theta$ is practically $= \theta$ (radian), then

$$gl_1\theta = (K^2 + l_1^2)\,\alpha$$

or, $$\frac{\text{angular acceleration } \alpha}{\text{angular displacement } \theta} = \frac{gl_1}{K^2 + l_1^2} = \text{constant.}$$

Thus the *restoring* angular acceleration of the pendulum is *proportional* to its angular displacement. Therefore its motion is *simple harmonic*, and the constant of proportionality is equal to $\dfrac{gl_1}{K^2 + l_1^2}$

$\therefore$ Time period $t_1 = \dfrac{2\pi}{\sqrt{\dfrac{gl_1}{K^2+l_1^2}}} = 2\pi\sqrt{\dfrac{K^2+l_1^2}{l_1 g}} \ldots \quad \ldots(1)$

$$= 2\pi\sqrt{\frac{\dfrac{K^2}{l_1}+l_1}{g}}$$

A point O, on the *other* side of G, in line with S and G and at a distance $\dfrac{K^2}{l_1}+l_1$ from S, and, therefore, at a distance $\dfrac{K^2}{l_1}$ from G, is called centre of *oscillation.* A horizontal axis passing through O is called axis of oscillation. Let GO be equal to l_2.

The time period t_2 about the axis of oscillation is given by

$$t_2 = 2\pi\sqrt{\frac{\dfrac{K^2}{l_2}+l_2}{g}} \quad \ldots \quad \ldots \quad \ldots(2)$$

$$l_2 = \frac{K^2}{l_1},$$

or
$$\frac{K^2}{l_2} = l_1$$

Adding the corresponding sides, we get

$$\frac{K^2}{l_2} + l_2 = \frac{K^2}{l_1} + l_1$$

$$\therefore \quad t_2 = t_1$$

Thus the axes of suspension and oscillation are interchangeable.

Determination of 'g'. On squaring and rearranging (1), we get

$$t_1^2 l_1 = \frac{4\pi^2}{g}(K^2+l_1^2) \quad \ldots \quad \ldots \quad \ldots(3)$$

Similarly, from (2)

$$t_2^2 l_2 = \frac{4\pi^2}{g}(K^2+l_2^2) \quad \ldots \quad \ldots \quad \ldots(4)$$

Subtracting (4) from (3) after putting $t_2 = t_1$, we have

$$t_1^2(l_1 - l_2) = \frac{4\pi^2}{g}(l_1^2 - l_2^2)$$ (Since $t_2 = t_1 = t$)

If l_1 is *not equal* to l_2, the above becomes

$$t_1^2=\frac{4\pi^2}{g}(l_1+l_2)$$

or
$$t_1=2\pi\sqrt{\frac{l_1+l_2}{g}} \quad \ldots \quad \ldots \quad \ldots(5)$$

Thus when the positions of the two horizontal axes, on the *opposite* sides of G and at *unequal* distances from it, are so adjusted that the time periods about them are *exactly equal*, accurate value of g can be calculated from the distance l_1+l_2 between them and the value of time period.

Usually it is very tedious to find the positions of the axes for the two time periods to be exactly equal, but they can be easily made nearly equal by adjusting the weights carried by the pendulum. Then subtracting (4) from (3),

$$(t_1^2l_1-t_2^2l_2)=\frac{4\pi^2}{g}(l_1^2-l_2^2)$$

Let $l_1 > l_2$, $l_1=l+x$, and $l_2=l-x$. $\therefore$ $l_1+l_2=2l$, $l_1-l_2=2x$, and $l_1^2-l_2^2=4xl$. Putting these values in the above equation,

$$\{t_1^2(l+x)-t_2^2(l-x)\}=\frac{4\pi^2}{g}\times 4xl$$

or
$$\left\{\frac{l(t_1^2-t_2^2)+x(t_1^2+t_2^2)}{4xl}\right\}=\frac{4\pi^2}{g}$$

or
$$\frac{4\pi^2}{g}=\frac{t_1^2-t_2^2}{4x}+\frac{t_1^2+t_2^2}{4l}$$

$$=\frac{t_1^2-t_2^2}{2(l_1-l_2)}+\frac{t_1^2+t_2^2}{2(l_1+l_2)}$$

$$\therefore\ g=\frac{4\pi^2}{\dfrac{t_1^2-t_2^2}{2(l_1-l_2)}+\dfrac{t_1^2+t_2^2}{2(l_1+l_2)}}$$

Hence l_1-l_2, the difference of the distances of the two axes from G, cannot be determined with great accuracy; but, as the two time periods are nearly equal, the first term $\dfrac{t_1^2-t_2^2}{2(l_1-l_2)}$

is very small as compared with the second term $\frac{t_1^2+t_2^2}{2(l_1+l_2)}$), and very little error is introduced.

Q. 12. Find an expression for the time of oscillation of a compound pendulum, and show that there are four points collinear with the centre of gravity the periods of oscillation about which are equal.

A heavy spherical bob of diameter 10 cm. is suspended by a very fine wire. If the distance from the point of suspension to the centre of the bob be 1 metre, calculate the length of the equivalent simple pendulum. (*B. U. 1935*)

Ans. See Q. 11 for expression for time period. The time period t of a compound pendulum about a horizontal axis at a distance l from a parallel axis through its centre of gravity is given by

$$t=2\pi\sqrt{\frac{K^2+l^2}{lg}},$$

where K is its radius of gyration about the parallel axis through its *centre of gravity* and g is the value of acceleration due to gravity.

$$t^2=\frac{4\pi^2(K^2+l^2)}{lg}$$

or

$$l^2-\frac{gt^2}{4\pi^2}l+K^2=0.$$

This is a quadratic equation in l and gives two values.

$$\therefore\ l=\frac{\frac{gt^2}{4\pi^2}\pm\sqrt{\frac{g^2t^4}{16\pi^4}-4K^2}}{2}$$

$$l=\frac{gt^2}{8\pi^2}+\sqrt{\frac{g^2t^4}{64\pi^4}-K^2},\ \text{or}\ \frac{gt^2}{8\pi^2}-\sqrt{\frac{g^2t^4}{64\pi^4}-K^2}$$

Similarly, there are two values of l on the *other side* of the centre of gravity for which t is the same as for the above two values of l on the first side,

Problem. The length of an equivalent simple pendulum is equal to $\frac{K^2+l^2}{l}$, and for a sphere of radius R, its radius of gyration about a diameter is $\sqrt{\frac{2}{5}R^2}$

Radius of bob = 5 cms.

$\therefore$ Radius of gyration $K=\sqrt{\frac{2}{5}\times 25}=\sqrt{10}$ cm.

Value of $l=100$ cms.

$\therefore$ Length of equivalent simple pendulum

$$=\frac{10+100^2}{100}=100{\cdot}1 \text{ cms.}$$

Q. 13. A unit cube is deformed by the application of equal perpendicular forces to its faces acting outwards. Prove that $k=3(\alpha-2\beta)$ when k, α and β have their usual meaning. Also express the rigidity modulus in terms of α and β, and hence deduce an expression for Young's modulus in terms of k and n.

(*P. U. 1921*)

Ans. Let a *unit* cube ABCDEFGH (Fig. 12*a*) be subjected to *equal* forces perpendicular to its faces and acting outwards, and let each force be equal to P per unit area. Each force produces *extension* in its *own* direction and *contraction* in directions *perpendicular* to it. Strain produced per *unit* stress in the direction of stress is denoted by α, while β denotes strain per unit stress in perpendicular directions. Therefore, due to one pull, an extension Pα is produced in its own direction, while the two sides perpendicular to it suffer a contraction of Pβ each. When all the forces act simultaneously, each side of the unit cube experiences an extension of Pα due to the pull in its own direction, a contraction Pβ due to the second pull, and a further contraction of Pβ due to the third pull.

Initial length of each side $=1$

,, Volume of the cube $=1$

Final length of each side $=1+P\alpha-P\beta-P\beta$

$$=1+P(\alpha-2\beta)$$

$\therefore$ Final volume of the cube $=\{1+P(\alpha-2\beta)\}^3$

$$=1+3P(\alpha-2\beta)+\text{negligible terms.}$$

As α and β are both *very small*, their higher powers are negligible.

Increase in volume $= 3P(\alpha - 2\beta)$

Strain $= 3P(\alpha - 2\beta)$

$$\therefore \text{ Bulk modulus } k = \frac{P}{3P(\alpha-2\beta)} = \frac{1}{3(\alpha-2\beta)}$$

$$\text{Rigidity modulus } n = \frac{1}{2(\alpha+\beta)},$$ [*see Q. 14.*]

$$\text{Young's modulus } y = \frac{\text{Stress}}{\text{Longitudinal strain}}$$

$$= \frac{\text{Stress}}{\text{Stress} \times \alpha} = \frac{1}{\alpha}$$

Rearranging the equations for k and n,

$$\alpha - 2\beta = \frac{1}{3k}$$

$$2\alpha + 2\beta = \frac{1}{n}$$

On adding,

$$3\alpha = \frac{1}{3k} + \frac{1}{n} = \frac{n+3k}{3kn}$$

or
$$\alpha = \frac{n+3k}{9Kn}$$

$$\therefore \quad \text{Young's modulus} = \frac{1}{\alpha} = \frac{9kn}{n+3k}$$

Q. 14. Define Poisson's ratio; and show that the rigidity n and Young's modulus y are connected by the relation

$$n = \frac{y}{2(1+\sigma)},$$

where σ is the Poisson's ratio. (*P. U. 1938*)

Ans. When a solid body is subjected to a force, the ratio of the *strain* produced in a *perpendicular* direction to the strain produced *in* the direction of the force is called **Poisson's ratio** for that solid,

If the lower face ABEF (Fig. 12 *a*) of a cube is kept fixed, and a shearing force T per unit area is applied to its upper face DCGH from left to right it is sheared clockwise through φ radian, and its upper face takes the position D′C′G′H′. The plane of shear is parallel to the faces ABCD and FEGH, and as φ is *very small*, the height of the solid remains practically the same, so that face D′C′G′H′ lies in the same plane as DCGH.

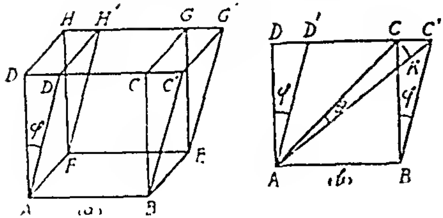


Fig. 12.

The front face becomes a parallelogram; diagonal AC and all lines parallel to it are extended and turned clockwise (Fig. 12 *b*), while all lines perpendicular to them and parallel to BD are compressed, but turned in the same direction. With A as centre, draw arc CK of radius AC. It is practically a straight line and perpendicular to AC′. Angle φ being *very small*, $\angle$ CC′B is almost equal to 90°, and $\angle$ CC′K equal to 45°, so that C′K and CK are equal.

$$AC = BC\sqrt{2},\ CK = C'K = CC'\cos 45^\circ = \frac{CC'}{\sqrt{2}},\quad \varphi = \frac{CC'}{BC}$$

$$\therefore \theta = \frac{CK}{AC} = \frac{CC'}{\sqrt{2}.BC\sqrt{2}} = \frac{CC'}{2BC} = \frac{\varphi}{2}$$

$$\text{Strain along } AC = \frac{C'K}{AC} = \theta = \frac{\varphi}{2}$$

Similarly, it may be shown that the diagonal BD is rotated clockwise through the same angle θ, the two diagonals remain perpendicular to each other, and the contraction of BD is *equal* to the extension of AC. Therefore the cube can also be sheared by applying to it, in the plane of shear, an *extending* force parallel to AC and an *equal contracting* force *perpendicular* to the first and parallel to BD, the two forces being inclined at 45° to the direction of the shearing force. Let the extending or contracting stress be equal to P.

When a body of length R and cross-section area a suffers longitudinal strain under a force F, work done is equal to the

product of contraction or extension x and the average force $\frac{F}{2}$, because stress increases with strain and is proportional to it.

$$\therefore \quad \text{Work done per unit volume} = \frac{F \times x}{2a \times l} = \frac{\text{Stress} \times \text{strain}}{2}$$

$$\text{Work done per unit volume by extending force} = \frac{P\theta}{2}.$$

$$\text{,, ,, ,, ,, ,, ,, contracting force} = \frac{P\theta}{2}$$

$$\text{Total work done per unit volume} = P\theta = \frac{P\varphi}{2}, \qquad [\because \theta = \frac{\varphi}{2}.$$

In the case of shear also, the shearing stress is proportional to the angle of shear. The shearing force F exerts moment F × DA about the fixed end AB (Fig. 12 *b*), and work done is equal to the product of average moment $\frac{F \times AD}{2}$ and the angle of shear ϕ. If a is the area of the upper face, tangential force per unit area is equal to $\frac{F}{a} = T$.

$$\therefore \quad \text{Total work done} = \frac{F \times AD \times \varphi}{2}$$

$$\therefore \quad \text{Work done per unit volume} = \frac{F \times AD \times \varphi}{2a \times AD} = \frac{T \times \varphi}{2}$$

$$= \frac{\text{stress} \times \text{strain}}{2}$$

As the above two methods produce the same shearing effect, work done per unit volume must be the same in the two cases,

$$\frac{P\phi}{2} = \frac{T\phi}{2}$$

$$\therefore \quad P = T$$

Let α and β respectively be the longitudinal and lateral *strain* per *unit longitudinal* stress. Then

$$\text{Young's modulus } y = \frac{1}{\alpha}$$

$$\text{Poisson's ratio} \quad \sigma = \frac{\beta}{\alpha}$$

Increase in AC due to extending force $= T \times AC \times \alpha$

" " " " " contracting force $= T \times AC \times \beta$

$\therefore$ Total increase in $AC = T \times AC(\alpha + \beta)$

$$\text{Extension in } AC = C'K = \frac{CC'}{\sqrt{2}} = \frac{BC.\varphi}{\sqrt{2}} = \frac{AC.\varphi}{\sqrt{2}.\sqrt{2}}$$

$$\therefore \quad T \times AC(\alpha + \beta) = \frac{AC.\varphi}{2},$$

$$\text{or } T = \frac{\varphi}{2(\alpha + \beta)}$$

$$\therefore \quad \text{Coefficient of rigidity } n = \frac{T}{\varphi} = \frac{\varphi}{2(\alpha + \beta)\varphi} = \frac{1}{2(\alpha + \beta)}$$

$$= \frac{1}{2\alpha\left(1 + \frac{\beta}{\alpha}\right)} = \frac{y}{2(1 + \sigma)}$$

Q. 15. What is meant by Young's modulus, the modulus of rigidity, and the bulk modulus ?

A straight cylindrical rod is fixed at one end and twisted by a couple applied to the other. Deduce the relation between the twisting couple and the coefficient of rigidity. *(P. U. 1936)*

Ans. When a solid is subjected to a deforming force, it experiences change of length, volume, or shape. The deformation brings into play a resisting force which is equal and opposite to the force applied. When the change is *very small*, the deformation is *proportional* to the deforming force. In all cases, stress is equal to the force applied per unit area of the surface.

Young's Modulus. When the deforming force is applied to the body along one direction only, the change of length per unit length in *that direction* is called longitudinal strain. Young's modulus is obtained by dividing stress by this strain.

Bulk Modulus. In this case change of volume is considered, and the strain is equal to change of volume per unit volume. The bulk modulus is equal to stress divided by strain.

Modulus of Rigidity. In this case there is no change of volume, and the deforming force changes only the shape of the body. In Fig. 13C, the upper face AF is kept *fixed*,

and a *tangential* force is applied to the lower face BE from left to right. The result is that all lines joining the two faces are turned through some angle. The strain is equal to the angle φ (in *radian*) through which a line originally *perpendicular* to the fixed face turns. It is also called angle of shear or shear strain. The coefficient of rigidity is equal to the tangential stress divided by shear strain.

Couple for twisting. A cylindrical rod of length l, radius r, and coefficient of rigidity n, is fixed at its upper end, and, by applying a couple, its lower end is twisted through θ radian. For equilibrium, the twisting and restoring couples are equal and apposite. The angle of twist is greatest at the free end, and decreases as the fixed end is approached. All the radii of the lower end are turned through the *same* angle, but the displacement of any radius is greatest at the rim and decreases to zero at the centre. Therefore the shearing stress is *not uniform*.

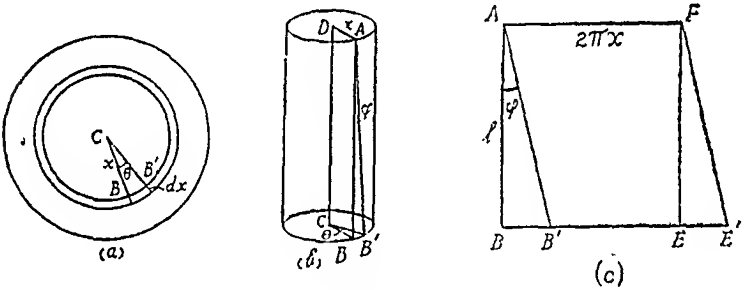


Fig. 13.

The cylinder may be considered to consist of a very large number of coaxial hollow cylinders, each of very small thickness dx (Fig. 13*a*). Consider a hollow cylinder of inner and outer radii x and $x+dx$ respectively. On twisting, a line AB (Fig. 13*b*), which is parallel to the axis DC, is turned through a very small angle φ radian, and takes the position AB$'$. Radius CB of the lower face is turned through θ radian into the position CB$'$. If this hollow cylinder is cut along AB before twisting, and flattened out, its cylindrical surface becomes a rectangle ABEF of sides l and $2\pi x$ (Fig. 13*c*). On twisting, AB takes the position AB$'$, and then on cutting the

hollow cylinder along AB′ and flattening it, a parallelogram AB′E′F is obtained. Thus this hollow cylinder has been sheared through φ radian.

$$\text{Arc } BB' = x\theta = l\varphi$$

$$\therefore \quad \varphi = \frac{x\,\theta}{l}.$$

The angle of shear φ is the same for any one hollow cylinder, but is different for different cylinders. It is greatest for the outermost cylinder and least for the innermost.

$$\text{Shearing stress} = n \times \varphi = \frac{nx\theta}{l}$$

$$\textit{Face}\text{ area of hollow cylinder} = 2\pi x \times dx$$

$$\text{Shearing force on this area} = 2\pi x dx \times \frac{nx\theta}{l} = 2\pi n \frac{\theta}{l} x^2.dx$$

$$\text{Moment of force about axis CD} = 2\pi n \frac{\theta}{l} x^2.dx \times x$$

$$\therefore \text{ Total twisting couple} = \int_0^r 2\pi n \frac{\theta}{l} x^3 dx$$

$$= 2\pi n \frac{\theta}{l} \left[\frac{x^4}{4}\right]_0^r$$

$$= 2\pi n \frac{\theta}{l} \cdot \frac{r^4}{4}$$

$$= \frac{\pi n\, \theta\, r^4}{2l}$$

Q. 16. What is meant by the coefficient of rigidity of a substance? Explain how it can be determined experimentally, deducing the formula used. (*P. U. 1935*)

Ans. For coefficient of rigidity see Q. 13.

Determination of Rigidity. A thin wire, whose coefficient of rigidity is required, is rigidly fixed to the middle of a heavy cylindrical rod, and is suspended vertically from a point. The rod is turned in a horizontal plane so as to twist the wire. When it is released, it executes torsional vibrations of definite time period about the axis of the wire.

It is proved in Q. 13 that when a wire (cylindrical) of length l, radius r, and coefficient of rigidity n, is twisted through θ radian, the restoring couple is equal to $\frac{n\theta\pi r^4}{2l}$. This *restoring* couple produces angular acceleration in the rod. Let the angular acceleration of the rod be α when the angle of twist is θ. All the particles of the rod have the same angular acceleration, but the linear acceleration of a particle is equal to the product of its distance from the axis of rotation and angular acceleration, and, therefore, is different for different particles.

Consider particles of masses $m_1, m_2, \ldots\ldots$, at distances $r_1, r_2, \ldots$ from the axis, and having linear accelerations $a_1, a_2, \ldots\ldots$

$$\text{Force acting on first particle} = m_1 a_1 = m_1 r_1 \alpha$$

$$\text{Moment of this force about the axis} = m_1 r_1 \alpha \times r_1 = m_1 r_1^2 \alpha$$

$$\text{Force acting on second particle} = m_2 a_2 = m_2 r_2 \alpha$$

$$\text{Moment of this force about the axis} = m_2 r_2^2 \alpha$$

....

$$\therefore \text{ Sum of moments for all particles} = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \cdots\cdots = \Sigma m r^2 \alpha = (\Sigma m r^2)\alpha = I\alpha$$

Here I is the moment of inertia of the rod about the axis of the wire.

$$I\alpha = \frac{n\theta\pi r^4}{2l} = c\theta, \quad \left[c = \frac{n\pi r^4}{2l}\right.$$

$$\therefore \quad \alpha = \frac{n\pi r^4}{2lI}\,\theta = \frac{c\theta}{I},$$

As the *restoring* angular acceleration α is *proportional* to the angular displacement θ, the motion of the rod is *simple harmonic.*

$$\therefore \text{ Time period } t = \frac{2\pi}{\sqrt{\frac{c}{I}}} = 2\pi\sqrt{\frac{I}{c}} = 2\pi\sqrt{\frac{2lI}{n\pi r^4}}$$

It is not easy to find I accurately. To overcome this difficulty, the rod is made hollow and is fitted with four cylinders,

two hollow (H, H) and two solid (S, S) of equal lengths. The experiment is performed first with the two solid cylinders in the inner position and hollow cylinders in the outer position (Fig. 14 a), and then with the hollow cylinders inner and the solid cylinders outer (Fig. 14 b).

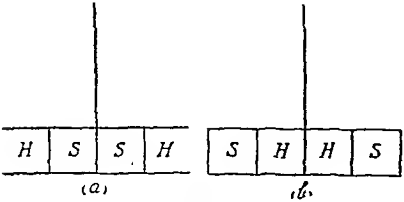


Fig. 14.

Let I_1 and I_2 be the moments of inertia in the first and second cases respectively, and t_1 and t_2 be the corresponding time periods. Then

$$t_1 = 2\pi\sqrt{\frac{I_1}{c}} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$t_2 = 2\pi\sqrt{\frac{I_2}{c}} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

On squaring and subtracting (1) from (2),

$$t_2^2 - t_1^2 = \frac{4\pi^2(I_2 - I_1)}{c} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

If each hollow cylinder be of mass m_1, each solid cylinder of mass m_2, length of the hollow tube $2a$, and, therefore, length of each cylinder $\frac{a}{2}$, then the distances of the centres of gravity of the inner and outer cylinders from the axis of oscillation are $\frac{a}{4}$ and $\frac{3a}{4}$ respectively. Each solid cylinder has mass $(m_2 - m_1)$ more than a hollow cylinder, and we may imagine that the change from the first adjustment to the second consists in transferring this excess of mass from each inner solid cylinder to each outer hollow cylinder. In this change the moment of inertia is increased, as on each side the centre of gravity of mass $(m_2 - m_1)$ is shifted from a distance $\frac{a}{4}$ to $\frac{3a}{4}$ from the axis. Therefore, according to the principle of parallel axes.

Increase of moment of inertia $= I_2 - I_1$

$$= 2(m_2 - m_1)\left\{\left(\frac{3a}{4}\right)^2 - \left(\frac{a}{4}\right)^2\right\}$$

$$= (m_2 - m_1)a^2$$

Putting this in (3),

$$t_2^2 - t_1^2 = \frac{4\pi^2(m_2 - m_1)a^2}{c}$$

$$= \frac{4\pi^2(m_2 - m_1)a^2 2l}{n\pi r^4}$$

$$\therefore\ n = \frac{8\pi l a^2(m_2 - m_1)}{\pi r^4(t_2^2 - t_1^2)}$$

Q. 17. Describe any experiment for finding the value of the gravitation constant G, and show how from this and other known quantities the mean density of the earth can be calculated. *(C. U. 1934)*

Ans. **Cavendish Method.** Two small spherical balls A and B (Fig. 15), each of mass m, are attached to a torsion rod which is suspended by a fine torsion wire. Two *equal* large spherical balls C and D, each of mass M, are suspended by another rod so that the centres of all the four balls are in the same horizontal plane. To each end of the torsion rod is attached a vernier which can move, without touching, over fine scales fixed to vertical stands. To avoid changes of temperarure, and the consequent air draughts, the apparatus is placed in a small closed chamber, and the observations are taken from outside with the help of telescopes, fixed in the walls of the chamber. For avoiding the action of the outside electric charges, the apparatus is enclosed in a gilded glass vessel.

Fig. 15.

The rod carrying the large balls is rotated, by an arrangement manipulated from outside, until the line joining their centres is perpendicular to the torsion rod, and the verniers are read. In this position there is no twist in the torsion wire. Then the big balls are brought near the small balls and on the *opposite* sides of the torsion rod in the position C, D. They are so adjusted that the lines joining the centres of the

near balls are *equal* in length and *perpendicular* to the torsion rod. Thus the forces exerted by the big balls on the near small balls are equal, parallel, and opposite, and the torsion rod is rotated under the action of the deflecting couple formed by these two forces. This produces twist in the wire and is opposed by the restoring torsional couple of the wire. Equilibrium is reached when the restoring couple becomes equal to the deflecting couple. Then the positions of the verniers are found out by the method of oscillations, as the torsion rod does not become stationary.

Then the big balls C and D are turned into the positions C′ and D′ respectively. The adjustment is made *exactly* the *same* as in the first experiment, keeping the *same* distance *d* between the centres of two near balls, but now the deflecting couple exerted on the torsion rod is in the opposite direction. The deflection of the torsion rod is again calculated from the position of the verniers, and the mean deflection is taken.

If G is the gravitation constant, the force of gravitation between two balls is equal to $\frac{GMm}{d^2}$.

$$\text{Length of torsion rod} = l$$

$$\therefore \quad \text{Deflecting couple} = \frac{GMml}{d^2}$$

$$\text{Angle of twist} = \theta \textit{ radian}$$

$$\text{Restoring couple} = c\theta$$

where c is the restoring couple per radian twist of the wire.

$$\therefore \quad \frac{GMml}{d^2} = c\theta$$

or

$$G = \frac{c\theta d^2}{Mml}.$$

To find the value of c, the torsion rod is set into torsional oscillations about the wire, and its time period t is measured. The moment of inertia I of the torsion rod and the small balls about the wire as axis is calculated. then

$$t = 2\pi\sqrt{\frac{I}{c}}$$

Corrections are applied for the force exerted by each large sphere on the distant small ball, the attraction between the torsion rod and the two large spheres, and the forces exerted by the rods carrying big balls.

Mean Density of the Earth. Let M be the mass of the earth, R its mean radius, and D its mean density. Then the force exerted by the earth on a mass m at its surface is equal to $\frac{GMm}{R^2}$. This force is also equal to mg, where g is the acceleration due to gravity at that place.

$$\therefore \quad \frac{GMm}{R^2} = mg, \text{ or } M = \frac{gR^2}{G}$$

But
$$M = \frac{4}{3}\pi R^3 D$$

$$\therefore \quad \tfrac{4}{3}\pi R^3 D = \frac{gR^2}{G}, \text{ or } D = \frac{3g}{4\pi RG}$$

Q. 18. What are the requisites of a balance? Obtain the general expression used for determining the conditions for these requisites, and show that the conditions for two of these are mutually contradictory.
(*Punjab, 1933*)

Ans. **Requisites of a Balance.** (1) Fig. 16 is a vertical section of a balance through its centre of gravity G. For stable equilibrium, the centre of gravity G should be vertically below the central knife-edge C when the beam is in its horizontal position.

(2) **Truth.** A balance is said to be true if its beam remains horizontal when equal masses are placed in, or removed from, the pans. For this, the clockwise moment must be equal to the anti-clockwise moment. Let P_1 and P_2 be the weights of the right and left pans respectively, l_1 and l_2 the lengths of the corresponding sides of the beam, and let equal weights W be placed in the pans. Then

$$(P_1+W)l_1 = (P_2+W)l_2 \quad \ldots \quad \ldots \quad (1)$$

When the weights are removed from the pans, the beam must again be horizontal.

$$\therefore \quad P_1 l_1 = P_2 l_2 \quad \ldots \quad \ldots \quad (2)$$

Subtracting (2) from (1),

$$Wl_1 = Wl_2$$

$$\therefore \quad l_1 = l_2$$

Putting this in (2), $\quad P_1 = P_2$

Thus a true balance should have pans of equal weight, and its arms should be of equal length.

(3) **Sensitiveness.** A true balance is sensitive if a very small difference between the loads in the pans produces a large deflection of the beam. The ratio of the deflection to the difference in loads is called its *sensitiveness.*

The end knife-edges A and B are higher than the central knife-edge C by h, and the depth of G below C is d. The mass of the beam and pointer is M, and acts vertically downward through G, while the mass of each pan and its load is m. When an additional and very small mass x is placed in the right pan, the beam and pointer are turned clockwise and make angle θ with their normal positions, and thereby the arms of the different forces producing moments about the central knife-edge are changed.

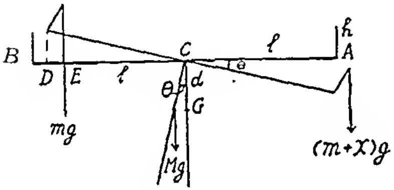


Fig. 16.

$$CD = l \cos \theta, \text{ and } DE = h \sin \theta.$$

Horizontal distance of A from C $= l \cos \theta + h \sin \theta$

" " " B " C $= l \cos \theta - h \sin \theta$

" " " G " C $= d \sin \theta$

The moments of the weight of the balance Mg and the left pan mg are anti-clockwise, while the moment of the weight of the right pan $(m+x)g$ is clockwise, and for equilibrium, the clockwise and anti-clockwise moments must be equal.

$$Mgd \sin \theta + mg(l \cos \theta - h \sin \theta) = (m+x)g(l \cos\theta + h \sin \theta)$$

The angle of deflection θ (radian) is very small, and, therefore, cos θ and sin θ are practically equal to 1 and θ respectively.

$$Md\theta + m(l - h\theta) = (m+x)(l + h\theta)$$

$$Md\theta + ml - mh\theta = ml + mh\theta + xl + h\theta x$$

As θ and x are both very small, their product is negligible.

$$\therefore \quad (Md-2mh)\theta = xl$$

or

$$\text{Sensitiveness } \frac{\theta}{x} = \frac{l}{Md-2mh}.$$

Thus a sensitive balance should have large arms l, small mass M, and small depth d of its centre of gravity below the central knife-edge. Its sensitiveness also depends on m and h; the greater m and h, the greater the sensitiveness. Conversely, when the outer knife-edges are below the central knife-edge, h is negative, and the sensitiveness decreases as the load is increased. When all the three knife-edges are *coplanar*, h is zero, and sensitiveness is *independent* of the load.

(4) **Quickness.** When the pans are equally loaded and the handle of the balance is raised, it begins to swing. For convenience in weighing, the time period of swing should be small. This requisite is also called *stability.* If all the knife-edges are coplanar, at every stage the moment of the right pan is equal and opposite to the moment of the left pan; the only restoring moment is due to the weight of the beam and pointer, and is equal to $Mgd \sin \theta$, or $Mgd\theta$ for very small θ. This accelerates the motion of the balance, and let the angular acceleration be α when the angular displacement is θ.

Let K be the radius of gyration of the beam and pointer about the central knife-edge, so that the moment of inertia about that axis is MK^2. To this should be added $2ml^2$, due to the two loaded pans, to get the total moment of inertia about the central knife-edge.

$$\text{Moment of inertia} = MK^2 + 2ml^2$$

$$\text{Restoring couple} = Mgd\theta$$

$$\therefore \text{ Angular acceleration } \alpha = \frac{Mgd\theta}{MK^2+2ml^2}$$

Hence, as the angular acceleration α is *proportional* to the angular displacement θ, the motion of the balance is *simple harmonic.*

$$\therefore \quad \text{Time period } t=\frac{2\pi}{\sqrt{\dfrac{Mgd}{(MK^2+2ml^2)}}}=2\pi\sqrt{\frac{MK^2+2ml^2}{Mgd}}$$

$$=2\pi\sqrt{\frac{K^2}{gd}+\frac{2ml^2}{Mgd}}$$

Thus, for small time of swing, K should be small, d large, l small, and M large. It also decreases as the load is increased. On the other hand, for sensitiveness, l should be large, and d and M both small. Therefore sensitiveness and quickness (or stability) require opposite conditions.

Q. 19. What is osmotic pressure? State the laws relating to it. Obtain the relation between the osmotic pressure and the vapour pressure of a solution.

(*Bombay, 1934*)

Ans. **Osmotic Pressure.** Certain membranes have the property of selective transmission : they allow some liquids to pass through them fully, but prevent the passage of others. The end of a thistle funnel is closed with such a membrane, and a solution whose solvent only can pass through it is put in it. The solvent is contained in a beaker, and the thistle funnel is placed in it in a vertical position. The solvent enters through the membrane ; the height of the solution in the funnel is raised, and the hydrostatic pressure due to the solution column increases.

After some time equilibrium is reached, and then there is no further rise in the level of the solution. In this condition the excess of hydrostatic pressure on the side of the solution just prevents any more of the solvent being added to the solution. This phenomenon is called *osmosis*, and the pressure which should be applied, at the beginning, on the solution side to prevent the entry of the solvent is called the **osmotic pressure** of the solution for that concentration.

Laws. (1) The osmotic pressure of a dilute solution is *proportional* to its concentration, referred to a definite volume of the *solvent*.

(2) It is proportional to its absolute temperature.

(3) It is equal to the gaseous pressure which would be exerted by the solute molecules were it possible for the solute

(non-electrolyte) to exist in the gaseous state at that temperature and occupy the volume of the *solvent*.

(4) Solutions of non-electrolytes, (in the *same* solvent, at the same temperature,) which have equal osmotic pressures, contain the same number of gram-molecules per unit volume.

(5) The solution of an electrolyte has a greater osmotic pressure, due to dissociation, than expected.

Vapour Pressure of Solutions. Two vessels, A and B, (Fig. 17) are separated by a membrane permeable to the solvent only. A contains the solution and B the solvent. The two vessels are placed under a bell-jar, and air is removed. Owing to osmosis, the level of the liquid in A rises, and after some time equilibrium is reached. Let the level of the solution in A be higher than the level of the solvent in B by a small height h. Let P and P′ be the vapour pressures of the solvent and the solution respectively, O the osmotic pressure of the solution, D the density of the solution in the final condition, and d the *average* density of the vapour under a pressure P. The whole space above the liquids is saturated with vapour, and the saturation pressure P′ of the vapour just above the solution is *smaller* than the saturation pressure P of the vapour just above the solvent by dgh.

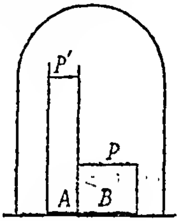


Fig. 17.

$$P - P' = dgh$$

$$O = Dgh$$

$$\therefore \quad \frac{P - P'}{O} = \frac{d}{D}$$

In this equation d is the average density of the vapour under a pressure P as the space is enclosed and exhausted. If d' be the density of the vapour pressure under the barometric pressure B, $\frac{d}{d'} = \frac{P}{B}$, or $d = \frac{d'P}{B}$.

$$\therefore \quad P - P' = \frac{Od}{D} = \frac{Od'P}{DB}.$$

Q. 20. What do you understand by the viscosity of a liquid? Define viscosity, and give an experimental

method for its determination. What is the effect of temperature on it ? (*Punjab, 1934*)

Ans. When the motion of a liquid over a horizontal solid surface is small and steady, its layer in contact with the solid surface is *stationary*, and the velocity of any other layer is *proportional* to its distance from the stationary layer. Consider any horizontal plane in the liquid. The liquid layer immediately above it is moving faster than the liquid layer immediately below it. The upper layer tends to accelerate the motion of the lower layer, while the lower layer tends to retard the motion of the upper layer. The two layers together tend to destroy their relative motion, as if there is a backward dragging tangential force.

An external force is required to overcome this backward drag and *maintain* relative velocity between two layers of a liquid, and when this force is withdrawn, relative motion ceases after some time. This property by virtue of which a liquid opposes *relative motion* between its different layers is called *viscosity*.

The backward force F acting on any liquid layer is proportional to its area A and velocity u and *inversely* proportional to its distance x from the stationary layer.

$$F = -\frac{nAu}{x} = -nA\frac{du}{dx},$$

where n depends on the nature of the liquid and is called its coefficient of viscosity, and $\frac{du}{dx}$ is the *velocity gradient* or the rate of change of velocity with distance. The negative sign shows that the force is acting backward opposite to the direction of velocity u. When A, and u, and x $\left(\text{or } \frac{du}{dx}\right)$ are each equal to one unit, n is equal to F. Therefore coefficient of viscosity of a liquid is equal to the tangential force per unit area required to maintain a unit velocity gradient (or a relative velocity of one unit between its two layers one unit distance apart).

Poiseuille's Method. A *long capillary* tube of *uniform circular* bore, of length l and radius r, is fixed *horizontally* near the bottom of a vessel [Fig. 18(a)]. The level of the liquid, whose coefficient of viscosity is to be found, is kept the *same* in the vessel so that a *constant* hydrostatic pressure P is applied to the end of the tube in the liquid. A weighed beaker is placed below the outer end of the tube, and the mass of the liquid that leaves the tube in a given time is found out. From this and the density of the liquid, its volume V flowing out in a *unit time* is calculated.

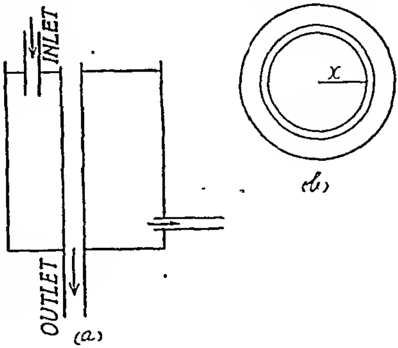


Fig. 18.

When the velocity of the liquid in the tube is small, its flow is steady and its particles travel in *straight* lines *parallel* to the axis of the tube. The layer of the liquid in contact with the walls of the tube is stationary ; velocity of flow increases as the axis is approached, and is the *same* at all the points at the same distance from the axis. As the liquid is incompressible, its amount passing across any section of the tube in a given time is the same.

Consider a cylindrical layer of the liquid, *coaxial* with the tube and of radius x. Its surface area (cylindrical) is equal to $2\pi xl$, and the velocity of flow at all the points on this cylindrical layer is the same. The liquid inside the cylinder is moving faster than the liquid outside it, and tangential force exerted by the outside liquid on the inside liquid *opposite* to the direction of flow is $\eta 2\pi rl \dfrac{du}{dx}$, where u is the velocity of flow at a distance x from the axis and $\dfrac{du}{dx}$ is the velocity gradient there. The difference between the pressure acting on the two ends of the tube is P, and the *forward* force due to it on the liquid *in* the imaginary cylinder is $P\pi x^2$. This tends to accelerate the motion of the liquid. As its motion is steady, the resultant of the two forces must be zero.

$$n 2\pi x l \frac{du}{dx} + P\pi x^2 = 0$$

$$n 2 l \frac{du}{dx} = -Px$$

$$\therefore \quad \int du = \int \frac{-Pxdx}{2nl}$$

$$u = \frac{-Px^2}{4nl} + c$$

where c is the constant of integration. When $x = r$, $u = 0$.

$$\therefore \quad 0 = \frac{-Pr^2}{4nl} + c$$

or
$$c = \frac{Pr^2}{4nl}$$

and
$$u = \frac{P(r^2 - x^2)}{4nl}$$

This gives the velocity of flow at a distance x from the *axis*. Consider a second coaxial layer of radius $x + dx$ (Fig. 18b). The velocity of flow of the liquid *between* the two cylindrical layers is u, and as $2\pi x.dx$ is the *cross-section* area between them, volume dV of the liquid flowing *per unit time* through this area is equal to $u \times 2\pi x dx$. The bore of the tube may be considered to consist of a very large number of such sections, and the volume of the liquid flowing through all of them in a unit time is obtained by integrating this expression.

$\therefore$ Volume V flowing per unit time $= \int_0^r u 2\pi x dx$

$$= \int_0^r \frac{P(r^2 - x^2) 2\pi x dx}{4nl}$$

$$= \frac{\pi P}{2nl} \int_0^r (r^2 x - x^3) dx$$

$$= \frac{\pi P}{2nl} \left[\frac{r^2 x^2}{2} - \frac{x^4}{4} \right]_0^r$$

$$=\frac{\pi P}{2nl}\cdot\frac{r^4}{4}$$

$$=\frac{P\pi r^4}{8nl}.$$

and $$n=\frac{\pi P r^4}{8Vl}.$$

As the temperature rises, the coefficient of velocity of a liquid decreases.

Q. 21. What is meant by the surface tension of a liquid? Explain this phenomenon, and find a relation between the surface tension of a liquid and its ascent in a capillary tube.

Ans. **Surface Tension.** The molecules of a liquid attract each other, but this force of cohesion becomes inappreciable if the distance between them exceeds a certain limit which is very small and is called the range of molecular attraction. Consider a sphere of this radius lying entirely in the liquid. A molecule at its centre is attracted equally in all directions; the resultant force on it is zero, and its motion in any direction is not opposed by any cohesive force. The condition of a molecule very near the free liquid surface is different. The lower half of its sphere of molecular attraction lies entirely in the liquid, but a part of the upper half is outside the liquid, and, therefore, contains a *smaller* number of liquid molecules than the lower half. The molecule at the centre experiences a *resultant force downward*, and *work has to be done* against this force if this molecule is to be carried to the free liquid surface. For molecules on the free surface, only the lower halves of their spheres of attraction are in the liquid, and so they experience the greatest cohesive force downward.

To increase the free surface of a liquid, more of the molecules have to be taken from its interior to its free surface. For this work has to be done, and their potential energy is *increased*. As a system tends to *decrease its potential energy*, this is opposed, and the surface tends to have the *least* surface area, so that it may have the minimum number of molecules on it. The free liquid surface behaves as if it were in *tension*.

If a straight line is imagined on the free surface of a liquid, the molecules lying just on its one side tend to *pull away* from the molecules lying just on the other side so as to decrease the surface area, but the rupture is prevented by the cohesive forces between them. The pulling away force exerted by one set of molecules on the other lies *in* the liquid surface and is *perpendicular* to the imaginary line, and its magnitude per *unit length* of the line is called the **surface tension** of the liquid.

Ascent of Liquid. A *capillary* tube of circular bore is dipped *vertically* in a liquid of surface tension T and density D. The liquid rises in it, and its meniscus is concave upward (Fig. 19). If R is the radius of the tube at this place, the liquid meniscus touches the tube along a circumference of length $2\pi R$. The angle of contact between the liquid and the tube is θ, and the force exerted by the liquid meniscus on the tube at A is T *per unit length* in the direction of the arrow. The tube exerts an *equal and opposite* force on the liquid meniscus. Its *vertical* component is $T \cos \theta$ *upward*, and the horizontal component is equal to $T \sin \theta$ outward. Considering the whole meniscus, the horizontal components cancel out, while its vertical components are added up. Therefore the total upward force exerted by the tube on the liquid meniscus is equal to $2\pi R \times T \cos \theta$, and this supports the weight of the liquid column.

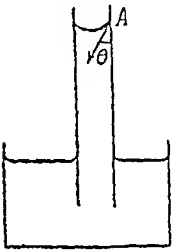


Fig. 19.

If h is the height of the liquid column up to the bottom of the meniscus, the volume of the cylindrical liquid column for this height is equal to $\pi R^2 \times h$. As the radius of the tube is very small, the liquid meniscus may be considered to be hemispherical of radius R, and the volume of the liquid meniscus is equal to the difference between a cylinder of radius R and length R and a hemisphere of radius R.

$$\text{Volume of the meniscus} = \pi R^2 \times R - \frac{2}{3}\pi R^3 = \frac{\pi R^3}{3}$$

$$\text{Total volume of the liquid column} = \pi R^2 h + \frac{\pi R^3}{3}$$

$$=\pi R^2\left(h+\frac{R}{3}\right)$$

$$\text{Weight of the liquid column}=\pi R^2\left(h+\frac{R}{3}\right)Dg$$

$$\therefore \quad 2\pi RT\cos\theta=\pi R^2\left(h+\frac{R}{3}\right)Dg$$

or
$$T=\frac{R\left(h+\frac{R}{3}\right)Dg}{2\cos\theta}.$$

Q. 22. How will you determine the angle of contact for mercury and glass?

Prove that the excess of pressure inside a soap bubble to the outside pressure $=\frac{4T}{R}$, where T stands for the surface tension of the soap solution and R for the radius of the sphere. (*Punjab, 1931*)

Ans. **Angle of Contact.** When a solid is placed in a liquid, the liquid in contact with the solid either rises above, or is depressed below, the rest of the free liquid surface. The angle which the tangent to the liquid surface, where it meets the solid, makes with the solid surface *in the liquid* is called the **angle of contact** of the two.

A small *spherical* glass flask of radius R and centre at C (Fig. 20) is about three-fourth filled with mercury. Its mouth is closed with a rubber cork, through which passes a glass rod, and is held inverted in a clamp. A sheet of printed paper is held against the flask, and its image is observed by light reflected at *grazing incidence* from the surface of mercury. By moving the rod, the level of mercury is adjusted so that the image of the print is *not distorted*. When this is the case, the surface of mercury AB is *plane* even where it touches the flask. The surface of mercury forms a circular sheet, whose diameter AB is then measured. Let the radius DA of this

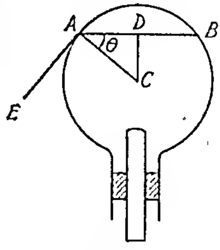


Fig. 20.

circle be equal to l.

$$\therefore \quad \cos\theta = \frac{DA}{CA} = \frac{l}{R}$$

or

$$\theta = \cos^{-1}\frac{l}{R}$$

If AE is the tangent at A, $\angle$ BAE is the angle of contact and is equal to $(90°+\theta)$

Excess of Pressure. A soap bubble tends to contract, and, thererore, the pressure inside must be greater than the external pressure. Let a soap bubble of radius R, and surface tension T, be considered to be divided into two *hemispheres* by a plane so that they meet each other at *right angles* to their common plane face ABDE, whose area is equal to πR^2 (Fig. 21). Let P and P' be the external and internal pressure respectively. Consider the equilibrium of the upper hemisphere. The thrust on it due to the internal pressure is equal to $P' \times \pi R^2$ and acts *upward* at right angles to the face ABDE. Similarly, the resultant thrust on it due to the external pressure is equal to $P \times \pi R^2$ and acts *downward* perpendicular to the face ABDE. The force due to surface tension on it is T per unit length. It acts *downward* perpendicular to the plane face and round its edge ABDE, and as the bubble has *two surfaces*, its magnitude is equal to $2 \times 2\pi R \times T$. As the upper hemisphere is in equilibrium, these three forces balance each other.

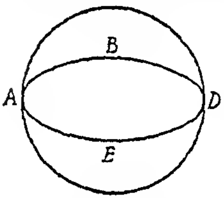


Fig. 21

$$\therefore \quad P'\pi R^2 = P\pi R^2 + 4\pi RT$$

$$(P'-P)R = 4T$$

$$\text{Excess of pressure} = P' - P$$

$$= \frac{4T}{R}$$

Q. 23. Why is the upper surface of mercury in a glass capillary tube convex upward, while for water it is concave?

Assuming the surface tension of rain water to be 72 dynes per cm., find the difference of pressure inside

and outside a rain-drop of diameter ˙02 cm. What would the difference of pressure amount to if the drop were to be decreased by evaporation to a diameter of ˙00002 cm. ? (*Punjab 1937*)

Ans. Let a glass capillary tube dip *vertically* in a liquid whose horizontal surface meets it at A (Fig. 22 *a*). A liquid molecule at A in contact with the solid experiences adhesive force due to the near molecules of the solid and cohesive force due to the near liquid molecules. The resultant force of adhesion acts perpendicular to the tube at A and is represented by the *horizontal* line AB, while the resultant force of cohesion is inclined at 45° to the vertical and is represented by AC.

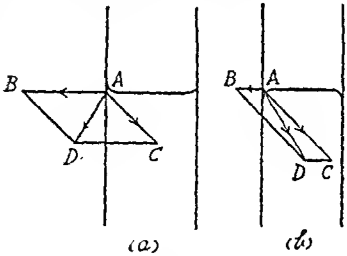


Fig. 22.

These two forces are inclined to each other at 135°. Their resultant is represented by the diagonal AD of the parallelogram ABDC, whose direction depends on the relative magnitude of the two component forces. It makes a smaller angle with the greater component than with the smaller component. If $\frac{AB}{AC}=\frac{1}{\sqrt{2}}$ ($=\cos 45°$), AD is along the vertical. Therefore AD lies in the liquid [Fig. 22 (*b*)] or out of it [Fig. 22 (*a*)] according as AC is greater or smaller than AB $\sqrt{2}$. Similarly, all other liquid molecules in contact with the solid are under the action of such forces. As a liquid cannot permanently withstand a shearing force, its surface, in equilibrium, is everywhere at right angles to the *resultant* force there.

In the first case, where AD lies outside the liquid, the liquid molecules near the tube are *raised* against the tube, and the surface is *concave upward*. This is the case with water in glass where the cohesive force AC is smaller than $\sqrt{2}$ times the adhesive force AB. In the case of mercury in glass, the cohesive force AC is greater than $\sqrt{2}$ times the adhesive force AB [Fig. 22 (*b*)]. AD lies in the liquid; the liquid molecules near the tube are *depressed*, and the surface is *convex* upward.

Problem. It is shown in Q. 22 that in the case of a soap bubble, of radius R and surface tension T, the pressure inside it is greater than the external pressure by $\frac{4T}{R}$. In the case of a liquid drop, unlike a soap bubble, there is only *one* liquid surface, and arguing as before, the excess of internal pressure is $\frac{2T}{R}$.

Surface tension of rain water $=72$ dynes/cm.

(*i*) Radius of the rain drop $= \cdot 02$ cm.

$\therefore$ Excess of pressure inside $= \frac{2 \times 72}{\cdot 02}$

$= 7200$ dynes/sq. cm.

(*ii*) Radius of the rain-drop $= \cdot 00002$ cm.

$\therefore$ Excess of pressure inside $= \frac{2 \times 72}{\cdot 00002}$

$= 72 \times 10^5$ dynes/sq. cm.

Q. 24. Find the vapour pressure over a curved liquid surface, and explain the use of dust particles in condensing a vapour.

Ans. A capillary tube dips vertically in a liquid of surface tension T and density D [Fig. 23(*a*)] The vessel is enclosed and air is removed. When equilibrium is reached, the space above the liquid becomes saturated with vapour. Let the liquid rise in the capillary tube to a height h above the outside level, R be the radius of its *concave* meniscus, P and P′ be the saturation vapour pressures just over the flat and concave liquid surfaces respectively, and d be the *average* density of the vapour (under pressure P). Evidently the difference between P and P′ is equal to the pressure of a vapour column of height h.

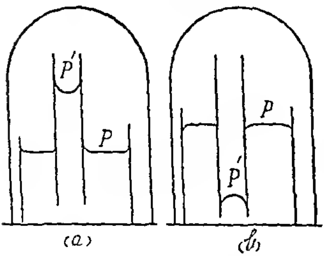


Fig. 23

$$P - P' = hdg$$

or $$hg = \frac{P - P'}{d} \qquad (1)$$

The pressure on the liquid side of the concave meniscus is *smaller* than that on the vapour side by $\frac{2T}{R}$, or is equal to $P' - \frac{2T}{R}$. The pressure of the liquid in the capillary tube is equal to hDg, and as the pressure at a point in the capillary tube in level with the flat liquid surface is P, the pressure at a height h above it and just below the liquid meniscus is equal to $P - hDg$.

$$\therefore \quad P' - \frac{2T}{R} = P - hDg$$

$$= P - D\frac{(P-P')}{d} \qquad \text{from (1)}$$

or
$$(P-P')\left(\frac{D}{d} - 1\right) = \frac{2T}{R}$$

or
$$P - P' = \frac{2Td}{R(D-d)}$$

As D is greater than d, $(D-d)$ is positive, and, therefore, saturation vapour pressure P′ over a concave liquid surface is smaller than that (P) over a flat surface of the same liquid, at the same temperature, by $\frac{2Td}{R(D-d)}$.

If the capillary tube is smeared with some oil so that the liquid does not wet it, instead of ascending in the tube it is depressed in it, and its meniscus is *convex* upward [Fig. 23(b)]. Proceeding as above, it may be shown that the saturation vapour pressure over the convex meniscus is *greater* than that over the flat liquid surface by $\frac{2Td}{R(D-d)}$. The *excess* of saturation vapour pressure is proportional to T and d, and is *inversely* proportional to $(D-d)$ and the radius of curvature R. The smaller the radius, the greater is the saturation vapour pressure, that is, *the saturation vapour pressure over a small drop of a liquid is greater than that over a large drop of the same liquid.*

If a very small drop of water is placed in water vapour whose vapour pressure is just saturated for *flat water* surface, it will *not* be saturated for the drop. The drop will, therefore, evaporate to increase the vapour pressure to its own saturation

value. As the radius of the drop decreases, its saturation vapour pressure increases, and it evaporates more and more rapidly. Hence it is not possible for the saturated water vapour to condense into drops, for as soon the formation of tiny drops starts, they begin to evaporate. The vapour may become supersaturated and still its condensation may not take place.

On the other hand, if dust particles are present in the saturated vapour, they serve as *nuclei* of appreciable thickness, and condensation starts *on them*. The radius of curvature of a drop, even at the very beginning, is *not very small*, and, therefore, its tendency to evaporate is very little. As its radius increases, the saturation vapour pressure for it becomes smaller and smaller, and its tendency to evaporate becomes negligible. Thus the dust particles help in condensing saturated vapour.

PART II

HEAT

Q. 25. Describe and explain Fizeau's interference method for finding the co-efficient of cubical expansion of a crystal.

Ans. This method is used for crystals and other substances which can be obtained in *small fragments* only. A metal disc AB (Fig. 24) is supported by 3 very fine *screws* which pass through it. The crystal is cut with its two opposite faces parallel to one another and perpendicular to the axis along which the coefficient of linear expansion is to be measured. These faces are polished; the upper face is made optically plane, and then piece C is placed on the plate AB. A glass plate DE, whose lower face is *optically plane*, is placed on the screws nearly parallel to the upper face of C and very near it.

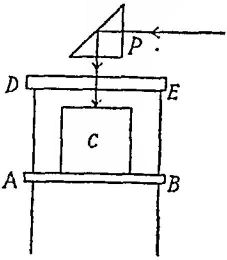


Fig. 24.

Monochromatic light is made to fall perpendicular to the glass plate with the help of a totally reflecting prism P. A part of the beam of light is reflected from the *lower* face of DE; the rest falls on the *upper* face of C, and then suffers successive reflections at the two surfaces bounding the thin air film. At each downward reflection from the lower face of DE, a part emerges upward, and the emerging rays produce parallel straight interference fringes, which are observed with a telescope fitted with cross-wires. If the crystal is transparent, its lower face is blackened so that no light is reflected from it.

The whole apparatus is placed in a double walled vessel which can be kept at any temperature. On heating the arrangement, the thickness of the air film increases or decreases according as the expansion of the parts of the screws *above* AB is greater or smaller than the expansion of the crystal. The change in the thickness of the air film is equal to the difference

between the two expansions. As the thickness of the air film changes, the interference fringes are displaced, and a displacement one of bright to bright fringe corresponds to a change of thickness of the air film by $\frac{\lambda}{2}$, where λ is the wave-length of light used. To find the expansion of the screws, the experiment is repeated without the crystal.

Let l be the thickness of the crystal and α the co-efficient of linear expansion in this direction, l' the length of the parts of the screws *above* the plate AB and α' their coefficient of linear expansion, $t°$ the rise of temperature, and n the number of fringes shifted,

Expansion of the crystal $= l\alpha t$

Expansion of upper parts of screws $= l'\alpha' t$

Change in the thickness of air film $= \frac{n\lambda}{2}$

$$\therefore \quad l\alpha t - l'\alpha' t = \frac{n\lambda}{2}$$

or

$$\alpha = \frac{\frac{n\lambda}{2} + l'\alpha' t}{lt} = \frac{n\lambda}{2lt} + \frac{l'\alpha'}{l}$$

The coefficient of linear expansion of a crystal is generally different in different directions. By repeating the above experiment, its values α_1, α_2, α_3 are determined for three mutually perpendicular directions. Consider a cube of each side l. On heating through $t°$, its three sides become $l(1+\alpha_1 t)$, $l(1+\alpha_2 t)$, and $l(1+\alpha_3 t)$.

$$\text{Initial volume} = l^3$$

$$\begin{aligned}\text{Final volume} &= l(1+\alpha_1 t) \times l(1+\alpha_2 t) \times l(1+\alpha_3 t)\\ &= l^3(1+\alpha_1 t+\alpha_2 t+\alpha_3 t+\cdots\cdots\cdots)\end{aligned}$$

As α_1, α_2, and α_3 are *very small*, terms involving their products are *negligible*.

$$\text{Increase of volume} = l^3(\alpha_1+\alpha_2+\alpha_3)t$$

$$\therefore \text{ Coefficient of cubical expansion} = \frac{l^3(\alpha_1+\alpha_2+\alpha_3)t}{l^3 \times t}$$

$$= \alpha_1+\alpha_2+\alpha_3.$$

Q. 26. Show that the coefficient of cubical expansion is approximately equal to three times the coefficient of linear expansion.

Describe a method of determining the absolute value of the coefficient of cubical expansion of mercury. *(Bombay, 1931)*

Ans. Consider a cube of each side l, coefficient of linear expansion α, and coefficient of cubical expansion γ. On heating it by $t°$, its each side becomes $l(1+\alpha t)$.

$$\text{Initial volume} = l^3$$
$$\text{Final volume} = \{l(1+\alpha t)\}^3$$
$$= l^3(1+3\alpha t+\cdots\cdots)$$

As α is *very small*, terms involving its higher powers are *negligible*.

$$\text{Increase in volume} = l^3 \times 3\alpha t$$

$$\therefore \text{ Coefficient of cubical expansion } \gamma = \frac{l^3 \times 3\alpha t}{l^3 \times t} = 3\alpha.$$

Regnault's Method. Two glass tubes, ABCD and EFGH, are joined by a flexible metal tube DE (Fig. 25), and almost completely filled with mercury. Arm CD is surrounded by a hot oil both, while ice-cold water is passed through the jackets surrounding the arms EF, GH, and BA. Axes of arms BC and GF are throughout kept in the *same horizontal* line. The oil bath is kept well stirred, and its temperature t°C is measured with an air thermometer.

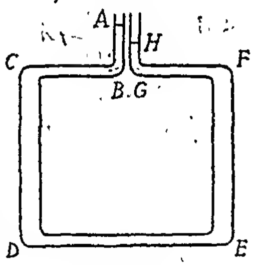


Fig. 25.

At the start, when arm CD is also at 0°C, arms EF and CD are both of the same height h, but, owing to rise of temperature, the length of CD becomes $h(1+\alpha t)$, where α is its coefficient of linear expansion, and D is depressed below E by $h\alpha t$. As the density of mercury in CD decreases from d at 0°C to d' at t°C, h_1 the height of mercury in AB, above the axis of CB, is greater than h_2 the height of mercury in GH, above the axis of GF, because considering the pressure at D, for equilibrium, the pressure of the column ABCD is equal

to the pressure of the column HGFED. E is slightly higher than D, and pressure due to this column DE is so small that no appreciable error is introduced if its temperature is taken to be 0°C.

Pressure of the left column at $D = h_1 dg + h(1+\alpha t)d'g$

" " " right " " " $= h_2 dg + hdg + h\alpha t dg$

$= \{h_2 + h(1+\alpha t)\} dg$

$\therefore \quad h_1 dg + h(1+\alpha t)d'g = \{h_2 + h(1+\alpha t)\} dg$

or $\qquad (h_1 - h_2)d = h(1+\alpha t)(d-d')$

If a mass M of mercury has volume V at 0°C and V′ at t°C, and its coefficient af absolute expansion is γ, then $M = Vd = V'd' = V(1+\gamma t)d'$, or $d' = \dfrac{d}{1+\gamma t} = d(1-\gamma t)$, as γt is very small as compared with 1. Putting this value of d' in the above equation, we get

$$(h_1 - h_2)d = h(1+\alpha t)\,(d - d + d\gamma t)$$
$$= h(1+\alpha t)\, d\gamma t$$
$$\therefore \quad \gamma = \frac{h_1 - h_2}{h(1+\alpha t)t}.$$

Q. 27. In a mercury thermometer, as well as in a constant pressure air thermometer, temperature is measured by the change in the volume of the indicating substance. Explain why for all standard measurements a gas thermometer is always preferred. Describe Callendar's form of constant pressure air thermometer. *(Calcutta, 1931)*

Ans. With change in temperature, the volume of the indicating substance changes, and this affords a measure of the change of temperature. For the following considerations a gas thermometer is preferred to a mercury thermometer :—

1. The coefficient of expansion of a gas is very great as compared with that of mercury, therefore a gas thermometer is *very sensitive.*

2. The expansion of the containing vessel is not exactly regular. As the expansion of the gas is very great, the expansion of the containing vessel need only be known approximately.

3. The expansion of mercury is *not regular*, but a gas expands at a *uniform rate*, that is, equal changes in temperature produce equal changes in its volume.

4. The range of temperature for a mercury thermometer is very much restricted, but for a gas thermometer it is *very wide*. It can be employed for both very high and very low temperatures.

5. The thermodynamic scale of temperature is the ultimate standard, and a perfect gas scale is identical with it. In all actual cases there is some deviation, but the correction can be calculated from the behaviour of the gas.

Callendar's Thermometer. In the ordinary constant pressure air thermometer, the connecting tube and other parts are not at the same temperature as the bulb. This difficulty is overcome in Callendar's compensated thermometer. Bulb A (Fig. 26) is connected with a calibrated mercury reservoir D, which is almost filled with mercury, and with an *exactly similar* bulb E through a sulphuric acid gauge G. A compensating tube C runs side by side with the connecting tube B and is *exactly similar* to it. The mass of dry air on the two sides of the gauge is exactly the same.

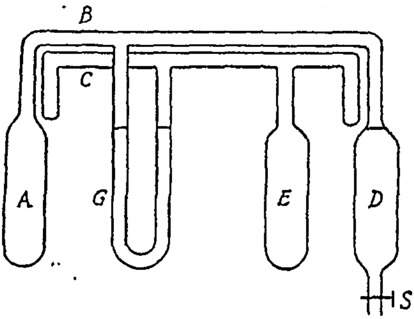


Fig. 26.

The instrument is placed in pure ice, and the level of mercury in D is adjusted for equality of pressure. Then A is placed in the substance whose temperature T° absolute is to be measured, but the rest of the instrument remains in ice at T_0° absolute. Air in A is heated and its pressure rises. Some mercury is withdrawn from D to allow this air to expand and come back to its original pressure.

Let V be the volume of A and V′ the volume of mercury withdrawn from D, both being measured at T_0°. Therefore, when the temperature of A is raised to T°, the mass of air

filling it *then* had a volume $V - V'$ at $T_0°$, that is, a mass of air occupies volume $V - V'$ at $T_0°$ absolute, and has volume V at T° absolute and at the *same pressure*. Any changes in the temprature of B and C are exactly equal, and their effects counterbalance each other.

$$\therefore \frac{V}{T} = \frac{V - V'}{T_0}$$

or

$$T = \frac{V\,T_0}{V - V'}$$

With rise of temperature, the volume of A increases, and a correction should be applied for this change.

Q. 28. Obtain a relation between the pressure of a gas and the speed of its molecules, and show that the absolute temperature of a gas is proportional to the square of the speed.

If the density of nitrogen at N. T. P. is 0·00125 gram per c.c., what is the speed of its molecules?

(*Bombay, 1935*)

Ans. A gas consists of very sparsely distributed *similar* molecules moving haphazardly with very great speeds in straight lines. Even a very small volume contains an enormous number of molecules, and their size is *negligible* as compared with the vacant space. They collide with each other very frequently, but, as it is assumed that there is no cohesive force between them, their collisions are instantaneous, and the whole of the time is spent in free flights. They are *perfectly elastic* spheres and, therefore, when a collision occurs, there is no loss of momentum or kinetic energy: equal and opposite changes of velocity take place, and each molecule continues the journey of the other as if no collision had taken place.

Let a hollow cube, of each side l, contain n molecules, each of mass m. Different molecules are moving with *different velocities* in different directions, but the velocity of any molecule may be considered to be resolved into three rectangular components, each being *perpendicular* to a pair of opposite faces. When a molecule strikes any inner face of the cube, the component of its velocity perpendicular to *that* face only is *reversed*, the other two components remain unaffected.

Consider a molecule moving with velocity c_1, whose rectangular components are x_1, y_1, and z_1, so that $c_1^2 = x_1^2 + y_1^2 + z_1^2$. When it strikes a face perpendicular to the component x_1, its velocity x_1 and momentum mx_1 are *reversed*, that is, its momentum is changed by $2mx_1$ inward, and that wall experiences an equal and opposite (*outward*) change of momentum. Then this molecule may be considered to fly to the opposite face; it strikes it after time $\frac{l}{x_1}$, imparts momentum $2mx_1$ to that face, and comes back to the *first* face after time $\frac{2l}{x_1}$. Thus it strikes each of these two faces $\frac{x_1}{2l}$ times in a unit time, and, therefore, the *rate* of change of momentum of either of these two faces, due to this molecule, is $2mx_1 \times \frac{x_1}{2l} = \frac{mx_1^2}{l}$, or for the two faces together its value is $\frac{2mx_1^2}{l}$.

Similarly, considering the other two components, the rate of change of momentum of the corresponding pairs of opposite faces is $\frac{2my_1^2}{l}$ and $\frac{2mz_1^2}{l}$. Treating the impacts of all the molecules, the rate of change of momentum of all the six faces, or the force (thrust) F on them, is given by

$$F = \frac{2m}{l}(x_1^2 + x_2^2 + \ldots\ldots + x_n^2) + \frac{2m}{l}(y_1^2 + y_2^2 + \ldots\ldots y_n^2)$$
$$+ \frac{2m}{l}(z_1^2 + z_2^2 + \ldots\ldots + z_n^2)$$
$$= \frac{2m}{l}(x_1^2 + y_1^2 + z_1^2 + x_2^2 + y_2^2 + z_2^2 + \ldots\ldots + x_n^2 + y_n^2 + z_n^2)$$
$$= \frac{2m}{l}(c_1^2 + c_2^2 + \ldots\ldots + c_n^2) = \frac{2mnc^2}{l}.$$

Here c^2 is equal to $\frac{c_1^2 + c_2^2 + \ldots\ldots + c_n^2}{n}$, and is called *mean square velocity*.

Area of six faces $= 6l^2$.

$$\therefore \text{ Pressure } P = \frac{\text{Thrust}}{\text{area}} = \frac{2mnc^2}{l \times 6l^2} = \frac{mnc^2}{3V}, \ldots\ldots\ldots\ldots (1)$$

where V $(=l^3)$ is the volume of the gas.

or
$$PV = \frac{mnc^2}{3}$$

But for a *perfect* gas having volume V and pressure P at T° *absolute*

$$PV = RT$$

where R is a constant,

$$\therefore \frac{mnc^2}{3} = RT$$

or
$$T = \frac{mnc^2}{3R}$$

Thus, as mn is the mass of the gas and is constant, its absolute temperature T is proportional to the *mean square* velocity c^2 of its molecules.

Problem. In equation (1), $\frac{mn}{V}$ is the density of the gas

$$\text{Normal pressure} = 76 \text{ cms. of mercury}$$
$$= 76 \times 13{\cdot}59 \times 981 \text{ dynes./sq. cm.,}$$

where 13·59 gms./c.c. is the density of mercury, and 981 cms./sec. sec. is the acceleration due to gravity.

$$c^2 = \frac{3PV}{mn} = \frac{3 \times 76 \times 13{\cdot}59 \times 981}{{\cdot}00125}$$

$$\therefore \text{ c at } 0°C = \sqrt{\frac{3 \times 76 \times 13{\cdot}59 \times 981}{{\cdot}00125}}$$
$$= 49290 \text{ cms./sec.}$$

Q. 29. Explain the failure of Boyle's law to account for the variation of volume with pressure in the case of the common gases.

Derive Vander Waal's equation, and explain the significance of the correction terms. (*Punjab, 1938*),

Ans. According to Boyle's law the volume V of a given mass M of a gas, at constant temperature, varies inversely as its pressure P, or PV is constant. In Q. 28, it is proved that

$PV = \frac{Mc^2}{3}$, where c^2 is the mean square velocity of its mole-

cules and changes with temperature only. In deriving this relation it is assumed that the gas molecules do not exert cohesive forces and that their volume is negligible as compared with the vacant space in which they roam about. In the case of common gases, even at ordinary pressure, these assumptions are not perfectly valid, but at high pressures they become more and more inadmissible, and volume does not vary with pressure in accordance with Boyle's law.

Force of Cohesion. Within a gas a molecule is attracted equally in all directions, and the cohesive forces produce no change in its speed. When it is very near the surface boundary, it is attracted *inward* only, and, therefore, in coming to the surface its speed is *decreased*. It strikes the walls of the containing vessel with this *decreased* speed, and hence the pressure *actually exerted* by the gas is *smaller* than that calculated for a perfect gas, where the force of cohesion is neglected.

This decrease in pressure is proportional to (1) the number of attracting molecules per *unit volume*, and (2) the number of attracted molecules striking *unit area* of the walls of the containing vessel in a *unit time*. But both these factors are proportional to the density of the gas. Therefore the decrease in pressure is proportional to the *square* of the density of the gas, or inversely proportional to the square of its volume, or equal to $\frac{a}{V^2}$, where a is a constant. Thus if the volume is made half, the number of attracting molecules is doubled, and the number of molecules hitting unit area of the walls in a unit time is also doubled; therefore the difference between the ideal and observed pressure is quadrupled.

$\therefore$ Pressure exerted by a *perfect* gas $=$ P(observed pressure) $+ \frac{a}{V^2}$.

Size of Molecules. The gas molecules have finite size, and this correction becomes more prominent when the gas is compressed. The *actual* free space in which the molecules move about is *smaller* than the internal volume of the containing vessel. When a gas is compressed, the total volume is decreased in a certain ratio, but, as the molecules are *incompressible*, the *free* space is decreased in a *greater* ratio. Similarly, when the total volume is increased in any ratio, the

free space is increased in a greater ratio. Therefore the *volume which changes with pressure* is $(V-b)$, where b is found to be four times the total volume of the molecules. That b should be greater than the total volume of the molecules is clear from the consideration that when all the molecules are moving about, they obstruct each other's motion more than if some are at rest and some in motion.

Making these corrections in the ideal gas equation $PV=RT$, **Van der Waals** obtained the equation

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT.$$

This equation is more in accordance with the experimental results than the ideal gas equation, but still it is an approximation only.

Q. 30. Derive the reduced equation of a gas, starting from the Van der Waals' equation of state; and show that if two gases have the same reduced pressure and volume, they also have the same reduced temperature. (*Punjab, 1934*)

Ans. **Reduced Equation.** Van der Waals' relation for a gas occupying volume V and exerting pressure P at T° absolute is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT, \qquad \text{...(1)}$$

where a, b, and R are constants.

On expanding, this equation may be put in the form

$$PV+\frac{a}{V}-Pb-\frac{ab}{V^2}=RT$$

or $$PV^3+aV-PbV^2-ab=RTV^2$$

or $$V^3-\left(b+\frac{RT}{P}\right)V^2+\frac{a}{P}V-\frac{ab}{P}=0 \qquad \text{...(2)}$$

This is a cubic equation in V, and has three roots for given values of P and T. At the *critical point* all the three roots are *equal*. Let the value of critical volume be x, then

$$V-x=0$$

or $$(V-x)^3=0$$

or $$V^3-3xV^2+3x^2V-x^3=0 \qquad \text{...(3)}$$

In equations (2) and (3), as *the coefficients of V^3 are equal, the coefficients of the other equal powers of V must also be equal.*

$$\therefore\ 3x = b + \frac{RT}{P} \qquad \ldots(4)$$

$$3x^2 = \frac{a}{P} \qquad \ldots(5)$$

$$x^3 = \frac{ab}{P} \qquad \ldots(6)$$

Dividing (6) by the corresponding sides of (5),

$$\frac{x^3}{3x^2} = \frac{ab}{P} \cdot \frac{P}{a}$$

$\therefore$ Critical volume $x = 3b$

From equation (5),

$$\text{Critical pressure } P = \frac{a}{3x^2} = \frac{a}{27b^2}$$

From equation (4),

$$\frac{RT}{P} = 3x - b = 8b$$

$$\therefore\ \text{Critical temperature } T = \frac{P8b}{R} = \frac{a}{27b^2} \cdot \frac{8b}{R}$$

$$= \frac{8a}{27Rb}$$

Two gases are said to be at *corresponding temperatures* if their temperatures bears the *same ratio* to their respective critical temperatures Similar is the meaning of corresponding pressures, or corresponding volumes.

Putting P, V, and T in equation (1) as fractions of the corresponding critical constants P_c, V_c, and T_c of the gas, and equal to αP_c, βV_c, and γT_c respectively,

$$\left(\alpha P_c + \frac{a}{\beta^2 V_c^2}\right)(\beta V_c - b) = R\gamma T_c$$

Then expressing P_c, V_c, and T_c in terms of a, b, and R, as found above,

$$\left(\alpha \frac{a}{27b^2} + \frac{a}{\beta^2 9b^2}\right)(\beta.3b - b) = R\gamma \frac{8a}{27bR}$$

or

$$\left(\alpha \frac{a}{27b} + \frac{a}{9\beta^2 b}\right)(3\beta - 1) = \frac{8\gamma a}{27b}$$

Dividing both sides by $\frac{a}{27b}$, we get

$$\left(\alpha+\frac{3}{\beta^2}\right)(3\beta-1)=8\gamma \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

This equation is called the **reduced equation of state.** It is *independent* of anything peculiar to a gas, and, therefore, *applies to all gases.* When two gases have the same values of any two of α, β, and γ, they must have the same value of the third. Hence if they have same reduced pressure and volume (same α and β), they also have the same reduced temperature (same γ).

Q. 31. Give an account of Andrew's experiments on carbon dioxide, showing by a sketch the general nature of the isothermals, above and below the critical temperature. (*Calcutta, 1934*)

Ans. **Andrew's Experiments.** A calibrated tube ABC, [Fig. 27(a)] whose upper part AB was very narrow, was filled with carbon dioxide by passing the dry gas through it for about 24 hours, and its both ends were then sealed. Its lower end was dipped in mercury and then broken, and by alternately heating and cooling its lower part was partially filled with mercury. A second tube was similarly filled with air, whose variation of volume with pressure was already accurately known.

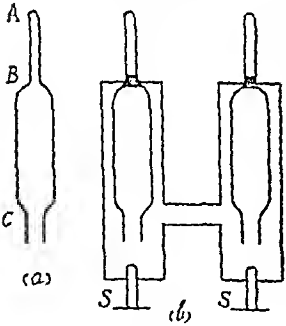


Fig. 27.

The volume, pressure and temperature of the two gases were measured, and then the two tubes were fitted in two metal cylinders [Fig. 27 (b)], which were connected with each other by a horizontal tube and filled with water. The cylinder's joints were made water-tight. The projecting parts of both the tubes were surrounded by water jackets. The temperature of the air tube was rupt the *same* throughout, but carbon dioxide was experimented on at different temperatures. By screwing the plungers S, S, the *same* pressure was applied to the two tubes, and was calculated from the observed compression of air. Readings were taken

after the mercury pellets had reached the projecting parts of the tubes, and were continued until the smallest volume which could be accurately measured was reached. The experiment was repeated with carbon dioxide at different temperatures.

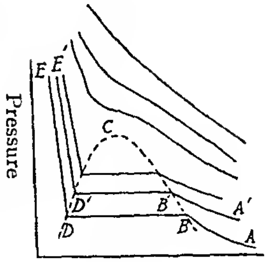


Fig. 28.

Fig. 28 shows the variation of the volume of a given mass of carbon dioxide with pressure at different temperatures. Each curve shows the variation at a constant temperature and is called an isothermal; the lower three curves are for temperatures below the critical temperature, while the upper three isothermals are for higher temperatures.

In the lowermost isothermal, the vapour is unsaturated from A to B; at B it is saturated, liquefaction commences and is indicated by a *sharp* change. This occurs under the saturated vapour pressure for this temperature. BD is *parallel* to the volume axis, and here there is a mixture of saturated vapour and liquid. Liquefaction becomes *complete* at D; further increase in pressure compresses the liquid very slightly, and, therefore, DE is almost parallel to the axis of pressure.

The higher isothermals are similar to the first, but here liquefaction commences at a *higher pressure and smaller volume* and is complete at a *greater volume* than at the lower temperature. This shows that with rise of temperature vapour (saturated) density increases and liquid density decreases. These two densities approach each other and become *equal* at the *critical temperature.* The dotted curve joining the extremities of the horizontal parts of these isothermals is called border curve, and the critical point C lies at its apex.

The fourth isothermal, for a temperature slightly higher than the critical temperature, shows *no discontinuity*, and there is a *continuous decrease* in volume. The whole mass remains *homogeneous throughout*, and the only change is a bend, which indicates a large decrease in V with a slight increase in P. The gas cannot be liquefied above its critical temperature.

This bend decreases with rise of temperatures, and higher isothermals have *no* such bend. They become more and more

similar to the isothermals of the permanent gases at ordinary temperatures.

Thus the border curve closely indicates the different states of a substance ; to its right the substance is unsaturated vapour, within it there is a mixture of saturated vapour and liquid, to the left it is entirely in the liquid state, and about C it is in the gaseous condition.

Q. 32. Explain why the specific heat of a gas at constant pressure is greater than its specific heat at constant volume.

Describe the method of determining the specific heat of a gas at constant volume by the calorimeter.

(*Punjab, 1929*)

Ans. The specific heat of a substance is equal to the amount of heat required to raise the temperature of a unit mass of the substance through one degree. When a gas is heated, its pressure as well as its volume may increase, but, for simplicity, we consider two cases : (1) when its volume is kept constant, and (2) when its pressure is kept the same throughout. In the first case heat supplied is used in increasing the speed and the kinetic energy of the molecules. As a result of this, the temperature and pressure of the gas are increased, but as there is no change of volume, no work is done by the gas.

When the pressure is kept constant, the gas has to be allowed to expand. During the expansion *work is done by* the gas in pushing back the atmosphere, and an equivalent amount of heat energy is *absorbed* for this work done. In addition to this the amount of heat required to raise the temperature of a given mass of the gas through a given range of temperature is the *same* as in the first case, when volume is kept constant. Therefore the specific heat of a gas at constant pressure is *greater* than its specific heat at constant volume.

Joly's Steam Calorimeter. Two hollow copper spheres of the *same* size and mass, that is, of *equal thermal capacities*, are suspended by fine platinum wires from the two pans of a balance, and are enclosed in a chamber in which *dry* steam

can be passed. Each sphere carries a small tray at its bottom in which condensed steam is collected. The two spheres are exhausted and counterpoised, and *dry* steam is allowed to enter the chamber. Steam condenses on both of them, and if, when *steady* condition is reached, the balance remains undisturbed, it shows that they have equal thermal capacities. The holes where the suspension wires enter the chamber are made *narrow* so that steam coming out of them may not disturb weighing. Their sides are covered with plaster of paris, and the suspension wires are surrounded by electrically heated coils; otherwise steam condenses on the holes, and, due to capillary action, weights cannot be found accurately.

Then the chamber and the spheres are dried. One of the spheres is filled with the dried gas at any desired pressure, and the mass of the gas is found by counterpoising the balance. The temperature of the chamber is found with a sensitive thermometer, and then *dry* steam is passed through it. More heat is required to raise the temperature of the sphere containing the gas than the other sphere, and, therefore, more steam is condensed on the former than the latter. When *steady* condition is reached, the difference between the two amounts of steam condensed is found. This difference represents the amount of steam whose latent heat set free on condensation has raised the temperature of the gas *only*, if the spheres have equal thermal capacities.

Mass of the gas $= M$

Specific heat of the gas at constant volume $= C_v$

Initial temperature $= t_1^\circ$

Temperature of steam $= t_2^\circ$

Additional mass of steam condensed $= m$

Latent heat of steam $= L$

$$\therefore \quad M \times C_v (t_2 - t_1) = mL$$

or

$$C_v = \frac{mL}{M(t_2 - t_1)}$$

Corrections are applied for the expansion of the sphere due to increase in temperature and pressure of the gas, and any slight inequality in the thermal capacities of the two spheres, and the weight of water condensed is found in vacuum.

Q. 33. Clearly distinguish between the specific heat at constant pressure and that at constant volume of a gas.

Describe an accurate method for measuring the specific heat of a gas under constant pressure, deducing the formula to be used. *(Cal. 1935)*

Ans. For first part see Q. 32.

Regnault's Method. The gas, whose specific heat at constant pressure is required, is *dried* and compressed in a reservoir R, (Fig. 29) which is surrounded by a water bath to keep its temperature constant at t°C. The gas is passed through a spiral tube dipping in a heated oil-bath B, kept at a constant temperature, and then through a spiral tube placed in the calorimeter C. Here it raises the temperature of the calorimeter and its contents, and then escapes into the atmosphere. The liquid in the calorimeter is constantly but gently stirred so that its temperature is the same as that of the escaping gas.

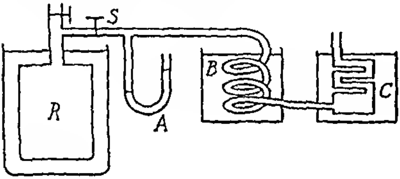


Fig. 29.

The stop cock S is so regulated that the gas flows at a *constant rate*, and this is indicated by the manometer A. Two manometers show the pressure of the gas on entering and leaving the calorimeter, and their difference is very small so that the gas cools in the calorimeter at almost constant pressure. The calorimeter is placed very near the oil-bath to ensure that the gas enters it at the temperature of the oil-bath, but is protected so that it may not receive heat directly from the bath.

Two sensitive thermometers are used to find the temperature t_1°C of the oil-bath and the initial t_2°C and final t_3°C corrected temperature of the calorimeter. When the gas passes, the temperature of the calorimeter and its contents rises progressively, and the temperature of the escaping gas becomes higher higher, but as the gas flows at a *uniform rate*, on an average, its temperature falls from t_1° to the *mean* temperature $\frac{t_2^\circ + t_3^\circ}{2}$ of the calorimeter.

Another manometer is used to find the initial pressure P_1 and the final pressure P_2 of the gas in the reservoir. Let D be the density of the gas at the normal pressure P and 0°C, and V be the volume of the reservoir.

Density of the gas at P_1 and t°C $= D \times \frac{P_1}{P} \times \frac{273}{(273+t)}$

" " " P_2 and t°C $= D \times \frac{P_2}{P} \times \frac{273}{(273+t)}$

Initial mass of gas in the reservoir $= V \times \frac{DP_1 \times 273}{P(273+t)}$

Final " " " " $= \frac{VD\ P_2 \times 273}{P(273+t)}$

$\therefore$ Mass of the gas passed out $= \frac{VD\ (P_1-P_2)\ 273}{P(273+t)}$

Let C_p be the specific heat of the gas at constant pressure and m be the thermal capacity of the calorimeter and its contents.

$$\therefore \frac{VD(P_1-P_2)273}{P(273+t)} \times C_p \times \left(t_1 - \frac{t_2+t_3}{2}\right) = m\ (t_3 - t_2)$$

or
$$C_p = \frac{m(t_3-t_2)(273+t)\ P}{VD\ (P_1-P_2)\ 273\left(t_1 - \frac{t_2+t_3}{2}\right)}$$

Q. 34. Show that for a perfect gas $C_p - C_v = R$, where C_p is the specific heat of a gram-molecule of a gas at constant pressure and C_v the specific heat at constant volume and R is the gas constant.

Calculate C_v for hydrogen, given that $C_p = 6{\cdot}85$ cal.

Density of hydrogen at N. T. P. $= 0{\cdot}0899$ gm./litre. and $J = 4{\cdot}19 \times 10^7$ ergs./cal. (*Punjab, 1938*)

Ans. The specific heat of a gas at constant pressure is greater than that at constant volume, because in heating a gas at constant pressure its volume increases and external work is done by the gas in pushing back the atmosphere, at the cost of heat energy, and in addition the *same* amount of heat is

required to heat the same mass of the gas through the same temperature when its volume is kept constant. As the pressure remains constant, the amount of work done is equal to the product of pressure and the increase in volume.

Let a gram-molecule of a gas have pressure P dynes/sq. cm. and volume V c.cs. at temperature T° absolute (centigrade). When its temperature is raised by $dT^\circ C$ at constant volume, the amount of heat absorbed by it is equal to $C_v.dT$ calories. If its pressure is kept constant, in raising its temperature by the same amount its volume increases by dV, and the amount of heat absorbed is equal to $C_p.\,dT$ calories, which is greater than $C_v dT$ by $P.dV$ ergs, or $\frac{P.dV}{J}$ calories, where J is the mechanical equivalent of heat in ergs per calorie.

$$C_p.dT = C_v.\,dT + \frac{P.\,dV}{J}$$

For a *perfect* gas, $PV = RT$, which on differentiating gives

$$P.dV = R.dT,$$

when P is kept constant. Putting this value of $P.dV$ in the above equation, we get

$$C_p.\,dT = C^v.\,dT + \frac{R.\,dT}{J}$$

or $$(C_p - C_v) = \frac{R}{J}$$

Here the gas constant R is taken in *mechanical* units. Its value in *thermal* units is equal to $\frac{R}{J}$.

In this treatment it is assumed that there are no intermolecular forces, and, therefore, the internal energy of the gas does not change when its volume is changed.

Problem.

Mechanical equivalent of heat $= 4{\cdot}19 \times 10^7$ ergs/cal.

Atomic weight of hydrogen $= 1{\cdot}008$

Molecular „ „ „ $= 2{\cdot}016$

Volume of ·0899 gm. at N. T. P. $= 1000$ c.cs.

„ „ 2·016 gms. „ „ $= \frac{1000 \times 2{\cdot}016}{0{\cdot}0899}$ c.cs.

Normal pressure = 75 cms. of mercury

$$= 76 \times 13{\cdot}59 \times 981 \text{ dynes/sq. cm.}$$

Putting these values in the gas equation, we get

$$R = \frac{PV}{T} = \frac{76 \times 13{\cdot}59 \times 981 \times 1000 \times 2{\cdot}016}{0{\cdot}0899 \times 273} \text{ ergs per gm.-mol. per } ^\circ C$$

$$\frac{R}{J} = \frac{76 \times 13{\cdot}59 \times 981 \times 1000 \times 2{\cdot}016}{{\cdot}0899 \times 273 \times 4{\cdot}19 \times 10^7}$$

$$= 1{\cdot}986 \text{ cals. per gm.-molecule per } ^\circ C$$

$$\therefore C = C_p - \frac{R}{J}$$

$$= 6{\cdot}85 - 1{\cdot}986$$

$$= 4{\cdot}864 \text{ cals. per gm.-molecule per } ^\circ C.$$

Q. 35. Define the term 'specific heat of saturated vapour.' Explain how this may in some cases be a negative quantity. *(Bombay, 1935)*

Ans. The specific heat of a saturated vapour is equal to the amount of heat supplied to raise the temperature of a unit mass of it through 1°, keeping it throughout just saturated. The vapour pressure of a liquid *increases* with temperature, and when the temperature of its saturated vapour is raised, it becomes *unsaturated*. To keep it just saturated its volume has to be *decreased* appropriately by subjecting it to greater pressure, and the energy used in doing work *on* the vapour is converted into heat. Thus the specific heat of a saturated vapour is neither at constant pressure nor at constant volume.

Let the two lowermost isothermals of Fig. 28 be for a unit mass of the substance, and their temperatures differ by 1°. At B a unit mass of the vapour is saturated at the lower temperature, and at B′ it is saturated at a 1° higher temperature. In order to keep it just saturated throughout, its pressure and volume have to be so adjusted that it travels along the border curve BB, and the amount of work done *on* the vapour is given by the area enclosed by the ordinates at B and B′, border curve BB′, and the axis of volume. Let the

amount of heat produced in this way be equal to H' units, and H units be the amount of heat required to raise the temperature of the vapour as explained above. Now three cases arise according as H' is equal to, smaller, or greater than H.

1. When H' is equal to H, heat produced by work done in compressing the saturated vapour to keep it saturated is *just sufficient* to raise the temperature of the vapour through 1°. In this case no *heat* need be supplied from outside for raising the temperature of the saturated vapour, and, therefore its specific heat is said to be zero. This does not mean that there is no change in the thermal energy of the vapour. The heat energy of the vapour *increases*, but this is due to the heat produced by the work done on it, and *no heat energy as such is supplied from outside.*

2. When H' is smaller than H, heat produced in compressing the vapour is not sufficient to raise its temperature by 1°. In this case some heat has to be supplied from outside, and the specific heat of the saturated vapour has a positive value.

3. When H' is *greater* than H, heat produced in compressing the vapour is *more than sufficient* to raise its temperature by 1°. In this case *not only no heat energy as such has to be supplied from outside, but some of the heat produced in compressing has to be removed* from the vapour so that the rise of temperature is just 1°. Here the specific heat of the saturated vapour is called *negative*, but this does not mean that the final heat energy of the vapour at the higher temperature is smaller than its value at the initial lower temperature. There is a definite increase in the heat energy of the saturated vapour, but this is due to a *part* of the heat produced by compression, and the rest of the heat produced has to be *removed* from it, though its temperature is *raised*.

Q. 36. Describe a method of determining the ratio of the specific heats of a gas at constant pressure and constant volume. How has the mechanical equivalent of heat been calculated from a knowledge of this ratio?
(Allahabad, 1929)

Ans. **Clement and Desormes' Method.** A *large* glass flask of *thick* walls is placed in a wooden box and

surrounded by a large amount of some *non-conducting* material so that appreciable heat cannot be *quickly* conducted to or from the flask (Fig. 30). It is connected by a side tube with a manometer in which a liquid of *low density* and *negligible vapour pressure* is used. It is filled with the *dry* gas at a pressure greater than the atmospheric pressure P, and the stop cock S is closed. After some time when the enclosed gas attains room temperature, its pressure P_1 is calculated from the readings of the manometer and a barometer.

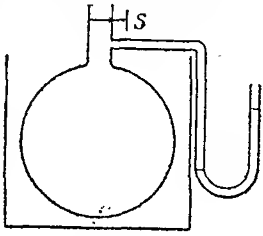


Fig. 30.

The stop-cock is opened for a *few seconds*, when some of the gas rushes out to equalise the pressure, and closed again. This expansion of the gas is *adiabatic* and its temperature *falls*. Then heat is absorbed by the flask; after some time the temperature of the gas rises to its *original* value, and its pressure *increases* to P_2. In this last process the level of the liquid in the manometer is depressed on the side of the flask, but as the volume of the flask is very large as compared with the volume of the manometer, the volume of the gas remains practically the *same*. Therefore the increase of pressure from P to P_2 takes place at *constant* volume.

Let the volume of the flask be V, and V_1 be the volume of that part of the compressed gas in it at pressure P_1 which on opening the stop-cock becomes V and fills the whole flask at pressure P.

$$\therefore \quad P_1V_1^{\gamma}=PV^{\gamma}$$

or
$$\frac{P_1}{P}=\left(\frac{V}{V_1}\right)^{\gamma} \quad \ldots \quad (1)$$

Here γ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume. As the final temperature is *equal* to the initial temperature, the change of volume V_1 at pressure P_1 to volume V at pressure P_2 is *isothermal*.

$$\therefore \quad P_1V_1=P_2V$$

or
$$\frac{V}{V_1}=\frac{P_1}{P_2} \quad \ldots \quad \ldots \quad (2)$$

Putting this value of $\frac{V}{V_1}$ in (1),

$$\frac{P_1}{P}=\left(\frac{V}{V_1}\right)^\gamma=\left(\frac{P_1}{P_2}\right)^\gamma$$

or $$\log P_1-\log P=\gamma\,(\log P_1-\log P_2)$$

$$\therefore\quad \gamma=\frac{\log P_1-\log P}{\log P_1-\log P_2}$$

Mechanical Equivalent of Heat. Let *one* gm. of a gas have volume V c.c. at a pressure of P dynes per sq. cm. and temperature T° absolute (centigrade). When its temperature is raised by dT° at constant volume, the amount of heat absorbed by it is equal to $C_v.dT$ calories, where C_v is its specific heat at constant volume. If its pressure is kept constant, its volume increases by dV when its temperature is raised by dT°, and the external work done by the gas, at the cost of its thermal energy, is equal to PdV ergs. If C_p is the specific heat of the gas at constant pressure, the amount of heat required in this case is equal to $C_p.dT$ calories, which is greater than $C_v.dT$ by $P.dV$ ergs, or $\frac{PdV}{J}$ calories, where J is the mechanical equivalent of heat in ergs per calorie.

$$C_p.\,dT=C_v.\,dT+\frac{P.dV}{J}$$

For a perfect gas, PV=RT, where R is the gas constant for *one gm.* of the given gas. This on differentiating gives

$$PdV=R.dT,$$

as P is kept constant. Putting this value of $P.dv$, the above equation becomes

$$C_p.dT=C_v.dT+\frac{R.dT}{J}$$

or $$J=\frac{R}{C_p-C_v}$$

If either of C_p or C_v is known, the other can be found from the known value of γ, and then the value of J can be calculated from the last equation.

Q. 37. Differentiate clearly between an isothermal and an adiabatic operation.

Show that the slope of the adiabatic through a point in $P-V$ diagram of a perfect gas is γ times that of the isothermal curve through the same point. (*Calcutta, 1936*)

Ans. In an **isothermal** operation the pressure and volume of a substance are changed, but its *temperature is kept the same throughout.* At each stage of the operation heat is supplied to the substance or is removed from it according as work is done by the substance or on it. In the first case work is done *by* the substance at the expense of its thermal energy, and to make up for this loss heat has to be supplied to it. If work is done *on* the substance, this mechanical energy is converted into heat, which has to be removed from it to keep its temperature constant. There is a transformation of energy, but the *thermal* energy of the substance remains the *same* throughout.

For a given mass of a perfect gas the product of its pressure P and volume V is constant at a given temperature T° absolute, and the equation of the isothermal for this temperature is given by

$$PV = RT, \quad \ldots$$

where R is the gas constant for the given mass.

In an **adiabatic** operation the pressure and volume of a substance are changed, but no heat energy *as such* enters or leaves it. If it is compressed, work is done *on* it ; its thermal energy is increased, and its temperature is raised. Owing to rise of temperature, the final volume is *greater* than would have been produced by the *same* increase of pressure under isothermal conditions. If it expands, work is done *by* it at the cost of its thermal energy, and this results in its fall of temperature. Therefore the final volume is *smaller* than would have been produced by the *same* decrease of pressure in an isothermal change. Thus in this operation, the heat energy of the substance is *increased* or *decreased*, but no energy is added to or removed from it in the form of *heat energy*.

For a given mass of a perfect gas the product of its pressure P and volume V changes, but PV^γ remains constant, where γ is

the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume.

Slope of an Adiabatic. Let A be a point on an isothermal BAB′ (Fig. 31) for a *unit mass* of a perfect gas. All points above this curve are for higher temperatures and all points below it are for lower temperatures than its temperature. If the gas is compressed adiabatically, its temperature rises and, the adiabatic AC lies *above* the isothermal AB. Similarly, when the gas expands adiabatically, its temperature falls, and the adiabatic AC′ lies *below* the isothermal AB′. Therefore the adiabatic curve passing through a point is *steeper* than the isothermal curve passing through the some point.

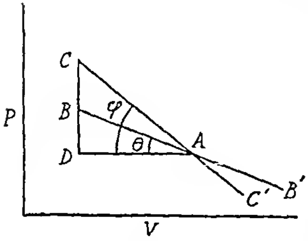


Fig. 31.

If the changes are *very small*, both the isothermal BAB′ and the adiabatic CAC′ are practically *straight* lines. Let the isothermal and adiabatic be inclined at θ and φ to AD, drawn parallel to the axis of volume, and CBD be parallel to P-axis. Instead of going directly form A to C or B, we may go first from A to D and then from D to C or B as the case may be. This makes no difference as far as the change in the thermal energy of the gas is concerned.

If the temperature of D is δT° below that of A, in compressing the gas from A to D at *constant pressure*, heat removed from it is equal to $C_p.\delta T$, where C_p is the specific heat of the gas at constant pressure. If this much heat is now returned to the gas, point C on the adiabatic CA is reached. In going from D to B, the temperature of the gas is raised, at *constant volume*, by δT°, as B lies on the isothermal BA, and the amount of heat absorbed by it is equal $C_v dT$, where C_v is the specific heat of the gas at constant volume.

Heat absorbed in going from D to C $= C_p.\ \delta T$.

" " " " D to B $= C_v.\delta T$.

$$\therefore \frac{\text{Slope of adiabatic}}{\text{Slope of isothermal}} = \frac{\tan\varphi}{\tan\theta} = \frac{DC}{AD}\cdot\frac{AD}{DB} = \frac{DC}{DB}$$

$$= \frac{C_p.\delta T}{C_v.\delta T} = \frac{C_p}{C_v} = \gamma,$$

where γ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume.

Q. 38. Derive for a perfect gas the relation connecting pressure and volume during an adiabatic change.

Calculate the rise in temperature when a gas, for which $\gamma = 1{\cdot}5$, is compressed to eight times its original pressure, *assuming the intial temperature to be* 27°C.

(*Punjab, 1937*)

Ans. **First Method.** See Q. 37 for proving that the slope of an adiabatic of a perfect gas at a point is γ times the slope of the isothermal passing at that point, where γ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume, and pressure P and volume V are taken along the Y-axis and X-axis respectively. For an isothermal, PV = constant, which on differentiating gives,

$$P.dV + V.dP = 0$$

$$\text{Slope of isothermal} = \frac{dP}{dV} = -\frac{P}{V}$$

$$\therefore \text{ Slope of adiabatic} = \frac{dP}{dV} = -\frac{\gamma P}{V}$$

or
$$\frac{dP}{P} + \frac{\gamma . dV}{V} = 0.$$

This on integrating gives

$$\log P + \gamma \log V = \text{constant}$$

or
$$\log PV^{\gamma} = \text{constant}$$

or
$$PV^{\gamma} = \text{constant}.$$

Second Method. When a unit mass of a gas is supplied with dH units of heat, a part of it is used in doing external work PdV, and the remaining increases its temperature by $dT°$ and is equal to $C_v.dT$.

$$C_v.dT + P.dv = dH.$$

Here dH and C_v are given in *mechanical* units. When the expansion is adiabatic, no heat energy is supplied to, or removed from, the gas, and the external work is done by the gas at the cost of its thermal energy.

$$\therefore \quad C_v.dT + P.dv = 0 \qquad \ldots \qquad \ldots \text{ (1)}$$

For a perfect gas, $PV = RT$, where R is the gas constant for *unit* mass, and this on differentiating gives

$$PdV + VdP = RdT$$

Putting this value dT in (2), we get

$$C_v\left(\frac{PdV + VdP}{R}\right) + PdV = 0$$

$$PdV(C_v + R) + C_v . VdP = 0$$

$$C_p . PdV + C_v . VdP = 0 \qquad [\because C_p - C_v = R$$

$$VdP + \frac{C_p}{C_v} P.dV = 0$$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Then integrating as in the first method, we get PV^{γ} $=$ constant.

Problem. Let P_1, V_1, and T_1 be the initial values of pressure, volume, and absolute temperature, and P_2, V_2, and T_2 be the corresponding final values.

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

or

$$\left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{P}{P_2}$$

or

$$\frac{V_2}{V_1} = \left[\frac{P_1}{P_2}\right]^{\frac{1}{\gamma}} \quad \dots \quad \dots \quad (3)$$

For the initial temperature,

$$P_1 V_1 = RT_1 \quad \dots \quad \dots \quad \dots \quad (4)$$

and for the final temperature,

$$P_2 V_2 = RT_2 \quad \dots \quad \dots \quad \dots \quad (5)$$

Dividing the corresponding sides of (4) and (5) and putting the value of $\frac{V_2}{V_1}$ from (3), we get

$$\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} = \frac{P_2}{P_1} \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{1-\frac{1}{\gamma}}$$

In this problem, $T_1 = 273° + 27° = 300°$ absolute,

$$P_2 = 8P_1, \quad \text{and} \quad \gamma = 1{\cdot}5$$

$$\therefore \; T_2 = 300\,(8)^{1-\frac{1}{1\cdot5}} = 300 \times 8^{\frac{1}{3}} = 600° \text{ absolute}$$

Hence the final temperature is $600 - 273 = 327°$C.

Q. 39. Describe and explain Rowland's method of measuring the mechanical equivalent of heat. Discuss the advantages of this method over that adopted by Joule. (*Calcutta, 1929*)

Ans. **Rowland's Experiment.** A calorimeter containing water was attached by its lid to a vertical rod and suspended by a torsion wire (Fig. 32). To the rod was attached a disc and a horizontal rod which carried two pulleys and two weights whose positions could be adjusted. The disc had a groove round which passed a string which left it *tangentially* at the opposite ends of a diameter. The string passed over the pulleys, and two *equal* masses M, M were suspended from its ends. A rod passed through the *bottom* of the calorimeter, and to it was fixed a *hollow perforated* spindle which carried four sets of perforated vanes. Similar perforated vanes were fixed to the inside of the calorimeter, and they alternated with the vanes of the spindle. A steam engine was used to rotate the spindle and its vanes *very rapidly*, and a chronograph was fixed to the bottom rod to determine the number of rotations of the spindle.

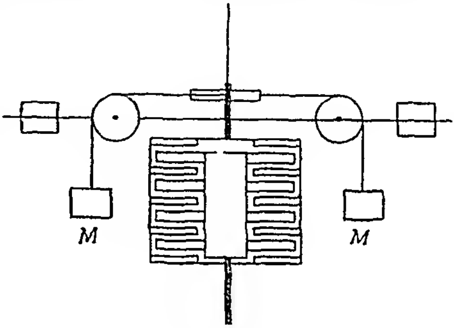


Fig. 32.

When the spindle and its vanes were rotated, water was set in motion, but was stopped by the fixed vanes. Water was thus vigorously churned, and thereby its kinetic energy was converted into heat. The suspended calorimeter tended to rotate with the spindle, but was prevented by the couple applied to the disc and the couple due to any twist produced in the torsion wire. The masses M, M were adjusted to apply the necessary opposing couple. Still the wire was twisted through a small angle. A *very sensitive* thermometer was suspended in the hollow perforated spindle to record the rise of temperature. The calorimeter was surrounded by a water jacket so that the radiation correction could be accurately determined. By a preliminary experiment the twisting couple for the torsion wire was found out.

Diameter of the disc $= d$
Opposing couple due to masses M, M $= Mgd$
" " " " torsion wire $= c$
Number of rotations $= n$
Work done in one rotation $= 2\pi(Mgd + c)$
" " " n rotations $= 2\pi n(Mgd + c)$
Thermal capacity of calorimeter and its contents
$= W$
Corrected rise of temperature $= t^\circ$
Heat produced $= W \times t^\circ$

$$\therefore \text{ Mechanical equivalent of heat J} = \frac{2\pi n(Mgd + c)}{Wt}.$$

Corrections were also applied for the true weight of masses M, M in vacuum, the expansion of the disc, heat conducted to the shaft, and the variation of the specific heat of water with temperature.

In Joule's experiment the mercury thermometer used was not compared with an air thermometer. He assumed the specific heat of water to be constant and equal to 1, and did not take into account its change with temperature. Moreover, the rate of rise of temperature was very slow —·62°C per hour—so that the radiation correction was large. Rowland standardised his thermometer by comparison with an air thermometer, and used the true specific heat of water at different temperatures. By using a steam engine, he obtained a very rapid rise of temperature at the rate of about 40°C per hour.

Q. 40. Describe the constant flow method of Callender and Barne's for the measurement of the mechanical equivalent of heat.

In an experiment, using this method, when the rate of flow of water was 11 grams per minute, the heating current 2 amperes, and the difference of potential between the ends of the heating wire 1 volt, the rise of the temperature of water was 2·5°C. On increasing the rate of flow to 25·4 grams per minute,

the heating current to 3 amperes and the corresponding P. D. to 1·51 volts, the rise of temperature was still 2·5°C. Deduce the value of mechanical equivalent of heat. *(Punjab, 1935)*

Ans. **Callendar and Barne's Experiment.** Fig. 33 shows a section of the apparatus. A fine platinum wire in the form of a *coil* is mounted along the axis of a narrow glass tube of about 2 mm. diameter and is connected at each end

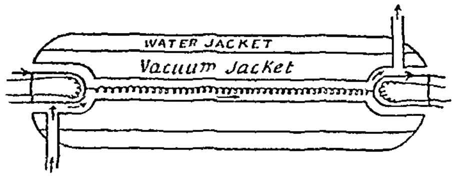


Fig. 33.

to a *thick* copper tube. Water is made to flow in this tube at a *constant rate*, and is heated on its way when electric current is passed in the platinum wire. As the wire is in the form of a coil, water is stirred thoroughly throughout. Two platinum resistance thermometers are placed in the two copper tubes to register the temperature of water at the two ends of the glass tube. The resistance of these thick copper tubes is negligible so that no heat is produced in them by the electric current, and as copper is a very good conductor of heat, the temperature indicated by each thermometer is not appreciably different from that of the surrounding water. These thermometers are very sensitive and can measure a change of about 0·001°C. To minimise the radiation correction and keep it constant, the glass tube is surrounded by an evacuated tube, and this in turn is enclosed in a water jacket, whose temperature is kept *constant*.

When a *steady condition* is reached, each thermometer indicates a *constant* temperature. Then the amount of water M flowing out in about 15 minutes is measured, the time t being indicated by an electric chronograph which reads to $\frac{1}{100}$ second. The potential difference E between the ends of the platinum wire is measured with an accurately calibrated potentiometer by comparison with a *standard* cell. The potential difference between the ends of a *standard* resistance, placed in series with the platinum wire, is measured, and by dividing this by the resistance the strength of the electric current C is found.

$$\therefore \quad ECt = J\{MS(\theta_2 - \theta_1) + H\} \ldots\ldots\ldots\ldots (1)$$

Here $(\theta_2 - \theta_1)$ is the rise of temperature, S is the specific heat of water at the mean temperature, H is the loss of heat due to radiation etc , and J is the mechanical equivalent of heat. As the temperature of the glass tube is not changing, no heat is absorbed by it.

To eliminate H, the experiment is repeated with a different strength of electric current, but the rate of flow of water is so adjusted that its rise of temperature is the *same*, and, therefore, H is the *same*. If now the corresponding values of E, C, t, and M become E′,C′,t', and M′,

$$E'C't' = J\{M'S(\theta_2 - \theta_1) + H\} \ldots\ldots\ldots\ldots (2)$$

Subtracting (2) from (1),

$$ECt - E'C't' = JS(M - M')(\theta_2 - \theta_1)$$

or

$$J = \frac{ECt - E'C't'}{S(M - M')(\theta_2 - \theta_1)}$$

Problem. *First Experiment.* Here E = 1 volt., C = 2 amperes, M = 11 grams, $(\theta_2 - \theta_1) = 2·5°C$, and $t = 60$ seconds. The specific heat of water is to be assumed to be equal to 1.

Heat absorbed by water $= 11 \times 2·5$ calories.

Electric energy converted into heat $= 1 \times 2 \times 60$ Joules

$$\therefore \quad 1 \times 2 \times 60 = J\{11 \times 2·5 + H\} \ldots\ldots\ldots\ldots (3)$$

Second Experiment. Here E = 1·51 volt., C = 3 amperes, M = 25·4 grams, $(\theta_2 - \theta_1) = 2·5°C$, and $t = 60$ seconds..

Heat absorbed by water $= 25·4 \times 2·5$ calories.

Electric energy converted into heat $= 1·51 \times 3 \times 60$ Joules

$$\therefore \quad 1·51 \times 3 \times 60 = J\{25·4 \times 2·5 + H\} \ldots\ldots\ldots (4)$$

Subtracting (3) from (4),

$$2·53 \times 60 = J \times 14·4 \times 2·5$$

or

$$J = \frac{2·53 \times 60}{14·4 \times 2·5}$$

$$= 4·217 \text{ Joules per calorie.}$$

Q. 41. Show that the efficiency of a reversible heat engine working between two specified temperatures is a maximum for those temperatures, and deduce an expression for the efficiency in terms of the temperature scale you adopt. (*Punjab, 1936*)

Ans. Fig. 34 shows Carnot's cycle for a reversible heat engine. Starting from A, at pressure P_1 and volume V_1, the working substance is allowed to expand, and push the piston outward, *along* the isothermal AB, at absolute temperature T_1, until it reaches B, at pressure P_2 and volume V_2. Work is done *by* the substance and an equivalent amount of heat H_1 is absorbed by it from the source.

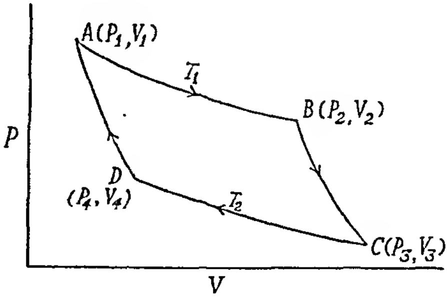


Fig. 34.

$$\text{Work done } by \text{ the substance} = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{RT_1 dV}{V}.$$

$$= RT_1 [\log V]_{V_1}^{V_2}$$

$$= RT_1 \log \frac{V_2}{V_1} \quad \cdot \cdot \cdot \quad (1)$$

Here R is the gas constant for the given mass of the working substance.

From B to C the working substance expands adiabatically from pressure P_2, volume V_2, and absolute temperature T_1 to pressure P_3, volume V_3, and absolute temperature T_2. Here again work is done *by* the substance at the cost of its heat, but no heat is absorbed or given out.

$$\text{Work done } by \text{ the substance} = \int_{V_2}^{V_3} P dV$$

$$= \int_{V_2}^{V_3} \frac{K.dV}{V^\gamma} [\because PV^\gamma = \text{constant K}$$

$$= \frac{1}{1-\gamma}\left[\frac{K}{V^{\gamma-1}}\right]_{V_2}^{V_3}$$

$$= \frac{1}{1-\gamma}\left[\frac{K}{V_3{}^{\gamma-1}} - \frac{K}{V_2{}^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{P_3V_3{}^{\gamma}}{V_3{}^{\gamma-1}} - \frac{P_2V_2{}^{\gamma}}{V_2{}^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\,(P_3V_3 - P_2V_2)$$

$$= \frac{1}{1-\gamma}\,(RT_2 - RT_1)$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1}$$

At C it is compressed along the isothermal CD at absolute temperature T_2 until it reaches D, at pressure P_4 and volume V_4, lying on the adiabatic AD. Here work is done *on* the substance and an equivalent amount of heat H_2 is given out to the condenser. Arguing as before, the amount of work done on the substance from C to D, on the isothermal for absolute temperature T_2, is equal to $RT_2 \log \frac{V_3}{V_4}$

On reaching D it is compressed adiabatically until it reaches its starting point A. Thus one cycle is completed, and the substance is exactly in the same state as at the beginning. The amount of work done *on* the substance from D to A is equal to $\frac{R(T_1 - T_2)}{\gamma - 1}$ and is converted into heat, though no heat is added to or removed from the substance. Work done by the substance from B to C is *equal* to the work done on it from D to A, and, therefore, their resultant is zero.

Thus in one cycle amount of heat H_1 is absorbed at T_1 and H_2 is given out at T_2, and the balance of heat $(H_1 - H_2)$ is converted into an equivalent amount of work. This engine is reversible, and when it is worked backward, its cycle is anti-clockwise: it absorbs heat H_2 at the temperature of the condenser and gives out greater amount of heat H_1 to the

source at higher temperature. In this case work is done *on* the substance.

The efficiency of the engine is equal to the ratio of the amount of heat converted into work to the total amount of heat absorbed.

$$\text{Efficiency} = \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1}$$

$$= 1 - \frac{RT_2 \log \frac{V_3}{V_4}}{RT_1 \log \frac{V_2}{V_1}} = 1 - \frac{T_2 \log \frac{V_3}{V_4}}{T_1 \log \frac{V_2}{V_1}} \quad . \quad . \quad (3)$$

For isothermal AB, $P_1V_1 = P_2V_2$

For adiabatic BC, $P_2V_2^{\gamma} = P_3V_3^{\gamma}$

For isothermal CD, $P_3V_3 = P_4V_4$

For adiabatic DA, $P_4V_4^{\gamma} = P_1V_1^{\gamma}$

Multiplying the corresponding sides of these four equations we get

$$P_1V_1 \times P_2V_2^{\gamma} \times P_3V_3 \times P_4V_4^{\gamma} = P_2V_2 \times P_3V_3^{\gamma} \times P_4V_4 \times P_1V_1^{\gamma}$$

or $$V_1V_2^{\gamma}V_3V_4^{\gamma} = V_2V_3^{\gamma}V_4V_1^{\gamma}$$

or $$V_2^{\gamma-1}V_4^{\gamma-1} = V_3^{\gamma-1}V_1^{\gamma-1}$$

or $$V_2V_4 = V_3V_1$$

or $$\frac{V_3}{V_4} = \frac{V_2}{V_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Putting this in (3),

$$\text{Efficiency} = 1 - \frac{T_2 \log \frac{V_3}{V_4}}{T_1 \log \frac{V_2}{V_1}}$$

$$= 1 - \frac{T_2}{T_1}$$

$$= \frac{T_1 - T_2}{T_1} \quad . \quad . \quad . \quad . \quad (5)$$

The efficiency of a *perfectly reversible* heat engine is the *greatest*, and is *independent* of the nature of the working

substance, depending only on the temperatures of the source and the condenser. If possible let another engine Q have greater efficiency E' than the efficiency E of a perfectly reversible Carnot's engine P. P takes heat H_1 from the source at T_1°, and gives out H_2 to the condenser at T_2°, converting $(H_1 - H_2)$ into work. Let Q absorb H_1' at T_1° and give out H_2' at T_2° so that the *same* amount of work is done by it as by P.

$$\therefore \quad H_1 - H_2 = H_1' - H_2'$$

Let Q work P *backward* through one cycle. Due to the work done by Q on P, P absorbs H_2 from the condenser at T_2° and gives out H_1 to the source at T_1. The work done by Q is *equal to the work done on* P, and, *therefore*, the work done in a cycle on, or by, the two engines taken together is zero.

$$E = \frac{H_1 - H_2}{H_1}, \quad E' = \frac{H_1' - H_2'}{H_1'}$$

$$\text{As } E' > E, \quad \therefore \quad \frac{1}{H_1'} > \frac{1}{H_1} \text{ or } H_1 > H_1', \text{ and } \quad \therefore \quad H_2 > H_2'$$

Heat absorbed from condenser at $T_2^\circ = H_2 - H_2'$,

Heat given to source at $T_1^\circ = H_1 - H_1' = H_2 - H_2'$.

Thus an amount of heat $(H_2 - H_2')$ is absorbed from the condenser at T_2° and given to the source at a *higher* temperature T_1° by the two engines working together *without the expenditure* of external energy. This is denied by the second law of thermodynamics. Therefore Q cannot be more efficient than P. Similarly, it may be shown that working between the same two temperatures no other engine has a greater efficiency than a perfectly reversible engine. Hence the efficiency of a perfectly reversible heat engine is the greatest for the given temperatures of the source and the condenser.

Q. 42. Explain Lord Kelvin's absolute scale of temperature, and show that its indications agree with those of the perfect gas. (*Bombay, 1927*)

Ans. The efficiency of a perfectly reversible heat engine is independent of the nature of the working substance, and is determined by the temperatures of the source and the condenser only. The lower the temperature of the condenser below the given temperature of the source, the greater is the

efficiency of the engine, and the greater is the amount of heat energy converted into work, for the two given adiabatics. If the engine absorbs heat H_1 at absolute temperature $T_1°$ (perfect gas scale) and gives out H_2 at $T_2°$, its

$$\text{Efficiency} = \frac{H_1 - H_2}{H_1} = \frac{T_1 - T_2}{T_1},$$ [See Q. 41 for proof.

or
$$1 - \frac{H_2}{H_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{H_2}{H_1} = \frac{T_2}{T_1}$$

$$\therefore \quad H_3 = H_1 . \frac{T_2}{T_1}$$

This shows that the amount of heat H_2 given to the condenser, and not converted into work, is proportional to the absolute temperature of the condenser. As the thermal energy of a body is due to the motion of its molecules, and as at the absolute zero of temperature its molecules have *no motion*, it has no heat energy. If the condenser is at the absolute zero of temperature, no heat is given to it; the whole of the heat absorbed from the source is converted into work, and the efficiency of the engine is equal to 1. As the working substance is left with no heat, no lower temperature is possible.

Lord Kelvin has proposed a new absolute scale of temperature where *temperature is measured in terms of energy*, which is independent of any peculiar property of a particular thermometric substance. It is also called **thermodynamic scale**, and according to it in a perfectly reversible heat engine, when working between two given adiabatics, *the amount of heat converted into work is proportional to the difference between the temperatures* (on this new scale) *of the source and the condenser*. In Fig. 35 the two given adiabatics are AH and BG. One engine absorbs heat H_1 at temperature θ_1 (thermodynamic scale) and gives out H_2 at θ_2, converting $(H_1 - H_2)$ into work, which is represented by the area

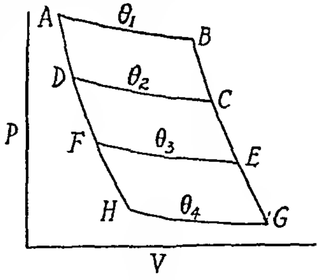


Fig. 35.

ABCD between the two adiabatics and the two isothermals. A second engine takes H_2 at θ_2 and gives out H_3 at θ_3, and converts (H_2-H_3) into work, represented by DCEF. A third engine takes H_3 at θ_3 and gives out H_4 at θ_4, and converts (H_3-H_4) into work, represented by FEGH. If $H_1-H_2=H_2-H_3=H_3-H_4$, then $\theta_1-\theta_2=\theta_2-\theta_3=\theta_3-\theta_4$, and ABCD $=$DCEF$=$FEGH.

The absolute zero of this scale is that temperature of the condenser at which no heat is given to it by the engine, and the whole of the heat absorbed from the source is converted into work. Thus the absolute zero of this scale is the *same* as the absolute zero of the perfect gas scale.

When an engine absorbs H_1 at θ_1 and gives out H_2 at θ_2,

$$H_1-H_2 \propto \theta_1-\theta_2$$

If $\theta_2=0$, H_2 is also equal to zero, and

$$H_1 \propto \theta_1.$$

Dividing the corresponding sides,

$$\text{Efficiency}=\frac{H_1-H_2}{H_1}=\frac{\theta_1-\theta_2}{\theta_1}$$

$$=\frac{T_1-T_2}{T_1} \qquad \text{[on perfect gas scale.}$$

$$\therefore \quad \frac{\theta_1-\theta_2}{\theta_1}=\frac{T_1-T_2}{T_1}$$

or

$$\frac{\theta_2}{\theta_1}=\frac{T_2}{T_1}$$

This shows that the ratio of any two temperatures is the same on the two scales. The interval between the boiling point and the freezing point of water under normal pressure is equal to 100°C on the perfect gas scale, and it is also made 100°C on the thermodynamic scale. If θ_1 and T_1, and θ_2 and T_2 represent these temperatures, $\theta_1-\theta_2=100=T_1-T_2$, and putting this in the above equation,

$$\frac{100}{\theta_1}=\frac{100}{T_1}$$

or

$$\theta_1=T_1$$

and

$$\theta_2=T_2$$

Thus the indications of the thermodynamic scale agree with those of the perfect gas thermometer, that is, the two scales are identical.

No thermometer has been constructed which depends on Carnot's reversible engine. As no actual gas is perfect, there is some difference between the gas scale and the thermodynamic scale. This difference is very small, and is determined by the Porous Plug experiment. Thus the thermodynamic scale of temperature can be realised.

Q. 43. Discuss theoretically the production of cold by expansion of gases through porous plugs. How has this principle been applied in machines for liquefying air ? (*Allahabad, 1931*)

Ans. If the molecules of a gas attract each other, work is done in separating the molecules when the gas expands ; the potential energy of the molecules increases at the cost of their kinetic energy (heat), and cooling is produced, even when no external work is done by the gas. In the absence of the cohesive forces, no work is done in separating the molecules, and there is no change of temperature. If the molecules repel each other, their potential energy decreases and kinetic energy increases when the gas expands, and, therefore, heating is produced.

In the middle of a long cylindrical tube, of non-conducting walls and cross-section area a, is placed a plate B with an aperture (Fig. 36). Pistons D and E move in this tube without any friction, and a gas is *maintained* at pressure P_1 and P_2 in the chambers A and C respectively. P_1 is greater than P_2, and as the gas passes through the aperture from A to C, piston D is pushed forward to keep the pressure in A constant. Similarly, piston E is pushed forward and pressure in C kept unchanged.

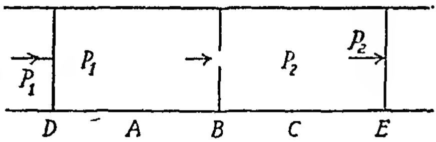


Fig. 36.

Force exerted on piston D is P_1a, and when the piston is moved forward through a distance l_1, work done *on* the gas is equal to P_1al_1, or P_1V_1, where V_1 is equal to al_1 and is the

volume of the mass of the gas at pressure P_1 that has passed through the aperture. At the same time piston E moves forward by l_2, and as the force exerted on it by the gas is P_2a, work done *by* the gas is equal to P_2al_2, or P_2V_2, where V_2 is equal to al_2 and is the volume of the mass of the gas at pressure P_2 that has passed through the aperture from A to C and had volume V_1 at pressure P_1. Work done on the gas, in forcing it through the aperture, is converted into heat, and the temperature of the gas should be raised. In chamber C, work is done by the gas, in expanding and pushing forward piston P_2, at the cost of its thermal energy, and the temperature of the gas should be lowered. Three cases arise according to the variation of the volume of the gas with pressure.

1. If the gas obeys Boyle's law, $P_1 V_1 = P_2V_2$, *work done on the gas is equal to the work done by the gas*, and there is no change in the total energy of a given mass of the gas in expanding from volume V_1 to volume V_2. Then, if no work is done in overcoming cohesive forces during expansion, the temperature of the gas on the two sides of the aperture and at sufficient distance from it is the *same*. In escaping through the aperture, the gas forms eddies and some of its heat is converted into kinetic energy of mass motion. These eddies subside after a short distance, and their kinetic energy is reconverted into heat. The temperature should, therefore, be measured at points where no eddies are present.

If cohesive forces are present, work is done in expanding the gas, and, therefore, the temperature of the gas in C is *lowered*. But there is *rise* of temperature in chamber C if the molecules of the gas repel each other, because when the gas expands, the potential energy of its molecules is decreased and their kinetic energy is increased.

2. If the gas does not obey Boyle's law and the product of pressure and volume of a given mass, at constant temperature, decreases with increase in pressure, P_2V_2 is greater than P_1V_1; *more work is done by the gas than is done on it*, the total energy of the gas *decreases*, and *cooling* is produced in chamber C. Presence of cohesive forces increases this cooling effect, but the repulsive forces between the molecules decrease it.

3. If the gas does not obey Boyle's law and the product of pressure and volume increases with pressure, P_1V_1 is greater than P_2V_2; *work done on the gas is greater than work done by it*, the total energy of the gas *increases*, and heating is produced in C. Cohesive forces decrease and repulsive forces increase this heating effect.

A porous plug consists of a very large number of long fine apertures placed in parallel, and the above treatment applies to it. If P_2 is equal to the atmospheric pressure, piston E may be removed and chamber C exposed to the atmosphere.

For second part see Q. 46 for Linde's method.

Q. 44. Describe the porous plug experiment. In what way have the results of the experiment been utilized? (*Punjab, 1929*)

Ans. **Porous Plug Experiment.** Joule and Thomson *compressed* the *dry* gas and passed it through a spiral placed in a water bath, whose temperature was kept constant. After it acquired the temperature of the bath, it was passed through a porous plug, and then it escaped into the atmosphere. The porous plug consisted of cotton-wool held between two perforated plates A and B (Fig. 37) in a non-conducting wooden tube. The tube was surrounded by a thick layer of asbestos, which in turn was surrounded by a water bath to prevent any gain or loss of heat. The cotton-wool plug provided a very large number of long fine orifices in parallel; it minimised the production of eddies, and decreased the velocity of the gas.

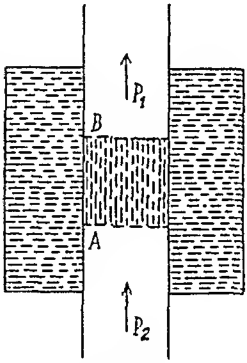


Fig. 37.

The gas was passed continuously through the porous plug at a *constant* pressure P_2 on entering and the atmospheric pressure P_1 on leaving the plug. When a *steady condition* was reached after about one hour, the temperature of the gas just before entering the plug and after leaving it was measured with two platinum resistance thermometers. The experiment was repeated with different gases, at different temperatures, and with different values of the initial pressure P_2. They observed cooling in all gases except hydrogen, where a slight heating was obtained.

These experiments have led to the following results :—

1. At a sufficiently low temperature all gases suffer cooling.

2. The fall in temperature is proportional to the decrease of pressure $(P_2 - P_1)$.

3. The fall in temperature, for a given drop of pressure, *decreases* as the initial temperature is increased. It becomes zero at a certain temperature, known as temperature of inversion of Joule-Thomson effect, and above that heating effect is produced. The temperature of inversion is different for different gases. and depends on the initial pressure P_2. For hydrogen at 100 atmospheres pressure it is $-80°C$, and working below this temperature, cooling effect has been obtained in the case of this gas also.

For the discussion of the effect see Q. 43.

Gases are liquefied by the combined action of high pressure and low temperature, which must be below the critical temperature. In the case of the permanent gases, as their critical temperatures are very low, they have to be cooled below their respective critical temperatures by evaporating some suitable liquid under reduced pressure, but this is not possible in the case of hydrogen and helium.

The temperature of *inversion* of a gas for Joule-Thomson effect is *much higher* than its critical temperature. The gas is compressed and cooled below this temperature and then is allowed to expand suddenly through a porous plug, where its temperature is lowered. Then the cooled gas is passed through a tube surrounding the inner tube through which the compressed gas is passing. Thus the temperature of the compressed gas is further lowered, and on suddenly expanding it is cooled to a still lower temperature. In this way the temperature of the gas issuing from the porous plug becomes lower and lower, and after it has become lower than its critical temperature, liquefaction begins.

For further details, see Q. 45.

Q. 45. Describe in detail the process of liquefaction of (*a*) carbon dioxide, (*b*) oxygen, (*c*) hydrogen. Why

are different methods necessary for liquefying these gases ? (*Bombay, 1934*)

Ans. Gases are liquefied by the combined action of low temperature and high pressure, but unless a gas is *below its critical temperature*, it can not be liquefied, however great the pressure may be. Temperature is usually lowered by using a suitable freezing mixture or evaporating a suitable liquid under reduced pressure. The critical temperatures of different gases are different, and when they are *widely different*, the same method cannot be used for bringing them below their critical temperatures. This is the case with carbon dioxide, oxygen, and hydrogen, whose critical temperatures are 31°C, −119°C, and −240°C respectively.

Carbon dioxide. The critical temperature of this gas is 31°C, and below that temperature it is liquefied by pressure not exceeding 76 atmospheres. A strong copper cylinder contains some sodium bicarbonate, and in it is held vertically a copper tube containing sulphuric acid. The cylinder is closed at the top and is connected with another empty cylinder through a stopcock, which is ordinarily closed.

On tilting the first cylinder, sulphuric acid comes in contact with sodium bicarbonate and carbon dioxide is produced. As the cylinder is closed, the high pressure produced is sufficient to liquefy the gas at ordinary room temperatures (below 31°C). When the stopcock is opened, liquid carbon dioxide distils over to the second cylinder, which is cooled by ice or a freezing mixture. If the temperature is not below 31°C, or it rises above that during the production of the gas, it is lowered below that temperature by cold or ice cold water.

Oxygen. In jacket J_1 (Fig. 38) cold water circulates, and it surrounds a tube which is connected, through pump P_1 on one side, with jacket J_2, which contains liquid methyl chloride. The central tube of this jacket in turn is connected, through pump P_2, with jacket, J_3, which contains liquid ethylene and surrounds a tube through which oxygen gas under pressure is passed.

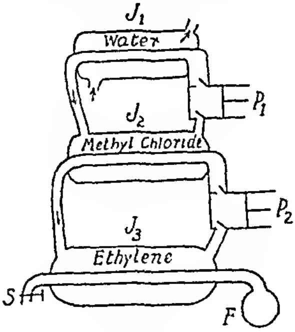


Fig. 38.

When the piston of pump P_1 is moved outward, pressure under it

is reduced; its lower valve opens, and methyl chloride evaporates absorbing heat from its central tube and its contents. On moving the piston of P_1 inward, the lower valve is closed and the upper valve opens, and methyl chloride vapour, under great pressure, is passed to the central tube of J_1. As its critical temperature is 143°C, the vapour is easily liquefied here and passes to J_2, and the heat set free is carried by the current of cold water. The normal boiling point of methyl chloride is −24°C, but on continually evaporating under reduced pressure a *lower* temperature of about −90°C is attained in J_2.

On moving the piston of P_2 outward, pressure under it is reduced, and its lower valve is opened. Liquid ethylene, whose normal boiling point is −104°C, evaporates briskly under lower pressure in J_3, and extracting heat from the corresponding central tube, lowers its temperature to about −160°C after some time. With the inward stroke of the piston, the lower valve closes and the upper valve opens, and *compressed* ethylene vapour is delivered to the central tube of jacket J_2, where it is liquefied, as its critical temperature is 9·5°C.

By heating potassium chlorate in a steel flask F oxygen is produced, and as the tube is closed by valve S, a very high pressure is attained. Under the combined action of high pressure and low temperature of about −160°C, produced in J_3, oxygen is liquefied as its critical temperature is −119°C. On opening the valve, liquid oxygen is withdrawn from the central tube of J_3 and is collected in a Dewar vacuum flask.

Hydrogen. The normal boiling point of oxygen is −183°C, and by evaporating it under reduced pressure the *lowest* temperature that can be produced is −218°C, which is *much higher* than −240°C, the critical temperature of hydrogen. Therefore the last method cannot be employed for the liquefaction of hydrogen.

At ordinary temperatures when compressed hydrogen is allowed to expand suddenly through a porous plug, a slight heating effect is produced. This heating effect decreases as the temperature of the compressed gas is lowered, and *cooling* effect is obtained below −80°C, with suitable initial pressure.

Hydrogen gas enters tube DE (Fig. 39) through a side tube T. When the piston of pump P is drawn outward, pressure below it is educed, the right valve opens, and hydrogen enters through it. On the inward stroke of the piston, the right valve closes and the left valve opens, and *compressed* gas, under *high* pressure, is passed on to the tube FAB. Tube AB is surrounded by jacket J, in which liquid air is evaporated to lower the temperature of hydrogen much below $-80°C$. It then passes through BC and on suddenly expanding through a valve V, its temperature is further lowered.

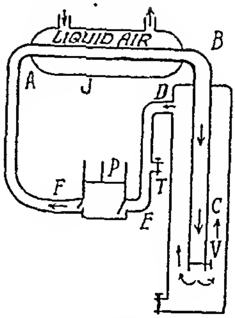


Fig. 39.

The gas, thus cooled, then passes upward through a tube surrounding BC, and lowers the temperature of the gas going downward through BC, which, therefore, is cooled to a *still lower* temperature on suddenly expanding through V. This in turn passes upward through the outer tube, and cools the gas in BC. The gas from the outer tube passes to the pump through DE. It is compressed there, cooled in AB, and then passed downward through BC, where it is further cooled by the upward going gas in the outer tube. Thus the temperature of the gas suddenly expanding through V becomes lower and lower and after it becomes lower than its *critical temperature* $-240°C$, liquefaction begins. Liquid hydrogen is then collected in a Dewar vacuum flask.

Q. 46. Describe Linde's method for liquefying gases and discuss the principle on which it is based.

(*Punjab, 1935*)

Ans. **Linde's Method.** See. Q. 45 for liquefaction of hydrogen. All gases cool when they are allowed to expand suddenly through an orifice provided they have already been cooled *below* their temperatures of *inversion* for Joule-Thomson effect. For each gas a different liquid is used in jacket J (Fig. 39) to cool the compressed gas below its temperature of inversion, which depends on its nature and pressure:

For discussion of the principle, see Q. 43.

Q. 47. Give an account of Forbe's method for determining the thermal conductivity of a metal. What is a steady state? (*Punjab, 1937*)

Ans. **Forbe's Method.** A long bar of the metal, whose thermal conductivity is to be determined, is held in a horizontal position and is heated by immersing its one end in some molten metal, whose temperature is kept *constant* (Fig. 40). The rod is passed through a non-conducting screen so that it may not receive heat directly from the crucible containing the molten metal. A number of holes are drilled along the rod, and thermometers are held in mercury or some fusible metal in these holes to indicate the temperature. At first temperature rises everywhere. After several hours a *steady state* is obtained when the temperature of different points ceases to increase and remains constant thereafter. For this reason, this is called *statical* experiment. When *steady* condition is reached, the temperature at different points along the rod is found, and a curve is drawn between θ, the *excess* of temperature at a point above that of the atmosphere, and x the distance of the point from the heated end The steepness of this curve changes and *decreases* as the distance from the hot end *increases*.

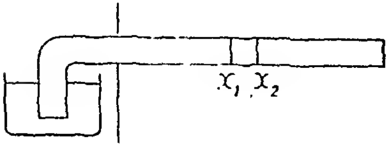


Fig 40.

From this curve the value of temperature gradient $\frac{d\theta}{dx}$ is found at different points along the rod. If K is the thermal conductivity of the rod and A is its cross-section area, the amount of heat that passes through any section in a *unit* time is equal to KA $\frac{d\theta}{dx}$, where $\frac{d\theta}{dx}$ is the temperature gradient at *that* point.

$$\text{Rate of flow of heat at } x_1 = \text{KA} \times \left(\frac{d\theta}{dx} \text{ at } x_1\right)$$

$$\text{,, \quad ,, \quad ,, \quad ,, \quad ,, \quad ,, } x_2 = \text{KA} \times \left(\frac{d\theta}{dx} \text{ at } x_2\right)$$

As $\frac{d\theta}{dx}$ decreases with distance from the hot end, the rate of flow of heat from left to right at x_1 is *greater* than that at x_2, and the difference between them is radiated from the exposed surface of the rod between these two sections.

$\therefore$ Rate of loss of heat from this surface

$$=\text{KA}\left(\frac{d\theta}{dx} \text{ at } x_1-\frac{d\theta}{dx} \text{ at } x_2\right). \quad . \quad .(1)$$

Then a rod of the *same material and cross-section*, but of smaller length, is heated uniformly above the highest temperature attained in the above static experiment. It is then allowed to cool under the *same* condition of the surrounding enclosure as in the first experiment, and its temperature is observed at different intervals. A curve is drawn between the *excess* of temperature θ above that of the surroundings and time t, and from it the value of $\frac{d\theta}{dt}$ is found at different temperatures. This is called *dynamical* experiment, as the temperature of the rod changes and does not remain constant.

Let M be the mass of the rod, S its specific heat, a its surface area (including the ends), and $\frac{d\theta}{dt}$ be the rate of fall of temperature when the temperature of the rod is θ above that of the surroundings.

Rate of radiation of heat at $\theta=\text{MS}.\frac{d\theta}{dt}$

" " " " " " " from unit area $=\frac{\text{MS}}{a}\cdot\frac{d\theta}{dt}$

If P is the perimeter of the rod, its surface area for length dx is Pdx, and the rate of radiation ot heat from it is equal to $\frac{\text{MS}}{a}\cdot\frac{d\theta}{dt}\cdot \text{P}.\,dx$. Then a third curve is drawn between the distance x from the hot end in the first experiment and the rate of fall of temperature $\frac{d\theta}{dt}$ at the corresponding temperatures as found from the second experiment. The area between this curve, x-axis, and the ordinates at x_1 and x_2

is equal to $\int_{x_1}^{x_2} \frac{d\theta}{dt}\, dx$, and may be found by integration or with a planimeter.

$\therefore$ Rate of radiation of heat from the surface

$$\text{between } x_1 \text{ and } x_2 = \frac{\text{MSP}}{a} \int_{x_1}^{x_2} \frac{d\theta}{dt} \,.\, dx \quad \ldots\ldots\ldots \quad (2)$$

Equating (1) and (2), we get

$$\text{KA}\left(\frac{d\theta}{dx} \text{ at } x_1 - \frac{d\theta}{dx} \text{ at } x_2\right) = \frac{\text{MSP}}{a} \int_{x_1}^{x_2} \frac{d\theta}{dt}\, dx$$

or

$$\text{K} = \frac{\text{MSP} \int_{x_1}^{x_2} \frac{d\theta}{dt} \,.\, dx}{a\,\text{A}\left(\frac{d\theta}{dx} \text{ at } x_1 - \frac{d\theta}{dx} \text{ at } x_2\right)}$$

In this way the value of K is found for different values of x_1 and x_2, and then its mean value is calculated.

Q. 48. Give an account of Lees' method for finding the conductivity of a solid. For what kind of conductors is it especially useful?

Point out summarily the difficulties that arise in the way of determining the conductivity of fluids, and how they can be overcome. (*Bombay, 1934*)

Ans. **Lees' Method.** This method is particularly useful for determining the thermal conductivity of *poor conductors*. If a bad conductor is used in the form of a bar, with a reasonable difference of temperature between its ends, it is very difficult to measure *accurately* the *small* rate of flow of heat through it. Moreover, the rate of loss of heat from its surface is considerable. It is not practicable to make the difference of temperature too much, but the rate of flow of heat, for a given difference of temperature, can be increased by increasing the face area and decreasing the distance between the faces, that is, by using a *thin* disc of *large* radius.

Two *thin* discs of the material (shaded), of about 4 cm. diameter and about 2 mm. thickness, are fastened with four copper discs, A, A′, B, B′, of the *same* diameter as shown (Fig. 41) and are suspended in a vessel whose temperature is kept constant by a water jacket. The central copper discs, A, A′, are fixed on the opposite faces of a flat coil of insulated wire through which electric current is passed. To ensure good thermal contact, the inter-faces of the discs are covered with a thin layer of glycerine.

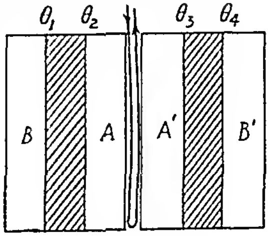


Fig. 41.

When a *steady* condition is reached, the temperature of the four faces of the shaded discs are measured with thermo-couples. In this steady condition, heat is radiated from the faces of the discs as fast as it is produced. As the edges of the discs are very thin, very little of heat is lost from them and almost the whole of it is lost from their faces.

If the current passing through the coil is C amperes and the potential difference between its ends is V volts, the amount of electrical energy used per second is CV joules, and the amount of heat produced per second is $\frac{CV}{4 \cdot 19}$ calories.

Thermal conductivity $= K$

Face area of each disc $= S$ sq. cm.

Thickness of the right disc $= d_2$ cm.

Thickness of the left disc $= d_1$ cm.

Temperature difference for „ „ $= (\theta_1^\circ - \theta_2^\circ)$C

„ „ „ right disc $= (\theta_3^\circ - \theta_4^\circ)$C

Heat flowing per sec. through the left disc $= \frac{KS(\theta_1 - \theta_2)}{d_1}$ calories.

„ „ „ „ „ „ right „ $= \frac{KS(\theta_3 - \theta_4)}{d_2}$ calories.

$$\therefore \ KS\left\{\frac{(\theta_1 - \theta_2)}{d_1} + \frac{(\theta_3 - \theta_4)}{d_2}\right\} = \frac{CV}{4 \cdot 19}$$

or $$K = \frac{CV}{4{\cdot}19S\left\{\frac{(\theta_1 - \theta_2)}{d_1} + \frac{\theta_3 - \theta_4}{d_2}\right\}}$$

For accurate work, a correction must be applied for the heat lost from the edges of the discs.

Liquids are usually poor conductors, and when they are heated convection currents are set up. To stop the convection currents, a liquid is heated from *above*, or a *thin* film of the liquid is used. The containing vessel should be made of a non-conducting material.

In the case of gases, not only convection currents are set up, but heat is also lost by radiation. A small hot body is placed at the centre of a spherical flask, containing the gas, and its rate of cooling is found The convection currents are minimised by using the gas at a *low* pressure, and the rate of loss of heat by radiation is found by evacuating the flask and noting the rate of cooling once more. The difference of the two gives the rate at which heat is lost by conduction.

Q. 49. State and explain the laws relating to the amount of radiation and the temperature of the radiating body. What do you understand by (1) a perfectly black body, (2) coefficient of absorption, and (3) coefficient of emission ?

In an experiment the mass of water together with the water equivalent of the calorimeter was 63 gms. and the surface area of the calorimeter forming the radiating surface was 100 sq. cms. The rate of cooling was 0·025 and the excess of temperature of the calorimeter and its contents over the surroundings was 50°C. Find the coefficient of emission ?

(*Bombay, 1926*)

Ans. **Laws of Cooling.** 1. According to the **Stefan's law**, the amount of heat energy radiated from a given surface in a unit time is proportional to the *fourth* power of its *absolute* temperature θ_1°, or equal to $K\theta_1^4$, where K is a constant depending on the nature and area of the surface. If the absolute temperature of the walls of the enclosure, in which the body is placed, is θ_2°, the rate of radiation of heat from them is proportional

to θ_2^4, and the rate of absorption of heat by the body is equal to $K\theta_2^4$, where K is the *same* constant as in the expression for the rate of radiation by the body, because the absorptive power of a body is *equal* to its emissive power. Combining the rate of radiation and the rate of absorption of heat of the body, its rate of cooling is equal to $K(\theta_1^4 - \theta_2^4)$. As θ_1 approaches θ_2, the rate of cooling decreases.

2. **Newton's Law.** The above expression for the rate of cooling of a body may be put in the form,

$$K(\theta_1^4 - \theta_2^4) = K(\theta_1^2 + \theta_2^2)(\theta_1^2 - \theta_1^2)$$
$$= K(\theta_1^2 + \theta_2^2)(\theta_1 + \theta_2)(\theta_1 - \theta_2)$$

If the temperature of the body is higher than that of the enclosure by a *few* degrees, $(\theta_1^2 + \theta_2^2)$ and $(\theta_1 + \theta_2)$ remain nearly constant during cooling, and the rate of cooling of the body is proportional to $(\theta_1 - \theta_2)$, its excess of temperature above that of the surroundings This is Newton's law of cooling, and is applicable if the excess of temperature is no more than, say, 25°C. The smaller the excess of temperature, the more accurately it gives the rate of cooling.

3. **Dulong and Petit's Law.** They performed many experiments on the rate of cooling of different bodies, at different temperatures, and found that it can be expressed by $K(a^{\theta_1} - a^{\theta_2})$, where a is a constant, K depends on the nature and area of the radiating surface, and θ_1 and θ_2 are the absolute temperatures of the radiating body and the enclosure respectively. This expression is applicable when the difference of temperature is small. It is nearly true for cases where the excess of temperature is *considerable* and cooling takes place in vacuum.

Perfectly Black Body. A body which absorbs *all* the radiation of any wave-length which falls on it is called a perfectly *black* body, because it neither reflects nor transmits any light and appears black, whatever the incident radiation. Conversely, when it is heated, it gives out radiation of all wave-lengths. The radiation given out is independent of the nature of the body and depends on its temperature only.

When radiation is incident on a body, a part of it is absorbed and the rest is reflected or transmitted. The amount of radiation

absorbed by the body depends on its nature and temperature and the wave-length of the incident radiation. The ratio of the radiation absorbed to the amount of radiation incident in the *same* time is called its **co-efficient of absorption.** The absorptive power of a perfectly black body is equal to one, but for other bodies it is less than one.

The amount of radiation given out by a unit area of a body in a unit time depends on its nature and temperature and the wave-length of the radiation. Good absorbers are good radiators. Under given conditions the amount of energy radiated by a perfectly black surface is the *greatest*. The **co-efficient of emission** of a body is equal to the ratio of the amount of energy radiated from its unit area to the amount of radiation given out from a unit area of a perfectly black surface under the same conditions (time, temperature, and wave-length). It is also defined as equal to the amount of energy radiated by a unit area of the surface in a unit time per degree excess of its temperature over that of the enclosure or surroundings.

Problem :

Water equivalent of calorimeter and its contents = 63 gms.

Surface area of calorimeter = 100 sq. cms.

Excess of temperature = 50°C.

Fall of temperature per second = ·025°C

Heat lost per second = $63 \times {}^{\cdot}025$ calories.

$$\therefore \text{ Co-efficient of emission} = \frac{63 \times {}^{\cdot}025}{100 \times 50}$$

$= {}^{\cdot}000315$ calories per sec. per sq. cm.

Q. 50. Explain Prevost's theory of exchanges, and show that the emissive power of a body is equal to its absorptive power.

Describe some instrument by which radiant heat can be measured. (*Punjab, 1933*)

Ans. **Prevost's Theory.** All bodies radiate heat at all temperatures. The rate of radiation from a given body depends

only on the nature and temperature of its surface, and is independent of the surrounding bodies. Similarly, the radiations emitted by the surrounding bodies depend only on the nature and temperature of their respective surfaces. Thus all bodies radiate heat and absorb a part of the heat that comes to them from the surrounding bodies. The temperature of a body rises or falls according as it absorbs *more* or *less* heat than it emits in the *same* time. When its rate of absorption is *equal* to its rate of emission of heat, its temperature remains constant, that is, its equilibrium of temperature is *dynamic* and not static.

Emissive and Absorptive Powers. If e_λ be the **emissive power** of a body at temperature $T°$, the amount of energy radiated by a unit area of it in a unit time at that temperature and lying between wave-lengths λ and $\lambda+d\lambda$ is equal to $e_\lambda . d\lambda$. The **absorptive power** a_λ of a body at temperature $T°$ and for wave-length λ is defined as equal to the *fraction* of the incident radiation, lying between λ and $\lambda+d\lambda$, which it absorbs at that temperature. Let a unit area of the body receive in a unit time amount of radiation dQ lying between λ and $\lambda+d\lambda$.

Amount of radiation absorbed $=a_\lambda . dQ$

" " " emitted $=e_\lambda . d\lambda$

As the body is in equilibrium, the amount of radiation absorbed by a unit area of it in a unit time is *equal* to the amount of radiation emitted by its unit area in the same time.

$$\therefore \; e_\lambda . d\lambda = a_\lambda . dQ \quad \cdots\cdots\cdots \quad (1)$$

A perfectly black body absorbs the whole of the radiation that fall on it. Therefore its absorptive power is equal to one. If its emissive power for the above temperature and range of wave length be denoted by E_λ,

$$E_\lambda . d\lambda = 1.dQ \quad \cdots\cdots\cdots \quad (2)$$

Dividing (1) by (2),

$$\frac{e_\lambda}{E_\lambda} = a_\lambda \quad \cdots\cdots\cdots \quad (3)$$

or
$$\frac{e_\lambda}{a_\lambda} = E_\lambda$$

This shows that the ratio of the emissive power of a body to its absorptive power for any temperature and range of wave-length is *constant* and is equal to the emissive power of a perfectly black body for the same temperature and the wave-length of radiation. Its emissive power is equal to its absorptive power if its emissive power is defined as the ratio of the energy radiated by it to that emitted by a perfectly black body, for the same time, area, temperature, and wave-length, that is, its emissive power is equal to $\frac{e_\lambda}{E_\lambda}$.

Pyroheliometer. Two thin *blackened* strips of platinum, $2 \times {}^{\cdot}15 \times {}^{\cdot}002$ cm, are attached to the two junctions of a very sensitive thermo-couple. When both the strips are protected from radiation, their temperature is the same, and a galvanometer connected to the thermo-couple indicates no thermo-electric current. Then one strip is exposed to the radiation to be measured, while the other is protected from it. The temperature of the exposed strip rises, and thermo-electric is indicated by the galvanometer. Electric current is passed through the protected strip to heat it, and its strength is adjusted so that *no* thermo-electric current is produced in the thermo-couple, which shows that the temperature of the two strips has gain become the *same*. Evidently, for this condition, the rate of production of heat in the protected strip by the electric current must be *equal* to the rate of absorption of heat by the exposed strip.

If the electric current in the protected strip is C ampere, and V volt is the fall of potential between its ends, CV joules of electric energy is converted into heat in one second; or $\frac{CV}{J}$ calories of heat is produced in it in one second, where J is the mechanical equivalent of heat in joules per calorie. This is also the rate of absorption of radiant heat by the exposed strip. The experiment is repeated with the second strip exposed to radiant heat and the first strip protected from radiation and heated by electric current.

LIGHT

Q. 51. Derive a formula expressing the relation between μ, U, V and R in the case of a refraction at a single spherical surface separating two transparent media. Using this result, show that $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ for a convex lens. What approximations have you made, and what limitations do they impose upon the use of this formula? (*Punjab, 1937*)

Ans. In Fig. 42, APA′ is a section of a concave spherical surface, whose centre of curvature is at C and pole at P so that CP is its principal axis. It separates a medium of refractive index μ_2 on the left from a medium of refractive index μ_1 on the right. From an object O on the principal axis a ray along OP strikes the surface normally and suffers no change in its path. Another ray OA makes angle i with the normal CAB at A, and if μ_2 is *greater* than μ_1, it is turned *towards* the normal and proceeds along AD so that the angle of refraction r is *smaller* than the angle of incidence. To an observer on the left of P, these two refracted rays appear to come from their point of intersection I when produced backward. Therefore I is the virtual image of O.

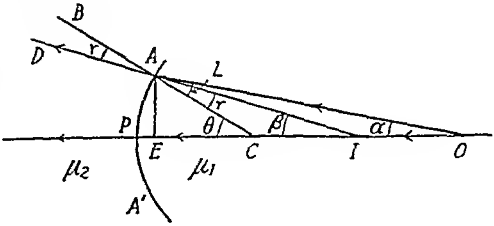


Fig. 42.

Let α, β, and θ be the angles made by OA, IA, and CA respectively with the principal axis, and AE be perpendicular to PO. It is assumed that the radius of curvature R of the surface is *very large* as compared with its aperture so that angles α, β and θ are *very small* and their sines are practically equal to their values in radians, and A is very close to P so that distances of O and I from A are practically equal to their

distances from the pole P. The second assumption is that the incident rays meet the refracting surface *very near* P, so that i and r are *very small.*

$$\frac{\mu_2}{\mu_1}=\frac{\sin i}{\sin r}\longrightarrow\frac{i}{r}$$

or $$\mu_1\, i=\mu_2\, r \quad . \quad . \quad . \quad . \quad (1)$$

But $i=\theta-\alpha$, and $r=\theta-\beta$

$$\therefore \quad \mu_1(\theta-\alpha)=\mu_2(\theta-\beta)$$

or $$(\mu_2-\mu_1)\theta=\mu_2\beta-\mu_1\alpha$$

Replacing these angles by their sines, we get

$$\frac{(\mu_2-\mu_1)\,\mathrm{AE}}{\mathrm{R}}=\frac{\mu_2 .\,\mathrm{AE}}{\mathrm{AI}}-\frac{\mu_1 .\,\mathrm{AE}}{\mathrm{AO}}$$

or $$\frac{(\mu_2-\mu_1)}{\mathrm{R}}=\frac{\mu_2}{\mathrm{PI}}-\frac{\mu_1}{\mathrm{PO}}$$

$$=\frac{\mu_2}{\mathrm{V}}-\frac{\mu_1}{\mathrm{U}}, \quad . \quad . \quad . \quad . \quad (2)$$

where V and U are respectively the distances of the image and object from the pole, and in this case both are positive.

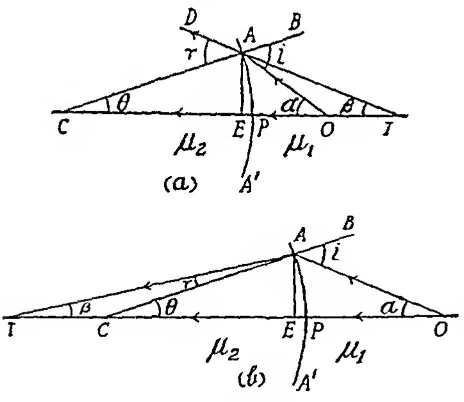


Fig. 43.

When the surface is convex, its centre of curvature C lies on the other side of P_1 and the radius of curvature is *negative.* Proceeding as before we get the position of I. In Fig. 43 (*a*), O is very close to P, the image is virtual, and U and V are both positive. When PO is *large* (Fig. 43, *b*), the refracted ray AI actually intersects the first ray along the principal axis at I, and to an observer on the left of I these rays diverge from I. Therefore I is the *real* image of O, and V is *negative.*

In Fig. 43 (*a*), $i=(\theta+\alpha)$, and $r=(\theta+\mathrm{B})$

Putting these values in (1),

$$\mu_1(\theta+\alpha)=\mu_2(\theta+B)$$

or

$$(\mu_2-\mu_1)\theta=-\mu_2\beta+\mu_1\alpha$$

Replacing these angles by their sines and putting AI = PI = V and AO = PO = U, we get

$$\frac{(\mu_2-\mu_1)\ AE}{AC=-R}=\frac{-\mu_2.\ AE}{AI=PI=V}+\frac{\mu_1.AE}{AO=PO=U}$$

or

$$\frac{(\mu_2-\mu_1)}{R}=\frac{\mu_2}{V}-\frac{\mu_1}{U} \quad . \quad . \quad . \quad (3)$$

In Fig. 43 (b), $i=(\theta+\alpha)$, and $r=(\theta-\beta)$. Proceeding as before,

$$\mu_1\ (\theta+\alpha)=\mu_2(\theta-\beta)$$

or

$$(\mu_2-\mu_1)\theta=\mu_2\beta+\mu_1\alpha$$

or

$$\frac{(\mu_2-\mu_1)AE}{AC=-R}=\frac{\mu_2\ .\ AE}{AI=PI=-V}+\frac{\mu_1\ .\ AE}{AO=PO=U}$$

or

$$\frac{(\mu_2-\mu_1)}{R}=\frac{\mu_2}{V}-\frac{\mu_1}{U} \quad . \quad . \quad . \quad . \quad (4)$$

Thus in all the three cases there is the same relation between μ_2, μ_1, R, U, and V, if they are given their proper signs. The refractive index of the left medium with respect to the medium on the right is equal to $\frac{\mu_2}{\mu_1}=\mu$, and dividing the above relation throughout by μ_1, we get

$$\frac{(\mu-1)}{R}=\frac{\mu}{V}-\frac{1}{U} \quad . \quad . \quad . \quad . \quad (5)$$

Convex Lens. Let a convex lens of refractive index μ_2 be placed in a medium of refractive index μ_1, and the radius of curvature of the incident and emergent faces be R_1 and R_2 respectively. For an object at a distance U, the image formed by the *first* surface is at a distance V′ from it, and is given by

$$\frac{(\mu_2-\mu_1)}{R_1}=\frac{\mu_2}{V'}-\frac{\mu_1}{U} \quad . \quad . \quad . \quad . \quad (6)$$

Then the rays proceed *from* a medium of refractive index μ_2 *to* a medium of refractive index μ_1 and suffer refraction at the second face of radius of curvature R_2. The image is the point to which the refracted rays converge, or appear to

diverge from, while the object is the point of intersection of the incident rays. The image for the first face is object for the second face, because the refracted rays for the first face are incident rays for the second face, and as the lens is *very thin*, the two faces are practically at the same distance from it. If the final image now formed is at a distance V from the lens,

$$\frac{\mu_1-\mu_2}{R_2}=\frac{\mu_1}{V}-\frac{\mu_2}{V'} \qquad \cdots \qquad \cdots(7)$$

Adding (6) and (7), and rearranging

$$(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{\mu_1}{V}-\frac{\mu_1}{U}$$

or

$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{V}-\frac{1}{U} \qquad \cdots(8)$$

When the object is at infinity, $\frac{1}{U}$ is zero; the incident rays are parallel to the principal axis of the lens, and on emerging from it converge to its *second* principal focus on its other side at a distance f from it.

$$\therefore \quad (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{V}-0=\frac{1}{f} \qquad \cdots(9)$$

Considering the assumptions made in deriving this formula, it is applicable to a *very thin* lens whose aperture is *very small* as compared with the radii of curvature of its faces, and when the incident rays are inclined to the principal axis of the lens at *very small* angle.

Q. 52. Obtain the relation between the distances of an object and its image formed by a lens in terms of the radii of curvature of the lens.

Show that the minimum distance between an object and its real image formed by a convex lens is four times the focal length of the lens. (*Punjab, 1934*)

Ans. See Q. 1. When the lens is concave, the rays emerging from the second face appear to diverge from a point on the side of the first face, and, therefore, the image is always *virtual.*

Minimum distance between Object and Image.

Putting $(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}$, (8) of that question becomes

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

For a convex lens, f is negative, and when it forms a *real* image, v is also *negative*,

$$\therefore \quad \frac{1}{f}=\frac{1}{v}+\frac{1}{u} \text{ (numerically)} \qquad \cdots(1)$$

Differentiating both sides, we get

$$0=-\frac{dv}{v^2}-\frac{du}{u^2}$$

or $$\frac{dv}{du}=-\frac{v^2}{u^2} \qquad \cdots \quad \cdots \quad \cdots(2)$$

Let $u+v=l$. For maximum or minimum distance $\frac{dl}{du}=0$.

Differentiating both sides,

$$1+\frac{dv}{du}=\frac{dl}{du}=0$$

or $$\frac{dv}{du}=-1$$

Putting this value of $\frac{dv}{du}$ in (2)

$$-\frac{v^2}{u^2}=-1$$

or $$v=u \text{ (numerically)} \qquad \cdots \quad \cdots(3)$$

and (1) becomes,

$$\frac{1}{f}=\frac{2}{v}=\frac{2}{u}$$

or $$v=u=2f$$

$$\therefore \quad u+v=4f$$

This is for *minimum* distance between an object and its real image ; the maximum distance between them is infinity.

Second Method.

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$\therefore\quad \left(\frac{1}{v}+\frac{1}{u}\right)^2=\frac{1}{f^2}=\left(\frac{1}{v}-\frac{1}{u}\right)^2+\frac{4}{uv}$$

or
$$\frac{4}{uv}=\frac{1}{f^2}-\left(\frac{1}{v}-\frac{1}{u}\right)^2$$

As $\left(\frac{1}{v}-\frac{1}{u}\right)^2$ is always positive, whether v is greater or smaller than u, and $\frac{1}{f^2}$ is constant, $\frac{4}{uv}$ is maximum, or uv is *minimum*, when $\frac{1}{v}-\frac{1}{u}=0$, or $v=u$.

But
$$\frac{uv}{u+v}=f=\text{constant}$$

When uv is minimum, $(u+v)$ is also minimum. Therefore for $(u+v)$ being minimum, $u=v$, and

$$f=\frac{uv}{u+v}=\frac{u}{2}$$

or
$$u=v=2f$$

$\therefore$ Minimum distance between object and image $=u+v$
$=4f$

Q. 53. Show that there are two positions for a convex lens for a sharp image of an object on a screen placed at a distance from the object greater than four times the focal length of the lens.

If m_1 and m_2 be the magnifications produced in these two positions and d the distance between them, prove that the focal length $f=\dfrac{d}{m_1-m_2}$. (*Calcutta, 1936*)

Ans. When a convex lens of focal length f forms a *real* image, the minimum distance between the object and its image is equal to $4f$, and the lens lies *midway* between them. For other positions of the lens, the distances of the object

and its image from the lens are *different*. In Fig. 44, O is an object and its image is formed at S by the lens placed in the position L_1. Using the usual notation,

Fig. 44.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

As f and v both are negative,

$$-\frac{1}{f}=-\frac{1}{v}-\frac{1}{u}$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \quad \text{(numerically)} \quad \ldots\ldots\ldots (2)$$

or $$f=\frac{u\times v}{u+v}=\frac{u(l-u)}{l}$$

where l is the fixed distance between the object and its image. Rearranging this we get a quadratic equation in u, which has *two* roots.

$$ul-u^2=fl$$

or $$u^2-lu+fl=0$$

$$\therefore \quad u=\frac{l\pm\sqrt{l^2-4fl}}{2}$$

$$=\frac{l}{2}\pm\sqrt{\frac{l^2}{4}-fl}$$

Thus u has two values; one is greater than $\frac{l}{2}$ and the other smaller than $\frac{l}{2}$ by the *same* amount. In the first case the image is diminished, but in the second case it is magnified. The values of v are :—

$$l-u=l-\left\{\frac{l}{2}\pm\sqrt{\frac{l^2}{4}-fl}\right\}$$

$$=\frac{l}{2}\mp\sqrt{\frac{l^2}{4}-fl}$$

This shows that the *value of v in the first case is equal to the value of u in the second case*, and *its value in the second case is equal to the value of u in the first case*.

From equation (1), $\dfrac{v}{f} = 1 + \dfrac{v}{u}$

$$= 1 + m_1, \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where m_1 is the linear magnification produced by the lens in this position, and is equal to $\dfrac{v}{u}$.

If the object is placed at S, its image is formed at O by the lens in the position L_1, but it is diminished instead of being magnified, as the distances of the object and its image from the lens are *interchanged*, and its magnification is equal to $\dfrac{u}{v}$. Instead of interchanging the positions of O and S, they are kept fixed, and the lens is moved towards S until in the position L_2 it forms a diminished image of O at S. Then $OL_2 = u' = v$, $SL_2 = v' = u$, and $d = v - u$.

$$\frac{1}{f} = \frac{1}{v'} + \frac{1}{u'}$$

$$\frac{v'}{f} = 1 + \frac{v'}{u'}$$

or

$$\frac{u}{f} = 1 + \frac{u}{v}$$

$$= 1 + m_2, \quad \ldots \quad \ldots \quad (3)$$

where m_2 is the magnification in the second position of the lens, and is equal to $\dfrac{v'}{u} = \dfrac{u}{v}$.

Subtracting (3) from (2),

$$\frac{v-u}{f} = m_1 - m_2$$

or

$$f = \frac{v-u}{m_1 - m_2} = \frac{d}{m_1 - m_2}$$

Q. 54. Find an expression for the focal length of a single lens which is equivalent to two thin lenses placed at a distance d from each other.

Ans. **Concave Lenses.** Two concave lenses L_1 and L_2 of focal lengths f_1 and f_2 respectively are placed at a distance d from

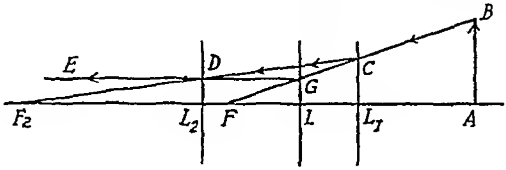


Fig. 45.

each other, and an object AB stands on their common principal axis (Fig. 45). A ray BC from the top of the object after passing through the first lens L_1 at C is directed towards the *first* principal focus F_2 of the second lens L_2; after passing through the second lens at D, it travels along DE *parallel* to the common principal axis, and, therefore, *determines the height* of the image (virtual), which is equal to the distance between this ray and the principal axis. Produce BC to meet the principal axis at F and line ED, when produced backward, at G. A *single* concave lens, of focal length f, which when placed at L has its *first* principal focus at F, turns the ray BCG in the direction GDE, and thus forms image (virtual) of AB of the *same height*, though not necessarily in the same position, as formed by L_1 and L_2 together. The top of the image formed by L_1 and L_2 together lies at the intersection of ED and CL_2, which is also the image of C formed by L_2, while that formed by the equivalent lens L lies at the intersection of EG and BL.

Triangles CFL_1 and GFL are similar,

$$\therefore \quad \frac{CL_1}{GL}=\frac{L_1F}{LF} \quad \ldots \ldots \ldots \quad (1)$$

Also triangles CF_2L_1 and DF_2L_2 are similar,

$$\therefore \quad \frac{CL_1}{DL_2=GL}=\frac{L_1F_2}{L_2F_2} \quad \ldots \ldots \ldots \quad (2)$$

Combining (1) and (2), we get

$$\frac{L_1F}{LF}=\frac{L_1F_2}{L_2F_2}$$

or

$$\frac{1}{LF}=\frac{L_1F_2}{L_2F_2\times L_1F} \quad \ldots \ldots \quad (3)$$

Here $L_1F_2 = -(d+f_2)$, and $L_2F_2 = -f_2$, because the focal length of a lens is the distance of its *second* principal focus from it, and is *equal* and *opposite* to the distance between the lens and its *first* principal focus.

If two rays are going along CF and L_1F to meet at F, the first lens makes them meet at F_2. Therefore F is a *virtual* object and F_2 is its real image formed by lens L_1.

$$\therefore \quad \frac{1}{L_1F_2} - \frac{1}{L_1F} = \frac{1}{f_1}$$

or

$$\frac{1}{L_1F} = \frac{1}{L_1F_2} - \frac{1}{f_1}$$

$$= -\frac{1}{(d+f_2)} - \frac{1}{f_1}$$

$$= \frac{f_1+d+f_2}{-(d+f_2)f_1}$$

Putting values of L_1F_2, L_2F_2, and L_1F in (3), we get

$$\frac{1}{LF} = \frac{-(d+f_2)}{-f_2} \times \frac{f_1+d+f_2}{-(d+f_2)f_1}$$

$$= \frac{f_1+d+f_2}{-f_1f_2}$$

$$\therefore \frac{1}{f} = -\frac{1}{LF} = \frac{f_1+d+f_2}{f_1f_2}$$

$$= \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1f_2} \quad \cdots \quad \cdots \quad (4)$$

Convex Lenses. Fig. 46 shows the case of two convex lenses, L_1 and L_2, and their equivalent lens L. A ray BC from the top of the object is deviated towards the principal axis by L_1, it then passes through the *first* principal focus F_2 of L_2, and emerges from L_2 along DE parallel to the principal axis. This ray determines the height of the image, whose top lies at the point of intersection of DE and CL_2.

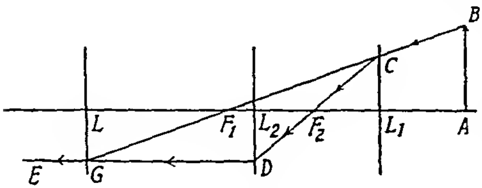


Fig. 46.

A single convex lens L, of focal length f, placed so that BC produced passes through its *first* principal focus F_1 and meets DE at a point G on it, turns ray BCG along GE, and produces image of the same height, though not necessarily in the same position, as formed by L_1 and L_2 together.

Proceeding as before, from similar triangles F_2CL_1 and F_2L_2D, and F_1CL_1 and F_1LG

$$\frac{CL_1}{L_2D}=\frac{L_1F_2}{L_2F_2}, \text{ and } \frac{CL_1}{LG=L_2D}=\frac{L_1F_1}{LF_1}$$

$$\therefore \quad \frac{L_1F_1}{LF_1}=\frac{L_1F_2}{L_2F_2}$$

or
$$\frac{1}{LF_1}=\frac{L_1F_2}{L_2F_2}\times\frac{1}{L_1F_1} \quad . \quad . \quad . \quad . \quad (5)$$

Giving proper signs, $L_2F_2=-f_2$, as F_2 is *first* principal focus of L_2, and $L_1F_2=-(d-L_2F_2)=-(d+f_2)$. Two rays BC and AL_1 would meet at F_1 but lens L_1 makes them meet at F_2, and, therefore, for it, F_1 is virtual object and F_2 its real image.

$$\frac{1}{L_1F_2}-\frac{1}{L_1F_1}=\frac{1}{f_1}$$

or
$$\frac{1}{L_1F_1}=\frac{1}{L_1F_2}-\frac{1}{f_1}=\frac{1}{-(d+f_2)}-\frac{1}{f_1}.$$

$$=\frac{-(f_1+d+f_2)}{(d+f_2)f_1}$$

Putting these values in (5), we get

$$\frac{1}{LF_1}=\frac{-(d+f_2)}{-f_2}\times\frac{-(f_1+d+f_2)}{(d+f_2)f_1}$$

$$=\frac{f_1+d+f_2}{-f_1f_2}$$

$$\therefore \quad \frac{1}{f}=-\frac{1}{LF_1}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{d}{f_1f_2} \quad . \quad . \quad . (6)$$

Q. 55. Explain the theory and application of achromatic combinations of prisms and lenses. (*Bombay, 1933*)

Ans. **Achromatic combination of Prisms.** When a ray of light passes through a prism, placed in a *rarer* medium, it suffers deviation towards its base. For a certain angle of incidence the deviation is minimum, and depends on the

refracting angle A of the prism and its refractive index μ for the wave-length of light used. The relation between these quantities is given by

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

When A is *very small*, D is also very small, and the above relation reduces to

$$\mu = \frac{A+D}{A}, \quad \text{or } D=(\mu-1)A \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The angle of minimum deviation, with a given prism, is different for lights of different wave-lengths. When the incident ray consists of different wave lengths, a number of differently coloured rays emerge from it at *different* angles. If μ_V and μ_R are the refractive indices of the prism for violet and red rays respectively, and D_V and D_R are the corresponding deviations, equation (1) gives

$$D_V=(\mu_V-1)A$$

$$D_R=(\mu_R-1)A$$

∴ Dispersion produced by the prism

$$=D_V-D_R=(\mu_V-\mu_R)A \quad . \quad . \quad (2)$$

To obtain an *achromatic combination*, that is, *deviation without dispersion*, a second prism of *different* kind of glass of suitable refracting angle A' and corresponding refractive indices μ'_V and μ'_R, is placed in contact with the first prism, but opposite to it, so that the refracting edge of each touches the base of the other. The second prism also deviates the violet rays *more* than the red rays towards its base, but *away* from the base of the first prism. If D'_V and D'_R are the corresponding deviations.

Dispersion produced by second prism

$$=D'_V-D'_R=(\mu'_V-\mu'_R)A' \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the two dispersions are *equal*, they neutralise each other, and violet and red rays emerge from the combination *parallel* to each other, and are brought to a single focus by a lens or

the eye of the observer. Therefore the condition for achromatic combination for *these rays* is :

$$D_V - D_R = D'_V - D'_R$$

or $$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A' \quad . \; . \; . \; . \quad (4)$$

If μ and D, and μ' and D', are the mean deviation and mean refractive index in the first and second case respectively, the deviation produced by the combination is given by

$$D - D' = (\mu - 1)A - (\mu' - 1)A' \quad . \; . \; . \; . \quad (5)$$

Achromatic Combination of Lenses. See Q. 56.

Applications. A prism is usually used, in preference to a plane mirror, for changing the direction of light, by total internal reflection. If light used is not monochromatic, it is dispersed, and the emergent beam is coloured. To overcome this difficulty a second prism, of different kind of glass and of suitable refracting angle, is used to neutralise the dispersion produced by the first, and still produce the desired change in the direction of light.

As the focal length of a lens depends on the wave-length of light used, and is *different* for light of different colours, unless monochromatic light is used, the image of a point object is drawn out along the axis of the lens, and is coloured for any position of the screen. Moreover, as the human eye is most sensitive to the yellow part of the spectrum, the position of the photographic plate is adjusted for focussing these rays in taking a photograph ; but the plate itself is most sensitive to the blue part of the spectrum, and, therefore, it is *out of focus* for these rays. Thus a single lens camera cannot give a sharp, distinct photograph. Therefore achromatic lenses are used in telescopes, microscopes, photographic cameras, and in all other instruments used for forming images free from chromatic aberration.

Q. 56. Explain what you understand by the dispersive power of a material.

Deduce the condition of achromatism for two lenses in contact with each other. (*Calcutta, 1933*)

Ans. **Dispersive Power.** The refractive index of any given medium is different for different wave-lengths of light

used, and they are deviated by different amounts. Consider the case of a prism of *very small* refracting angle A that produces minimum deviation D (very small) in a ray of light for which its refractive index is μ.

$$\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \longrightarrow \frac{A+D}{A}$$

or
$$D = (\mu - 1)A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the refractive indices of this prism for violet and red rays are μ_V and μ_R respectively, and the corresponding deviations are D_V and D_R, then

$$D_V = (\mu_V - 1)A$$

and
$$D_R = (\mu_R - 1)A$$

$$\therefore \text{ Dispersion} = D_V - D_R = (\mu_V - \mu_R)A \quad . \quad . \quad (2)$$

Let D be the *mean* deviation and μ the corresponding *mean* refractive index. Dividing (2) by (1),

$$\frac{D_V - D_R}{D} = \frac{(\mu_V - \mu_R)}{(\mu - 1)} = w, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where w is called the **dispersive power** of the material of the prism *for* violet and red rays, and it indicates *how the angular dispersion (or refractive index) varies for a given mean deviation (or mean refractive index).* For different media w is different, that is, for the same value of D (or μ), $D_V - D_R$ (or $\mu_V - \mu_R$) is different, and for the same value of $D_V - D_R$ (or $\mu_V - \mu_R$), D (or μ) is different. For any particular wave-length its value is given by $\frac{\delta D}{D}$, or $\frac{\delta\mu}{\mu - 1}$, where D is the deviation and μ is the refractive index for that wave-length, and $\delta\mu$ is a very small change in refractive index for a very small change δD in the deviation.

Equation (3) gives $D_V - D_R = w.D$ . . . . (4)

This shows that the angular dispersion for any two wave-lengths is equal to the product of their mean deviation and the dispersive power of the medium for *those wave-lengths.*

Achromatic Combination of Lenses. A thin lens may be considered to consist of a very large number of prisms of

varying refracting angles placed end to end from its pole outward, the refracting angle decreasing at a uniform rate from its pole towards its periphery. The deviation produced by the different parts of the lens goes on increasing from its pole outward, and all the rays of light coming from a point on its principal axis converge to, or appear to diverge from, a point after passing through it. But the deviation produced at a point is different for different wave-lengths, and, therefore, the focal length of a lens *depends* on the colour of light used.

To get an achromatic combination for any two colours of light, two lenses of *different* kinds of glass and of suitable focal lengths are placed in contact so that *the focal length of the combination is the same for those two colours.* Let f_V and f_R be the focal lengths of one lens for violet light and red light respectively and f be its mean focal length, μ_V, μ_R, and μ be the corresponding refractive indices, and R_1 and R_2 be the radii of curvature of its two faces.

$$\frac{1}{f} \doteq (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(\mu - 1)f}$$

$$\therefore \frac{1}{f_V} = (\mu_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{(\mu_V - 1)}{(\mu - 1)f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly,

$$\frac{1}{f_R} = \frac{(\mu_R - 1)}{(\mu - 1)f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Let the corresponding terms for the second lens be denoted by a dash. Then proceeding as before,

$$\frac{1}{f'_V} = \frac{(\mu'_V - 1)}{(\mu' - 1)f'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\frac{1}{f'_R} = \frac{(\mu'_R - 1)}{(\mu' - 1)f'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The reciprocals of the *combined* focal lengths for violet and red rays are equal to $\frac{1}{f_V} + \frac{1}{f'_V}$ and $\frac{1}{f_R} + \frac{1}{f'_R}$ respectively, and

these should be *equal* for achromatism for violet and red. Putting their values from (6)......(9), we get

$$\frac{(\mu_V-1)}{(\mu-1)f}+\frac{(\mu'_V-1)}{(\mu'-1)f'}=\frac{(\mu_R-1)}{(\mu-1)f}+\frac{(\mu'_R-1)}{(\mu'-1)f'}$$

or $$\frac{(\mu_V-1)}{(\mu-1)f}-\frac{(\mu_R-1)}{(\mu-1)f}+\frac{(\mu'_V-1)}{(\mu'-1)f'}-\frac{(\mu'_R-1)}{(\mu'-1)f'}=0$$

or $$\frac{(\mu_V-\mu_R)}{(\mu-1)f}+\frac{(\mu'_V-\mu'_R)}{(\mu'-1)f'}=0$$

or $$\frac{w}{f}+\frac{w'}{f'}=0,$$

where w and w' are the dispersive powers of the first and second lens respectively for violet and red. As w and w' are both positive, f and f' must be of *opposite* signs, that is, one lens must be convex and the other concave, and their mean focal lengths should be in the ratio of their dispersive powers. Further, the two surfaces in contact should have the same radius of curvature.

Second Method. Differentiating (5), we get

$$\frac{-df}{f^2}=d\mu\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Dividing (10) by (5), $\frac{-df}{f}=\frac{d\mu}{(\mu-1)}=w \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$

If the two lenses have their mean focal lengths f_1 and f_2, for any given region of the spectrum, then their combined mean focal length f is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

This on differentiating gives,

$$\frac{-df}{f^2}=\frac{-df_1}{f_1^2}-\frac{df_2}{f_2^2}$$

$$=\frac{w_1}{f_1}+\frac{w_2}{f_2}, \qquad \text{[from (11)}$$

where w_1 and w_2 are the corresponding dispersive powers for the given region of the spectrum. As the *combined* focal length is to be *constant* for that region, its differential should

be zero. Therefore the condition of achromatism is

$$\frac{w_1}{f_1}+\frac{w_2}{f_2}=0,$$

for any given region of the spectrum.

Q. 57. What is meant by an achromatic combination of lenses, and what are the principles underlying its construction? Prove the formula involved.

(*Punjab, 1933*)

Ans. See Q. 56 for the meaning of an achromatic combination of lenses and the principles underlying its construction when the lenses are placed in *contact*.

Another method is to use two thin lenses of the *same kind* of glass placed at a distance equal to their average focal length. This is usually done in the case of eye-pieces. The rays dispersed by the first lens are made *parallel* by the second lens. The coloured images are neither of the same size nor are they at the same distance, but all of them subtend the *same angle* at the eye, and, therefore, a white image is seen.

The focal length f of the combination of two lenses of focal lengths f_1 and f_2, and placed a distance a apart, is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1 f_2} \quad \ldots\ldots \quad (1)$$

Let f_{V_1} and f_{R_1} be the focal lengths of the first lens for violet and red rays and f_1 be its mean focal length; f_{V_2}, f_{R_2}, and f_2 be the corresponding values for the second lens; μ_V and μ_R be their refractive indices for violet and red rays, and μ be their mean refractive index. If R_1 and R_2 are the radii of curvature of the two faces of the first lens, then

$$\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_{V_1}}=(\mu_V-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\frac{(\mu_V-1)}{(\mu-1)f_1} \quad \ldots\ldots \quad (2)$$

$$\frac{1}{f_{R_1}} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{(\mu_R - 1)}{(\mu - 1)f_1} \quad \ldots \ldots \ldots \ldots \quad (3)$$

Similarly for the second lens,

$$\frac{1}{f_{V_2}} = \frac{(\mu_V - 1)}{(\mu - 1)f_2} \quad \ldots \ldots \ldots \ldots \quad (4)$$

$$\frac{1}{f_{R_2}} = \frac{(\mu_R - 1)}{(\mu - 1)f_2} \quad \ldots \ldots \ldots \ldots \quad (5)$$

Let f_V and f_R be the focal lengths of the combination for violet and red rays respectively.

$$\frac{1}{f_V} = \frac{1}{f_{V_1}} + \frac{1}{f_{V_2}} + \frac{a}{f_{V_1} f_{V_2}}$$

$$= \frac{(\mu_V - 1)}{(\mu - 1)f_1} + \frac{(\mu_V - 1)}{(\mu - 1)f_2} + \frac{a(\mu_V - 1)^2}{(\mu - 1)^2 f_1 f_2}$$

$$= \frac{(\mu_V - 1)}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \left(\frac{\mu_V - 1}{\mu - 1}\right)^2 \times \frac{a}{f_1 f_2} \quad \ldots \quad (6)$$

$$\frac{1}{f_R} = \frac{1}{f_{R_1}} + \frac{1}{f_{R_2}} + \frac{a}{f_{R_1} f_{R_2}}$$

$$= \frac{(\mu_R - 1)}{(\mu - 1)f_1} + \frac{(\mu_R - 1)}{(\mu - 1)f_2} + \left(\frac{\mu_R - 1}{\mu - 1}\right)^2 \times \frac{a}{f_1 f_2}$$

$$= \frac{(\mu_R - 1)}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \left(\frac{\mu_R - 1}{\mu - 1}\right)^2 \frac{a}{f_1 f_2} \quad \ldots \quad (7)$$

For achromatic combination, f_V should be equal to f_R. Equating (6) and (7), we get

$$\frac{(\mu_V - 1)}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \left(\frac{\mu_V - 1}{\mu - 1}\right)^2 \times \frac{a}{f_1 f_2}$$

$$= \frac{(\mu_R - 1)}{(\mu - 1)}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \left(\frac{\mu_R - 1}{\mu - 1}\right)^2 \times \frac{a}{f_1 f_2}$$

or $$\frac{(\mu_V-\mu_R)}{\mu-1}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)=\frac{(\mu_R-1)^2-(\mu_V-1)^2}{(\mu-1)^2}\times\frac{a}{f_1f_2}$$

$$=\frac{(\mu_R-\mu_V)(\mu_R+\mu_V-2)}{(\mu-1)^2}\times\frac{a}{f_1f_2}$$

$$=\frac{-(\mu_V-\mu_R)2(\mu-1)}{(\mu-1)^2}\times\frac{a}{f_1f_2}$$

$[\because \mu_R+\mu_V=2\mu$

or $$\frac{1}{f_1}+\frac{1}{f_2}=\frac{-2a}{f_1f_2}$$

Multiplying both sides by f_1f_2,

$$f_1+f_2=-2a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

As a is always positive, hence (f_1+f_2) is always negative: either both the lenses are convergent, or one is convergent and of greater focal length than the other divergent lens. This combination is achromatic for *all colours*, because μ is the same for both the lenses for any wave-length of light used.

Second Method. Differentiating (1) we get

$$-\frac{df}{f^2}=\frac{-df_1}{f_1^2}-\frac{df_2}{f_2^2}-\frac{a}{f_1}\cdot\frac{df_2}{f_2^2}-\frac{a}{f_2}\cdot\frac{df_1}{f_1^2}$$

For achromatic combination f must be the same for light of different colours, and, therefore, its differential df is equal to zero. Also, as shown in equation (11), Q. 56, dispersive powers w_1 and w_2 are respectively equal to $\frac{-df_1}{f_1}$ and $\frac{-df_2}{f_2}$

$$\therefore \quad \frac{w_1}{f_1}+\frac{w_2}{f_2}+\frac{aw_2}{f_1f_2}+\frac{aw_1}{f_2f_1}=0$$

But in this case $w_1=w_2$,

$$\therefore \quad \frac{1}{f_1}+\frac{1}{f_2}+\frac{2a}{f_1f_2}=0$$

or $$f_1+f_2+2a=0$$

or $$f_1+f_2=-2a$$

Q. 58. What is meant by an achromatic combination of lenses? How will you correct the chromatic

aberration of a plano-convex, crown glass lens of 30 cm. mean focal length, *the following data being given?* Refractive indices for red and blue light respectively are 1·520 and 1·540 for the crown glass and 1·630 and 1·660 for flint glass, various lenses of which material are available and with any desired focal length and radii of curvature. (*Punjab, 1936*)

Ans. For achromatic combination of lenses, see Q. 56.

Problem. Two lenses of dispersive powers w_1 and w_2 for two given colours form an achromatic combination for them (colours) when placed in *contact*, if their mean focal lengths f_1 and f_2 for those two colours are given by

$$\frac{w_1}{f_1}+\frac{w_2}{f_2}=0$$

(*a*) *Crown glass convex lens.*

Mean focal length $f_1=-30$ cm.

Refractive index for blue light $=1·54$

" " " red light $=1·52$

Mean refractive index $=1·53$

$\therefore$ Its dispersive power $w_1=\dfrac{1·54-1·52}{1·53}=\dfrac{·02}{1·53}$

(*b*) *Flint glass lens.*

Refractive index for blue light $=1·660$

" " " red light $=1·630$

Mean refractive index $=1·645$

$\therefore$ Its dispersive power $w_2=\dfrac{1·66-1·63}{1·645}$

$$=\frac{·03}{1·645}$$

For achromatic combination,

$$\frac{·02}{1·53\times(-30)}+\frac{·03}{1·645\times f_2}=0$$

or $$f_2=\frac{·03\times 1·53\times 30}{1·645\times ·02}=41·85 \text{ cm.}$$

Therefore the correcting flint glass lens should be *concave* and of focal length 41'85 cm.

Q. 59. A compound achromatic lens was constructed, having focal length 50 cm.; the surface of contact of the crown and flint glasses having a common radius of 30 cm. The dispersive powers of crown and flint glasses being taken as 0'22 and 0'46 and the refractive indices of the middle of spectrum assumed as 1'52 and 1'63 respectively, calculate the radii of curvature of the second faces of the two lenses. (*Punjab, 1932*)

Ans. Two lenses placed in contact form an achromatic combination for two colours if the dispersive power w_1 of one, for the two given colours, divided by its mean focal length f_1 for them is *equal* and *opposite* to the corresponding ratio $\frac{w_2}{f_2}$ for the same two colours for the second lens.

Dispersive power of crown glass $= {}^{\cdot}22$

" " " flint glass $= {}^{\cdot}46$

Mean refractive index of crown glass $= 1{}^{\cdot}52$

" " " flint glass $= 1.63$

Focal length of the combination (*assumed to be convex*) $= -50$ cm.

Common radius of curvature $= 30$ cm.

Let f_1 cm. and f_2 cm. be the mean focal lengths for crown glass and flint glass lenses respectively. Then for achromatism,

$$\frac{{}^{\cdot}22}{f_1} = -\frac{{}^{\cdot}46}{f_2}$$

or

$$f_2 = -\frac{{}^{\cdot}46}{{}^{\cdot}22} f_1 = -\frac{23}{11} f_1 \quad . \quad . \quad . \quad (1)$$

As the reciprocal of the focal length of the combination is equal to the sum of the reciprocals of the focal lengths of the components,

$$-\frac{1}{50} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$=\frac{1}{f_1}-\frac{11}{23f_1} \qquad \text{[from (1)}$$

$$=\frac{12}{23f_1}$$

or $$f_1=\frac{-12\times 50}{23} \text{ cm.}$$

and $$f_2=-\frac{23}{11}\times\frac{-12\times 50}{23}$$

$$=\frac{12\times 50}{11} \text{ cm.}$$

The focal length f of a lens of refractive index μ and radii of curvature R_1 and R_2 is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots\ldots \quad (2)$$

(*a*) *Crown glass convex lens.* If the convex lens is the *first* lens, the common radius of curvature $R_2=+30$ cm. Putting the values of f_1, μ, and R_2 in (2), we get

$$-\frac{23}{12\times 50}=(1{\cdot}52-1)\left(\frac{1}{R_1}-\frac{1}{30}\right)$$

or $$\frac{1}{R_1}-\frac{1}{30}=\frac{-23}{12\times 50\times{\cdot}52}$$

or $$\frac{1}{R_1}=\frac{-23}{12\times 50\times{\cdot}52}+\frac{1}{30}$$

$$=\frac{-23+10{\cdot}40}{12\times 50\times{\cdot}52}$$

and $$R_1=\frac{12\times 50\times{\cdot}52}{-12{\cdot}6}=-24{\cdot}76 \text{ cm.}$$

Thus its first surface is *convex* and of radius of curvature 24·76 cm.

(*b*) *Flint glass concave lens.* The concave lens being the second lens, the common surface is its *first* surface, that is,

$R_1 = +30$ cm. Putting the known values in (1), we get

$$\frac{11}{12 \times 50} = (1{\cdot}63 - 1)\left(\frac{1}{30} - \frac{1}{R_2}\right)$$

or
$$\frac{1}{30} - \frac{1}{R_2} = \frac{11}{12 \times 50 \times {\cdot}63}$$

or
$$\frac{1}{R_2} = \frac{1}{30} - \frac{11}{12 \times 50 \times {\cdot}63}$$
$$= \frac{12{\cdot}6 - 11}{12 \times 50 \times {\cdot}63}$$

and
$$R_2 = \frac{12 \times 50 \times {\cdot}63}{1{\cdot}6}$$
$$= +236{\cdot}25 \text{ cm.}$$

The second surface of the flint glass concave lens is of radius of curvature 236·2 cm. and of the *same* (positive) sign as its first surface, that is, it is convex and bulges outward.

Q. 60. (*a*) State and explain the essential conditions for the formation of a primary rainbow. Show clearly the points of distinction between it and the secondary one.

(*b*) A luminous source situated 30 cm. from a double convex lens forms a real image on the axis such that the blue part of the image is at a distance of 50 cm. from the lens, the axial length of the image from blue to red being 4·11 cm. If the refractive index for the blue is 1·6337, determine the refractive index for the red. (*Bombay 1935*)

Ans. See Q. 61 for the formation of a primary rainbow and the following points of distinction between it and the secondary rainbow :—

(1) The primary rainbow is formed by rays which have suffered *one* internal reflection, while the secondary rainbow is due to the rays which have been *twice* internally reflected. Therefore the secondary rainbow is *fainter* than the primary rainbow.

(2) The secondary rainbow is *higher* than the primary rainbow.

(3) The primary rainbow is violet on its inner (lower) edge and red on its outer edge, but in the secondary rainbow the opposite is the case, and the spectrum colours are in the *reverse* order.

Problem. If a lens of radii of curvature R_1 and R_2 has refractive indices μ_B and μ_R for blue and red rays respectively, its corresponding focal lengths f_B and f_R are given by

$$\frac{1}{f_B}=(\mu_B-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

and
$$\frac{1}{f_R}=(\mu_R-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Dividing the first equation by the corresponding sides of the second equation, we get

$$\frac{f_R}{f_B}=\frac{(\mu_B-1)}{(\mu_R-1)} \quad \ldots\ldots\ldots \quad (1)$$

Refractive index for blue rays $= 1{\cdot}6337$
Distance of object from the lens $= 30$ cm.
" " blue image " " " $= 50$ cm.
" " red image " " " $= 54{\cdot}11$ cm.

Using lens formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, where v is *negative* for real image,

$$\frac{1}{f_B}=-\frac{1}{50}-\frac{1}{30}=\frac{-8}{150}$$

$$\frac{1}{f_R}=-\frac{1}{54{\cdot}11}-\frac{1}{30}=\frac{-84{\cdot}11}{54{\cdot}11\times 30}$$

Putting the values of f_R, f_B, and μ_B in (1) we get

$$\frac{54{\cdot}11\times 30}{-84{\cdot}11}\times\frac{-8}{150}=\frac{1{\cdot}6337-1}{\mu_R-1}$$

or
$$(\mu_R-1)=\frac{{\cdot}6337\times 84{\cdot}11\times 5}{54{\cdot}11\times 8}$$

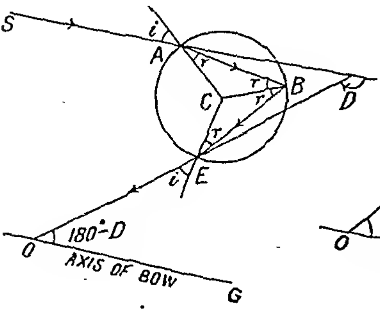
$$={\cdot}6156$$

$$\therefore\ \mu_R=1{\cdot}6156.$$

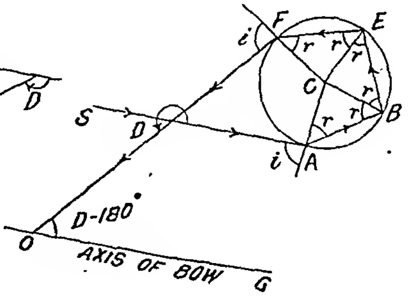
Q. 61. Explain the formation of Primary and Secondary rainbows. Why is the latter red in its inner and violet on its outer edge? (*Punjab, 1929*)

Ans. **Rainbows.** They are seen in the sky due to the dispersion which sun's rays suffer when they are refracted and internally reflected by the spherical rain drops. The primary rainbow is formed by the rays which have suffered *one* internal reflection, while the secondary rainbows are due to the rays which have been *twice* internally reflected before emerging from the water drops. The intensity decreases with the number of internal reflections, and, therefore, usually primary, and sometimes secondary, rainbows only are seen by an observer with his *back* towards the sun.

Primary Rainbows. Parallel rays of the sun fall on spherical water drops in the sky. As they are incident at different points on a drop, making different angles of incidence, they undergo different deviations. A ray incident at A



(a)



(b)

Fig. 47.

[Fig. 47 (*a*)], at angle of incidence i, makes angle of refraction r and is turned clockwise; at B on internal reflection it is turned clockwise through $180° - 2r$, and it emerges at E along EO suffering a further clockwise deviation of $i - r$. As all the three deviations are in the *same direction*, the total deviation D is equal to their sum and is given by

$$D = (i - r) + (180° - 2r) + (i - r)$$
$$= 180° + 2i - 4r$$

A ray which is directed towards the centre of the drop falls on it along the normal, and suffers the greatest deviation of 180° on being internally reflected back on its path. From the refractive index of water and different angles of incidence, the corresponding angles of refraction can be calculated, and the value of D in each case can be found. Working in this way the minimum value of deviation is found to be about 138°. Near this point deviation changes *very slowly* : the deviation of the rays on either side of the minimum-deviation ray is almost the *same*, and, therefore, all these emergent rays are almost *parallel* to one another, so that *the intensity of the emergent beam remains almost the same.* All other rays are divergent, or after crossing each other become divergent, and their intensity *decreases* with distance.

If the observer's eye is in the direction of the least deviated rays, he receives *copious* light from the drop; but in all other directions the emergent light from the drop is *faint.* All the drops of water which lie on a cone, whose apex is at the observer O, axis parallel to the rays of the sun, and line OE on its surface, send out least deviated rays to the observer, and, therefore, appear *much more bright* than other drops. (The incident rays on all these drops are parallel to one another, but as their planes of refraction and internal reflection are *different*, the emergent rays are not parallel and can pass through O.) In this way a bright circular arc, whose centre is in line with the sun and the observer, is seen in the sky.

Light of the sun is dispersed by the water drops: angle of minimum deviation is the greatest for violet and smallest for red, and for other colours lies between them. As the angle between OE and the axis of the cone OG is the *supplementary* of the angle of minimum deviation, the value of this angle is *greater* for red rays than for violet rays, and, therefore, the observer receives red light from a *higher* circular arc of water drops than violet light. Thus the primary rainbow is coloured violet on its *inner* edge and red on the *outer* and other colours lie between them. Angle EOG is 40·2° and 42·1° for violet and red bows respectively.

Secondary Rainbows. In Fig. 47 (*b*) a ray of light undergoes two refractions and two internal reflections. All the four

deviations are in the anti-clockwise direction, and the total deviation D is given by

$$D = (i-r) + (180^\circ - 2r) + (180^\circ - 2r) + (i - r)$$
$$= 360^\circ + 2i - 4r$$

Working as before, the angle of minimum deviation is found to be 232°. In this case the angle between the line of sight OF and the axis of the cone OG is equal to $D - 180^\circ$, and as the angle of minimum deviation for violet light is greater than for red light, the angle between OF and OG is greater for violet light than for red light. Therefore the observer O receives violet light from a *higher* arc of water drops and red light from a *lower* arc of drops, the respective angles being 54·5° and 51·8°. Thus the secondary rainbow, unlike the primary rainbow, is red on its inner edge and violet on its outer.

As the angle between the line of sight and the axis OG of the bow is greater for secondary rainbow than for primary rainbow, the secondary rainbow is *higher* than the primary rainbow. Fig. 48 is a vertical section of the rainbows by a plane in the line of sight. Water drops S_V and S_R send violet

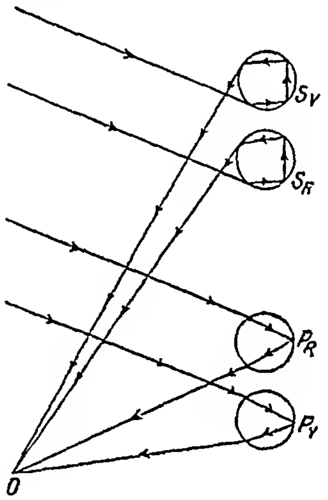


Fig 48.

and red rays respectively of the secondary rainbow, and the corresponding drops for the primary rainbow are P_V and P_R.

The region between P_R and S_R appears *darker* than the rest of the sky, because light reflected internally from the water drops here can not reach the observer, as, for both single and double internal reflection, the angle of deviation would have to be smaller than the angle of minimum deviation, which is not possible.

Q. 62. Explain the theory of a direct-vision spectroscope. (*Punjab, 1935*)

Ans. When a prism of refracting angle A produces minimum deviation D in a ray of light, its refractive index μ is equal to $\frac{\sin \frac{1}{2}(A+D)}{\sin (A/2)}$. When A is small, D is also small, and

$$\mu = \frac{A+D}{A},$$

or

$$D = (\mu - 1)A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the incident light is not monochromatic, it is dispersed, and different wave-lengths are deviated by different amounts. If D_V and D_R are the deviations for the extreme parts, violet and red, of the spectrum, and μ_V and μ_R the corresponding refractive indices,

$$D_V = (\mu_V - 1)A$$

and

$$D_R = (\mu_R - 1)A$$

$$\therefore \text{ Dispersion} = D_V - D_R$$

$$= (\mu_V - \mu_R)\, A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If μ is the *mean* refractive index and D the corresponding devation, equation (1) expresses the relation between them.

For a second prism of a *different* kind of glass, refracting angle A', refractive indices μ', μ'_V, and μ'_R, and the corresponding deviations D', D'_V and D'_R,

$$\text{Mean deviation } D' = (\mu' - 1)A' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\text{Dispersion} = D'_V - D'_R$$

$$= (\mu'_V - \mu'_R)A' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If A and A' are so selected that $(\mu - 1)\,A$ is *equal* to $(\mu' - 1)A'$, and the prisms are placed with their refracting edges *opposite* to one another, that is, the refracting edge of each is along the base of the other, the *mean* deviation due to one is *equal* and *opposite* to the mean deviation produced by the

other, and, therefore, the combination produces no deviation in the *central* rays. As the dispersive powers of the two prisms are *different*, for the *same* mean deviation, they produce *different* degrees of dispersion. The resultant dispersion is equal to the difference between the two dispersions, and it lies on the two sides of the undeviated central rays. Thus with this combination the spectrum is seen by looking *directly* through it.

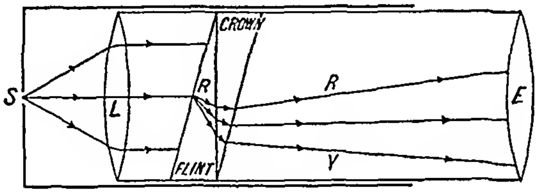


Fig. 49.

Two prisms, first of flint glass and second of crown glass, and greater refracting angle, are held with their refracting edges in opposite directions, in a tube, which carries a lens L at one end and an eye-piece at the other and can slide in an outer tube (Fig. 49.). The outer tube has a narrow slit S placed *parallel* to the refracting edges of the prisms and at the first principal focus of lens L, so that the rays emerging from the lens are parallel to its principal axis.

Light passing through the slit is dispersed by the first prism; it is then dispersed by the second prism, in the opposite direction, and emerges from it with the central part of the spectrum undeviated. The dispersive power $\frac{D_V - D_R}{D}$ of the first prism is *greater* than the dispersive power $\frac{D'_V - D'_R}{D'}$ of the second prism. As D, the mean of D_V and D_R, is *equal* to D', the mean of D'_V and D'_R, the dispersion $(D_V - D_R)$, due to the first prism, is *greater* than the dispersion $(D_V' - D'_R)$, produced by the second prism, and D_V is *greater* than D'_V and D_R *smaller* than D'_R. Thus the combination produces spectrum whose central part is undeviated; the part of the spectrum on the violet side of the central rays is deviated towards the base of the flint glass prism, and that on the red side of

the central rays is deviated towards the base of the crown glass prism. The spectrum is examined with an eye-piece E. Usually, to obtain a *large* spectrum, two flint glass prisms are combined alternately with three crown glass prisms of appropriate refracting angles.

Q. 63. An objective consists of two lenses A and B in contact. A is plano-concave, and B is bi-convex. The radii of curvature of the objective are thus: ∞, $-r_1$; $-r_1, +r_2$. The optical data for the glasses are :—

Refractive indices : (A), $\mu=1{\cdot}614$; (B), $\mu=1{\cdot}522$

Dispersive powers : (A), $w=0{\cdot}027$; (B), $w=0{\cdot}017$. Calculate r_1 and r_2 so that the objective may be achromatic and have a focal length of 300 mm.

(*Bombay, 1932*)

Ans. A lens A of mean focal length f_1 and dispersive power w_1 when placed *in contact* with another lens B of mean focal length f_2 and dispersive power w_2 forms an achromatic combination if

$$\frac{w_1}{f_1}+\frac{w_2}{f_2}=0$$

or
$$f_1=-\frac{w_1 f_2}{w_2}.$$

Putting the values of w_1 and w_2,

$$f_1=-\frac{{\cdot}027\, f_2}{{\cdot}017}$$
$$=-\frac{27\, f_2}{17}$$

The focal length f of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$
$$=\frac{-17}{27f_2}+\frac{1}{f_2}=\frac{10}{27f_2}$$

As the objective is a *convex* combination, $f=-30$ cm.

$$\therefore \quad \frac{10}{27f_2}=-\frac{1}{30}$$

or $$f_2 = -\frac{300}{27} = -\frac{100}{9} \text{ cm.}$$

and $$f_1 = -\frac{27f_2}{17} = -\frac{27}{17}\left(\frac{-100}{9}\right)$$

$$= \frac{300}{17} \text{ cm.}$$

The focal length f of a lens of refractive index μ and radii of curvature R_1 and R_2 is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

For lens A, $f = \frac{300}{17}$ cm, $\mu = 1{\cdot}614$, $R_1 = \infty$, and $R_2 = r_1$, so that

$$\frac{17}{300} = (1{\cdot}614 - 1)\left(\frac{1}{\infty} - \frac{1}{r_1}\right)$$

$$= -\frac{{\cdot}614}{r_1}$$

$$\therefore \quad r_1 = -\frac{300 \times {\cdot}614}{17} = -\frac{184{\cdot}2}{17}$$

$$= -10{\cdot}84 \text{ cm.}$$

For lens B, $f = -\frac{100}{9}$ cm., $\mu = 1{\cdot}522$, $R_1 = r_1 = -\frac{184{\cdot}2}{17}$ cm., and $R_2 = r_2$. Putting these values in the lens formula, we get

$$-\frac{9}{100} = \left(1{\cdot}522 - 1\right)\left(-\frac{17}{184{\cdot}2} - \frac{1}{r_2}\right)$$

$$= {\cdot}522\left(-\frac{17}{184{\cdot}2} - \frac{1}{r_2}\right)$$

or $$\frac{17}{184{\cdot}2} + \frac{1}{r_2} = \frac{9}{52{\cdot}2}$$

or $$\frac{1}{r_2} = \frac{9}{52{\cdot}2} - \frac{17}{184{\cdot}2}$$

$$= \frac{184{\cdot}2 \times 9 - 52{\cdot}2 \times 17}{52{\cdot}2 \times 184{\cdot}2}$$

$$\therefore \quad r_2 = \frac{52{\cdot}2 \times 184{\cdot}2}{184{\cdot}2 \times 9 - 52{\cdot}2 \times 17}$$

$$= 12{\cdot}47 \text{ cm.}$$

Q. 64. Describe with necessary diagrams the optical parts of an astronomical telescope which is fitted with a micrometer eye-piece. How can you use it to measure the angular distance between stars? In what respects does an opera glass differ from an astronomical telescope? (*Calcutta, 1931*)

Ans. In an astronomical telescope an objective of *long* focal length is fitted at one end of a tube in the other end of which can slide a smaller tube carrying an eye-piece of *short* focal length, and cross-wires in front of it (Fig. 50). The objective consists of a crown glass convex lens placed in contact with a flint glass concave lens of appropriate focal length so that the combination is *achromatic* and *convex*. The eye-piece consists of two plano-convex lenses of *equal* focal length and the *same* kind of glass, placed apart at a distance equal to two-third the focal length of either and their convex faces towards each other. By this arrangement spherical and chromatic aberrations are reduced. For avoiding chromatic aberration altogether, each lens is made an achromatic doublet, and their faces are made of proper radii to minimise spherical aberration. The inner lens F is called field lens, and the second lens E is called eye lens.

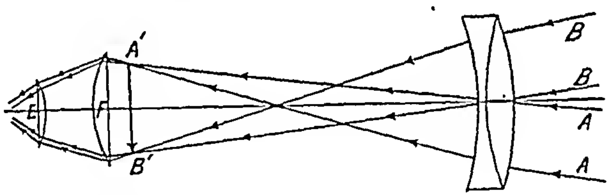


Fig. 50.

The rays of light coming from the top B of a *very distant* object AB are *parallel* to one another, but after passing through the objective become convergent. The lower ray passes through the optical centre of the objective and passes out without any deviation, while the upper ray is turned by the objective toward its principal axis and meets the first ray at B', in the second principal focal plane of the objective. Therefore B' is the real image of B. Then these rays pass through the field lens; they are turned towards its principal axis so that they appear to come from a point in the first

principal *focal plane* of the eye lens E, and, therefore, on passing through lens E become parallel to one another. For this to be possible, the distance of $A'B'$ from F should be equal to one-fourth the focal length of either the field or eye lens. Similarly, the real image of A is formed at A', and $A'B'$ is the real image of AB. The final image seen through the eye-piece is at infinity.

The angular distance between two stars can be measured with this telescope if they are not wide apart and both of them can be seen *at the same time* with it. By moving the micrometer screw the point of intersection of the cross-wires is made to coincide first with the centre of the image of one star and then with that of the other. The distance between the two positions divided by the focal length of the objective gives the angle in *radian* subtended by the two images at the objective, and is equal to the angular distance between the two stars.

An opera glass consists of *two* small telescopes, one used for each eye of the observer and thus *stereoscopic* effect is obtained. The eye-piece of each telescope consists of a *concave* lens; it is placed *between* the objective and its second principal focus, and, therefore, an *erect* image is seen. By using a concave lens the length of the telescope is *shortened* but the field of view is very much *decreased*. Moreover, an opera glass cannot be used for *quantitative work*.

Q. 65. Define the magnifying power of a telescope. Give a neat diagram of an astronomical telescope, showing the paths of the principal rays. How is its magnifying power for infinity practically determined?
(*Calcutta, 1935*)

Ans. See Q. 64 for a diagram of an astronomical telescope and the paths of the principal rays forming image. *The magnifying power of a telescope is equal to the ratio of the angle that the image seen through it subtends at the eye of the observer to the angle which the object subtends at the eye when seen directly.* When the object is at a very great distance, the objective of the telescope forms its real image at its second principal *focus*. The magnifying power is equal to the focal length F of the objective divided by the focal length

f of the eye-piece, and the distance between the objective and the eye-piece is equal to $F+f$ (*numerically*). The telescope is focussed for infinity, and then its objective is replaced by a disc having a vertical rectangular slit of length O. The eye-piece forms a real diminished image of this slit, whose length I is measured with a travelling microscope, fitted with cross-wires.

If u is the distance of the siit in front of the eye-piece and v the distance of the image from it,

$$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f},$$

as u is positive and f is negative.

$$\text{or} \qquad \frac{1}{v}=\frac{1}{u}-\frac{1}{f}$$

$$=\frac{1}{(F+f)}-\frac{1}{f} \qquad [\because\ u=F+f$$

$$=\frac{-F}{f(F+f)}$$

$$\text{But} \quad \frac{\text{Height of object}}{\text{Height of image}}=\frac{O}{I}=\frac{u}{v}=\frac{F+f}{v}$$

$$=\frac{-F}{f} \qquad [\text{from last equation.}$$

$$\therefore \text{Magnifying power}=\frac{F}{f}=\frac{O}{I}$$

$$=\frac{\text{diameter of objective}}{\text{diameter of its real image formed by eye-piece}}$$

Q. 66. Explain Foucault's method of measuring the velocity of light. How did it help the wave theory of light to get firmly established? (*Punjab, 1931*)

Ans. **Foucault's Method.** Light from a narrow illuminated slit S, with a fine platinum wire stretched across it, falls on a glass plate G (Fig. 51) A part of it is transmitted through G, and an *achromatic* lens L makes the rays converge to S_1. In the path of these rays is placed a vertical plane mirror M which can be rotated about a vertical axis passing through O. This mirror reflects the rays and makes them meet at a point

C on a concave mirror whose centre of curvature is at O. These rays are reflected back by the concave mirror; then on

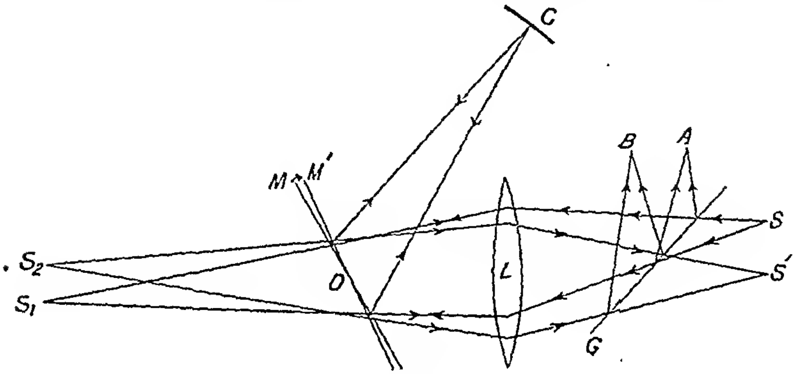


Fig. 51.

being reflected from the stationary plane mirror M they appear to come from S_1 so that OS_1 is *equal* to OC. They follow back their original course, are made convergent by the lens L, and, on being reflected from G, meet at A, which is at the *same* distance from G as S. Here the image of the platinum wire is observed with a micrometer eye-piece. The rest of light passes through G and converges to S.

When the plane mirror is rotated about a vertical axis through O, an intermittent image is seen once in each rotation when light falls on the concave mirror and is reflected back by it. With more than 16 rotations per second, due to *persistence of vision*, a *continuous* image is seen, but it is less bright than when the plane mirror is stationary. The velocity of light is so great that when the rays of light reflected from C come back to the plane mirror, they do not find it *appreciably* rotated from the position in which they left it; they are reflected back along their original course, and, therefore, the image is seen at A. With *very great* speed of rotation, the plane mirror turns through an *appreciable* angle θ into the position M' before the rays, reflected from C, return to it. Then they do not go back along their original course, but are deflected through 2θ, so that they appear to come from S_2

and the lens L makes them meet at B. Thus the image is displaced through a distance AB, which is measured with the micrometer eye-piece. The rest of light passes through G and converges to S′. The distance of S′ from G is the *same* as that of B and SS′ is equal to AB.

Calculation of Velocity. Let the plane mirror make n rotations per second or its angular velocity be equal to $2\pi n$ radians per second, SL equal to a, LO equal to b, OC or OS_1 equal to d, AB or SS′ equal to l (all measured in centimetres), and angle MOM′ equal to θ *radian*. If the velocity of light is V cm. per second, it takes $\frac{2d}{V}$ second to go from O to C and back to O, and, therefore, θ is equal to $2\pi n \times \frac{2d}{V}$ radian. As the plane mirror is turned through θ radian, the rays reflected from it are turned through 2θ radian, and S_1S_2 subtends angle 2θ radian at O, that is, S_1S_2 is equal to $d \times 2\theta$ cm. Also S and S_1, and S′ and S_2 are conjugate foci for the lens L,

$$\therefore \quad \frac{SS' = AB}{S_1S_2} = \frac{SL}{LO + OS_1}$$

or

$$\frac{l}{d \times 2\theta} = \frac{a}{b+d}$$

or

$$\frac{lV}{2d \times 4\pi nd} = \frac{a}{b+d} \qquad \left[\because \quad \theta = \frac{4\pi nd}{V}\right]$$

or

$$V = \frac{8\pi nad^2}{l(b+d)} \text{ cm./sec.}$$

Foucault placed a long glass tube full of water in the path of the rays between M and C and found the displacement of the image to be slightly *greater*. This proves that the velocity of light in water is *smaller* than in air, because the *smaller* the velocity, the *greater* the time taken by light to return to M from C, and the *greater* the angle of rotation of the plane mirror M, the *greater* the displacement of the reflected rays and the image. This result finally decided the fate of the corpuscular theory, for according to this theory the velocity of light in an optically denser medium (water) should be *greater* than in a rarer medium (air). On the other hand, the wave theory of light

indicates a *smaller* velocity in an optically denser medium than a rarer medium. Thus the corpuscular theory was finally rejected and the wave theory got firmly established.

Q. 67. Explain in brief the phenomenon of "Spherical aberration". Under what conditions is spherical aberration produced in telescopes and microscopes? Show the construction of principal types of eye-pieces that can be used to reduce it. Illustrate your answer with neat geometrical diagrams. (*Bombay, 1935*)

Ans. **Spherical Aberration.** (*a*) **Reflection.** When a spherical mirror has a *small* aperture as compared with its radius of curvature, *all* the rays of light coming from a point on its axis after reflection from its surface converge to, or appear to diverge from, a *single* point on its principal axis, called the image of the first point. With a wide aperture, the rays incident *near the pole* of the mirror still intersect, after reflection, at a single point, but the rays reflected from the *remote parts*, that is, incident rays which do not make *small* angle with the principal axis, intersect the principal axis at points nearer the pole. The more remote is the point of reflection from the pole of the mirror, the nearer to the pole is the point of intersection of the reflected rays with the principal axis. Any two rays reflected from two near points on the remote surface intersect each other before reaching the principal axis, and each such point of intersection is an image of the point object from which the rays started.

Thus the focal length of the mirror is *different* for points of incidence at different distances from the pole, and the *more remote* the point of incidence from the pole, the *smaller* is the focal length of the mirror. When an object subtends a *large* angle at a mirror, the rays coming from its outer parts make *large angles* with the principal axis, and for these *oblique* centric rays the focal length of the mirror is smaller than for centric rays. The result is that a curved and distorted image is produced.

(*b*) **Refraction.** Similar is the case for the refraction of rays through a *thick* lens of large aperture. With a thin lens of small aperture, all the rays coming from it converge to, or appear to diverge from, a single point on its principal

axis. When the lens is thick, that is, its aperture is not small as compared with the radii of curvature of its faces, or an object subtends a *large angle* at the lens, the rays coming from a point of the object, after passing through different parts of the lens converge to, or appear to diverge from, different points. The *more remote* the point of incidence from the middle of the lens, and *more oblique* the rays, the *smaller* is the focal length of the lens, and the image is curved and distorted.

In a telescope the object is at a *very great* distance; and the rays coming from it are incident at the large objective at a *very small* angle. As the refraction is centrical, there is no appreciable spherical aberration. The case of a microscope is different. Here the object is *very near* the objective, and subtends a *large* angle on it. The refraction is excentrical, and, therefore, to minimise spherical aberration, the objective is made of a large number of appropriate radii of curvature. In both the instruments the rays coming from the objective form a real image and then fall *obliquely* on the eye lens, as the real image is very near this lens. Here the refraction is excentrical and oblique, and a good deal of spherical aberration is produced. To minimise spherical and chromatic aberrations and increase the field of view an eye-piece, consisting of two lenses, of appropriate focal length and radii of curvature, and placed at a suitable distance from each other, is used.

For eye-pieces see Q. 68.

Q. 68. Explain the construction and theory of different types of compound eye-pieces, and discuss their advantages as compared with simple ones.

(*Bombay, 1926*)

Ans. The image formed by the objectives of telescopes and microscopes is observed with an eye-piece which *magnifies* it, as it is of *short* focal length and the image formed by the objective is *close* to it and subtends a *large* angle on it. If it consists of a *single* lens, a good deal of *spherical aberration* is introduced, as the refraction of the rays is *excentrical*, and *chromatic aberration* is also produced. Moreover, the rays passing through its outer parts do not enter the eye of the observer, and thus the field of view is *narrow*. A

compound eye-piece consists of two convex lenses placed apart. The first lens on the objective side is called **field-lens**, and the second lens on the side of the observer is called **eye-lens.** The rays of light passing through the outer parts of the field lens are turned by it *towards* its principal axis and made to pass through the middle of the eye-lens. Thus all the rays coming from different points of the object enter the eye of the observer at the same time and the field of view is *increased.*

Spherical aberration is decreased as the deviation is now spread over four refracting surfaces, and, therefore, it is smaller on each surface than when a single lens is used. It is further decreased and minimised by using lenses of appropriate radii of curvature and making the deviation at the two lenses *equal* by adjusting the distance between them. A ray of light parallel to the principal axis of the field lens *F* (Fig. 52), of focal length f_1 is deviated at A through angle θ

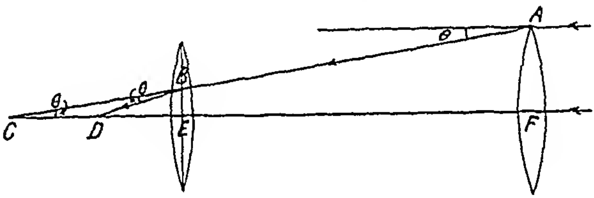


Fig. 52.

and converges to C. At B it is again deviated by the eye-lens *E*, of focal length f_2, by the same amount and then passes through D. As the deviation produced by a lens is equal to the distance of the point of incidence from its principal axis divided by its focal length, if AF and BE are equal to h_1 and h_2 respectively and EF is equal to d,

$$\frac{h_1}{f_1}=\frac{h_2}{f_2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

and as triangles AFC and BEC are similar,

$$\frac{AF=h_1}{CF=f_1}=\frac{BE=h_2}{EC=f_1-d} \quad . \; . \; . \; . \; . \; . \quad (2)$$

Equating the right sides of (1) and (2),

$$\frac{h_2}{f_2}=\frac{h_2}{f_1-d}$$

$$f_2=f_1-d$$

$$\therefore\ d=f_1-f_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The two lenses are of the *same* kind of glass, and the condition for achromatism is (see Q. 57) that

$$f_1+f_2+2d=0$$

or

$$f_1+f_2=2d \text{ (numerically)}. \quad . \quad . \quad . \quad (4)$$

Combining this with (3), we get

$$f_1+f_2=2(f_1-f_2)$$

$$\therefore\ f_1=3f_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus the focal length of the field lens should be three times the focal length of the eye-lens, and the distance between them should be equal to the difference between their focal lengths.

Huygens' Eye-piece. The above arrangement is used in this eye-piece, and two plano convex lenses are placed with their convex faces towards the incident rays (Fig. 53). In the

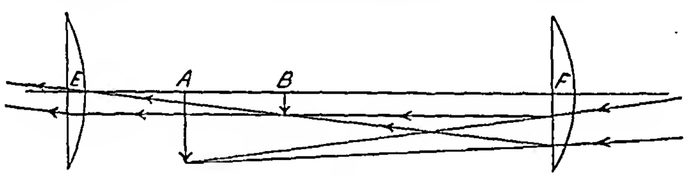


Fig. 53.

absence of the field lens F, rays coming from the objective would form the image at A, but this lens deviates them towards its principal axis, and the image is formed at B. After passing through the eye-lens E, the rays become *parallel* to one another ; therefore, B must be at the first principal focus of lens E.

This eye-piece is called *negative*, because the image at A formed by the objective is on the negative side of the field lens F and is not real. Cross-wires or a micrometer scale (in the focal plane of the objective) cannot be used in this eye-piece, and, therefore, it is not employed in telescopes. No

cross-wires can be placed at B for the points of this image do not necessarily have the same relative position as the corresponding points of the virtual image at A. The cross-wires (or scale) between E and F are magnified by lens E only, while the image is magnified by lens F also. As greater magnification leads to greater distortion, no cross-wires are used.

The rays of different colours for which this eye-piece is achromatic suffer equal deviations, and on emerging from the eye-lens are *parallel* to one another, but are *not necessarily coincident*. As the eye treats all parallel rays as coincident, this eye-piece is practically achromatic. Moreover, as the dispersive power of the two lenses is the same, the eye-piece is equally achromatic for *all colours*. A ray incident on the field lens is dispersed—violet being deviated more towards its principal axis than red. At the eye-lens red ray is incident at a *greater* distance from its principal axis, and suffers greater deviation, than violet, and the two emerge from lens E parallel to one another.

Kellner's Eye-piece. The two plano-convex lenses are of *equal* focal lengths, placed with their convex faces towards the incident light, and each is at the principal focus of the other, so that the condition for achromatism is satisfied. The objective forms an image at the field lens on the side of the objective. The pencils of rays from this image are deviated by the field lens towards its principal axis so that pencils of rays coming from the different points can simultaneously enter the eye of the observer. By using two lenses and spreading deviation over four surfaces, spherical aberration is decreased. Like a simple magnifying glass, this eye-piece is achromatic in the sense that red and violet images subtend the *same angle* on the eye of the observer. Its disadvantage is that all smears, scratches, or dust particles on the field lens are also visible and the image becomes indistinct.

Ramsden's Eye-piece. In this combination the two plano-convex lenses are of *equal* focal lengths f, and their convex faces are *towards* each other (Fig. 54). For perfect achromatism the distance between them should be equal to $\frac{f+f}{2}=f$; but in this case the field lens would be at the

principal focus of the eye-lens, and this would lead to indistinctness especially if the field lens is not clean, or has some dust particles, smears, or scratches on its surface. As a compromise, the distance between the lenses is made equal to $\frac{2}{3}f$. This introduces some chromatic aberration though the image is not indistinct; and to avoid it each lens may be made an achromatic doublet. Spherical aberration is minimised by a proper selection of the radii of the faces of the lenses.

The objective forms a real and inverted image at A. The

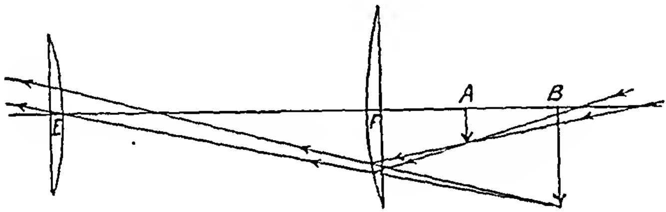


Fig. 54.

rays then pass through F and appear to come from the *focal* plane B of lens E, so that after passing through E they may become *parallel* to one another. Thus the field lens forms a virtual image at B of the object (real image) at A. As EF is equal to $\frac{2}{3}f$, FB is equal to $\frac{f}{3}$. Using lens formula,

$$\frac{1}{FB}-\frac{1}{FA}=-\frac{1}{f}$$

or

$$\frac{1}{FA}=\frac{1}{f}+\frac{3}{f}=\frac{4}{f}$$

or

$$FA=\frac{f}{4}$$

Hence the cross-wires should be placed at a distance $\frac{f}{4}$ in *front* of the field lens. The eye-piece magnifies *equally* both the image at A and the cross-wires or transparent micrometer scale placed there, and, therefore, it can be easily focussed and there is very little distortion in the image.

Q. 69. Calculate the focal length of a Ramsden's and a Huygens' eye-piece. Which one of them is more achromatic? (*Punjab, 1938*)

Ans. See Q. 68 for Ramsden's and Huygens' eye-pieces. The condition for an achromatic combination of two lenses, of the same kind of glass and of focal lengths f_1 and f_2, is that the distance between them should be equal to $\frac{f_1+f_2}{2}$. In Huygen's eye piece this condition is satisfied as the focal length of its field lens is three times the focal length f of its eye-lens and the distance between them is equal to $\frac{3f+f}{2} = 2f$. In Ramsden's eye-piece the two lenses are of equal focal lengths f, but the distance between them is equal to $\frac{2}{3}f$ and not f, and thus the condition for achromatism is not satisfied. Therefore *Huygens' eye-piece is more achromatic than Ramsden's.*

When two lenses of focal lengths f_1 and f_2 are separated by a distance d, the focal length f of an equivalent lens is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2}$$

or
$$f = \frac{f_1 f_2}{f_1 + f_2 + d},$$

where f_1 and f_2 are to be given their *proper* signs.

(*a*) Huygen's Eye-piece.

Focal length of convex eye lens $= -f$

" " " " field lens $= -3f$

Distance d between them $= 2f$

$\therefore$ Focal length of eye-piece $= \frac{(-f)\times(-3f)}{-f-3f+2f}$

$$= -\frac{3f}{2}$$

(*b*) Ramsden's Eye-piece.

Focal length of convex eye lens $= -f$

" " " " field lens $= -f$

Distance d between them $= \frac{2}{3}f$

$$\therefore \text{ Focal length of eye-piece} = \frac{(-f)\times(-3f)}{-f-f+\frac{2f}{3}}$$

$$= -\frac{3f}{4}.$$

Q. 70. What are the different types of spectra? Explain how they are produced. What are Spectrum lines? How would you deduce the constitution of a star from its spectrum? (*Punjab, 1930*)

Ans. When a substance is heated in a flame, or an electric arc is produced between its electrodes, or an electric spark is passed through it, its atoms *acquire energy* by collision with the hotter atoms of the flame, or the fast moving ions in the electric field or spark, and become *excited*. Their electrons are disturbed and displaced from their normal orbits to the *outer* orbits of *greater* energy, and when they return to their original, or intermediate, orbits, energy is given out in the form of radiation. There are three types of spectra : (*a*) continuous spectra, (*b*) band (or fluted) spectra, and (*c*) line spectra. The same substance may give different spectra under different conditions.

Continuous Spectra. When the temperature of a solid or a liquid (with few exceptions) is raised, radiant energy corresponding to every wave-length is given out. The intensity is maximum for some region, which shifts towards *violet* with rise of temperature, and decreases on its two sides. Continuous spectra may be produced by both atoms and molecules, but do not commonly occur in the case of vapours and gases mildly excited at normal or low pressure. High pressure of a gas, or high density of a vapour, helps in the emission of continuous spectra.

Discontinuous Spectra. The substance is first obtained in the gaseous condition, and then excited by flame, arc, or spark. **Line spectra** are produced by *atoms*, while *molecules* are responsible for **band spectra.** Fluted or band spectra consist of a number of bands, sharp at one end and gradually fading off at the other. When examined with a spectrometer of high resolving power, each band is found to consist of a large number of lines of varying intensities, *close* together near the

sharp end and comparatively *far apart* near the other. All the three spectra may be found together. Thus in the spectrum of carbon arc lamp, continuous spectrum due to electrodes, line spectra due to metallic impurities and band spectra in the violet due to nitrogen are found. The characteristic colour of the arc is due to the band spectra.

Line spectra consist of sharp lines. Each element gives its own *characteristic* spectrum lines, whether in combination or not, and the wave-length of each line is *independent* of temperature and chemical composition. The spectrum lines of an element can be arranged in **series** given by definite formulae. In the case of atomic hydrogen, one of these series is called **Balmer** series, in which the frequency n of a line is given by

$$n = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right).$$

where R is the **Rydberg constant** and $m = 3, 4, 5, 6$ for the four lines in the visible spectrum. Lines in the ultra-violet region are given by Lyman's series,

$$n = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right), \quad m = 2, 3, 4, \ldots\ldots$$

and Paschen's series in the infra-red part of the spectrum is expressed by

$$n = R\left(\frac{1}{3^2} - \frac{1}{m^2}\right), \quad m = 4, 5, 6, \ldots\ldots$$

In a series intensity decreases with decreasing wave-length or increasing frequency; the lines converge and become less defined. These series show that there is something fundamental in the hydrogen atom which is responsible for this radiation. For other elements, the formulae are similar but much more complex. The spectrum of an ionised atom is different from that of the normal atom, and may resemble that of the normal atoms of another element. The spectra of the elements of the same chemical family are similar, but with increasing *atomic number* each line is displaced towards the violet part of the spectrum. All this shows that *atoms* are responsible for the line spectra and that they are produced when the outer electrons are disturbed.

See. Q. 72 for absorption spectra.

Spectrum and Constitution of a Star. When different elements are heated to the luminous vapour condition, they

give out light of *definite* frequencies characteristic of each element. If continuous spectrum light, emitted by a body at a *higher* temperature, is passed through a vapour, it is robbed of lines which the vapour may emit when at a high temperature. A star is at a high temperature, and the spectrum of light coming from it is crossed by a large number of absorption lines, which correspond to the frequencies of light given out when different elements are heated to become luminous vapour. Therefore the light from the star must have passed through a cloud of such vapours in its atmosphere or in the atmosphere of the earth. As the atmosphere of the earth contains very few elements in the vapour condition, most of the absorbing vapours must be present in the star. When its spectrum is compared with the spectra emitted by various elements heated to luminous vapour condition, it becomes possible to identify the elements present in the star, and thus its constitution is found out. All the elements that are found on the earth are present in the stars also. A new element helium was first found in the atmosphere of sun and later on discovered on the earth.

Q. 71. Describe in detail the methods of investigating the infra-red and the ultra-violet spectra.

(*Bombay, 1934*)

Ans. **Ultra-Violet and Infra-Red Spectra.** The **visible** part of the spectrum extends from about 4000 to 7500° Angström units, its violet and red ends respectively. The region below the violet is called **ultra-violet** and lies between about 136 and 4000° A.U., while the part of the spectrum comprised between 7500 and 400,000° A.U. (or ·75—40μ) is called **infra-red.** Both these parts of the spectrum do not excite the sensation of sight, and, therefore, for detecting them other methods are employed. As glass is mostly opaque to these radiations, lenses and prisms of other materials are used in the spectrometer for forming spectrum. For the ultra-violet region, glass is opaque below 3000° A.U., quartz is used upto 1900° A.U., and fluorite (calcium fluoride) upto 1200° A.U. Glass is not transparent beyond 3μ in the infra-red part. Quartz is used upto 6·5μ, rock salt upto 18μ, sylvine (potassium chloride) upto 22μ, and again quartz is used for higher wave-lengths. Quartz produces *double refraction*, but this difficulty is overcome by holding together two right-angled 30° prisms, one *left-handed*

and second *right-handed*, with a thin layer of glycerine between them, to form a 60° prism. These prisms and lenses must be kept safe from moisture.

When a blackened thermometer bulb is held in the infra-red part of the spectrum, it indicates rise of temperature. This shows that the radiation is absorbed by the black surface and converted into heat. The instruments used for investigating this part of the spectrum are thermopile, radio-micrometer, and bolometer. A bolometer consists of two blackened strips of platinum, about 1 cm. long, ·1 mm. wide, and ·005 mm. thick, connected to form two arms of a Wheatstone bridge. The other two resistances of the bridge are adjusted so that the galvanometer shows *no* deflection when the two platinum strips are at the *same* temperature. The galvanometer is *very sensitive*, and a ray of light reflected from its mirror, attached to its deflecting part, falls on a photographic plate.

The radiation is made to fall on the narrow slit of a collimeter and made parallel by a lens. It is then dispersed by a prism, placed with its refracting edge *parallel* to the slit, and focussed by another lens on *one* strip of the bolometer, the other strip being screened. The exposed strip of the bolometer is *parallel* to the refracting edge of the prism and the slit. Owing to the absorption of radiation and the consequent rise of temperature, the resistance of the exposed bolometer wire is *increased* and the equilibrium of the bridge is disturbed; the ray of light reflected from the galvanometer is displaced and falls at a different point on the photographic plate. The prism is rotated slowly by a clock-work about an axis parallel to its refracting edge, so that different parts of the spectrum are in turn focussed on the bolometer strip, and the photographic plate is rotated in a perpendicular plane by the same clock-work. In this way a curve is produced on the photographic plate in which the displacement of the ray of light reflected from the galvanometer is proportional to the energy of the radiation falling on the bolometer strip.

The ultra-violet rays produce fluorescence in some substances, and to detect them a filter paper soaked in quinine sulphate solution, slightly acidulated with sulphuric acid, and dried may be used. When they fall on a negatively charged zinc plate,

electrons are given out, and this photo-electric effect is measured with a quadrant electrometer or some other sensitive electroscope. These rays are particularly active in producing chemical action in silver salts, and, therefore, their action on a photographic plate is usually used for investigating this part of the spectrum. The dispersed radiation is focussed by a lens on a specially prepared photographic plate, which is rotated along with the prism as in the case of infra-red spectra. As some of these rays are absorbed by the atmospheric air to some extent, the experiment is usually conducted in vacuum.

Q. 72. (*a*) Give a general explanation of Fraunhofer's lines in the solar spectrum and describe an experiment to verify the explanation.

(*b*) If the earth were moving rapidly through space, what would be the general effect on the spectra of stars which it was (*i*) approaching; (*ii*) receding from? Give reasons for your answer. (*Bombay, 1935*)

Ans. (*a*) **Absorption Spectra.** When continuous spectrum light passes through a transparent solid or liquid, or a gas, some of the possible emission lines of the substance are absorbed, due to absorption of energy. The transmitted light is found to consist of some lines or bands which are *dark as compared with* the rest of the spectrum. These lines and bands are called **absorption spectrum** and are characteristic of the absorbing substance though they change considerably with temperature and pressure. According to Kirchoff a substance which emits light of definite wave-length when heated absorbs selectively light of the *same* wave-length when *cold*. The absorbing substance also emits its characteristic light, but as its temperature is *lower* than that of the source of continuous spectrum light, the intensity of light absorbed by it is *greater* than that of light emitted by it, and, therefore, in the transmitted light the intensity of these characteristic lines is feeble, and they appear dark, as compared with the unabsorbed parts of the spectrum. Nearly all the lines of absorption spectra of a substance can be emitted under suitable conditions of excitation. The converse is not true, for only those lines are absorbed which correspond to the transition from various orbits to the *normal* orbit.

The sun is at a very high temperature, and its central incandescent mass emits light of continuous spectrum. The temperature of its atmosphere is comparatively *lower*, but high enough to have substances in the gas condition. The light emitted by the central incandescent mass on passing through the comparatively colder atmosphere of the sun is robbed of light of certain particular wave-lengths characteristic of the elements present there. The transmitted light when examined with a spectrometer is found to consist of many lines which are dark as compared with the rest of the spectrum. These absorption lines are called **Fraunhofer's lines,** and are due to light absorbed selectively by substances present in the atmosphere of the sun or the earth.

Experiment. The light given out by a sodium flame consists of two lines in the yellow part of the spectrum. If a solid is heated in an arc lamp to emit light of continuous spectrum, and this light is passed through sodium vapour (sodium flame of an ordinary burner) at a *lower* temperature, the transmitted light is found to have two lines in the yellow part of the spectrum which are *darker as compared* with the rest of the spectrum. These comparatively dark lines are exactly in the *same* position as those given out by a sodium flame.

Doppler's Principle. When there is no relative motion between an observer and a source of light, the rate at which any particular spectrum line is received by the observer is the *same* as that at which it is produced by the source. The interval between the receiving of two consecutive waves is *equal* to the interval between their production at the source, and the frequency of reception is *equal* to the frequency of emission. Any relative motion between them increases or decreases the frequency of reception according as the distance between them decreases or increases. This is called **Doppler's Principle** and applies to all kinds of wave motion.

If the earth *approaches* a star with a velocity v, and V is the velocity of light in the intervening medium, an observer on the earth receives light waves of a particular wave-length *more quickly* than if there is no relative motion,

$$\text{Frequency of emission} = n$$

Wave-length of light emitted $= \dfrac{V}{n}$

Velocity of light with respect to the *observer*

$$= V + v$$

$\therefore$ Frequency of reception of light

$$= \frac{\text{Velocity of light with respect to the observer}}{\text{Wave-length of light}}$$

$$= \frac{(V+v)n}{V}.$$

As the frequency of reception is *increased*, the spectrum line is *displaced towards the violet part of the spectrum.*

When the earth is *receding* from the star, the interval between the reception of any two consecutive waves is *greater* than if there is no relative motion, and the frequency of reception of light waves decreases. In this case the velocity of light with respect to the observer is $V - v$; the frequency of reception is decreased to $\dfrac{(V-v)n}{V}$, and the spectrum lines are displaced towards the *red* part of the spectrum.

Q. 73. How is the rectilinear propagation of light explained on the wave theory ? Calculate the velocity of light in diamond, the refractive index from air to diamond being 2·5. (*Punjab, 1936*)

Ans. **Rectilinear Propagation of Light.** According to the wave theory, light is propagated as a transverse wave motion in the ether. A point source of light gives out spherical waves in a homogeneous medium, and each particle of ether on being disturbed by a wave acts as a new source and gives out secondary waves. After any time the envelope of these secondary waves gives the position of the wave front at that instant.

A wave front coming from a very *distant* point source of light is *plane*. Let ABCD be a part of the plane wave front

perpendicular to the plane of paper and going from left to right, and O be a point in the plane of paper where the resultant effect of the secondary disturbances produced by the points in this wave front is to be found (Fig. 55). Draw OP perpendicular to the wave front, and let this distance be equal to d. With O as centre draw spheres of radii $d+\frac{\lambda}{2}$, $d+\lambda$, $d+\frac{3\lambda}{2}$,......,

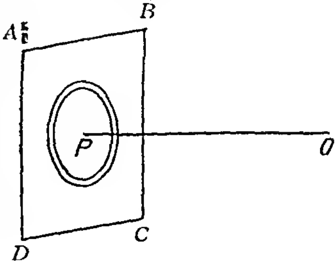


Fig 55.

where λ is the wave-length of light. These spheres cut the wave front in circles and divide it into a series of rings or half period zones.

$$\text{Outer radius of } n\text{th zone} = \sqrt{\left(d+\frac{n\lambda}{2}\right)^2 - d^2} = \sqrt{n\lambda d},$$

as λ is *very small*, and $\left(\frac{n\lambda}{2}\right)^2$ is negligible.

$$\text{Inner radius of } n\text{th zone} = \sqrt{\left\{d+\frac{(n-1)\lambda}{2}\right\}^2 - d^2}$$
$$= \sqrt{(n-1)\lambda d}$$
$$\therefore \text{ Area of } n\text{th zone} = \pi n\lambda d - \pi(n-1)\lambda d$$
$$= \pi\lambda d$$

This shows that each half period element has practically the *same* area, and, therefore, each gives out the same amount of light energy. All the points in this wave front are in the same phase, but as their distances from O are different, the secondary disturbances from them arrive at O in *different* phases. With increase in the distance of the outer zones from O, the inclination of the wave front to the line joining it with O also increases, and, therefore, the disturbances at O due to these zones are in *descending* order and almost form an arithmetical progression.

The distance of O from the points of the first half period element varies from d to $d+\frac{\lambda}{2}$, and from the second zone

from $d+\frac{\lambda}{2}$ to $d+\lambda$. As the average path difference between any two consecutive half period elements from O is equal to $\frac{\lambda}{2}$, the secondary disturbances from them at O are in *opposite* phases, and denoting these by $a_1, a_2, \ldots\ldots$, the resultant disturbance A at O is given by

$$A = a_1 - a_2 + a_3 - a_4 + \ldots\ldots + a_n$$

$$= \frac{a_1}{2} + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2}\right) + \ldots\ldots$$

$$+ \left(\frac{a_{n-2}}{2} - a_{n-1} + \frac{a_n}{2}\right) + \frac{a_n}{2}.$$

$$= \frac{a_1}{2} + \frac{a_n}{2}$$

If the wave front is large, n is very large, and a_n is *very small* as compared with a_1, so that the resultant disturbance at O is practically equal to *half* of the disturbance due to the first half period zone alone, and the intensity of light at O is one quarter of the intensity due to the first zone, whose area is very small. If an obstacle is placed at P, the resultant disturbance at O is equal to half the disturbance due to the *first exposed* zone. As even a small obstacle at P covers a *very large number* of zones, the disturbance due to the first exposed zone is almost negligible, and, therefore, no light energy is received at O. This result is the same as if light travels in *straight* lines.

Velocity of light in diamond. The refractive index of a medium A with respect to a medium B is equal to the velocity of light in B divided by the velocity of light in A. The refractive index of diamond is 2·5 with respect to vacuum.

Velocity of light in vacuum $= 3 \times 10^{10}$ cms. per sec.

$$\text{Refractive index of diamond} = \frac{\text{Velocity of light in vaccum.}}{\text{Velocity of light in diamond.}}$$

$$\therefore \text{ Velocity of light in diamond} = \frac{\text{Velocity of light in vacuum}}{\text{Ref. index of diamond.}}$$
$$= \frac{3 \times 10^{10}}{2 \cdot 5}$$
$$= 1 \cdot 2 \times 10^{10} \text{ cms. per sec.}$$

Q. 74. (*a*) Clearly explain Huygen's principle and deduce the law of refraction from it. What is meant by refractive index and how is it related to the velocities of light in different media?

(*Calcutta, 1935*)

(*b*) Prove on the wave theory of light that the angle of incidence is equal to the angle of reflection.

(*Calcutta, 1930*)

Ans. (*a*). **Huygens' Principle.** According to Huygens light consists of transverse waves in the ether. A point source of light gives out spherical waves in a homogeneous medium, and they travel with a uniform velocity in all directions. In any position of the wave front all the ether particles in it become new sources of secondary disturbances which travel out with velocity V. After a time t these secondary disturbances are at a distance Vt from their respective centres, and the bounding surface of all these secondary wavelets is the wave front at that instant. The illumination at any point is due to the combined effect of all the secondary disturbances. A ray indicates the direction of propagation of light, and at any point it is normal to the wave front there. At a *great* distance from the source, the radius of curvature of the wave is very large, and it is practically a *plane* wave. In this condition the rays are *parallel* to one another. The velocity of light is *greater* in an optically rarer medium than a denser medium, and this change in velocity is the cause of refraction of light.

Refraction of light at a plane surface. XY is a section of the plane refracting surface, and AC of the plane wave front, both being perpendicular to the plane of paper (Fig. 56). The refracting surface separates the upper optically rarer medium of refractive index μ_1 from the lower denser medium of refractive index μ_2. The velocity of light

in the upper and lower media is equal to V_1 and V_2 respectively. When the wave front strikes the refracting surface at A, secondary waves are given out from there both in the upper and lower media. For refraction we consider

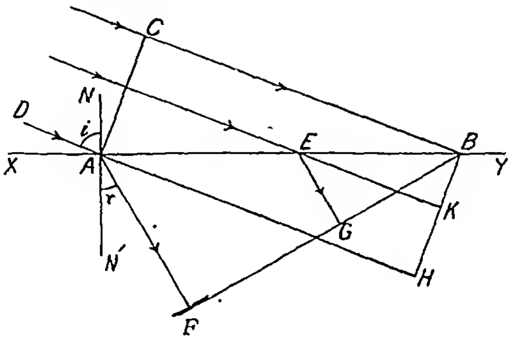


Fig. 56.

the waves in the lower medium only. Similarly, the other points of the refracting surface become in turn sources of secondary waves when the incident wave front reaches them. In this figure the wave front is just incident at B, and a secondary wave is just starting from there in the lower medium.

Had the refracting surface been absent, the wave front would have occupied the position BKH. Owing to the presence of the refracting surface, the secondary wave from A has travelled the distance AF equal to V_2t in time t, while the incident wave has travelled the distance CB equal to V_1t in the same time. From B draw a tangent plane touching the secondary wave from A at F.

In the right-angled triangles ABH and ABF,

$$\frac{AF}{AH}=\frac{V_2t}{V_1t}=\frac{V_2}{V_1} \quad \ldots \ldots \quad (1)$$

From E draw EG perpendicular on BF.

$$\frac{EG}{AF}=\frac{EB}{AB}=\frac{EK}{AH}$$

$$\therefore \quad \frac{EG}{EK}=\frac{AF}{AH}=\frac{V_2}{V_1}, \text{ from (1)}$$

or

$$EG=\frac{EK}{V_1}\cdot V_2$$

But the distance travelled by the secondary wave from E is equal to this value of EG, therefore the tangent plane through B and F touches the secondary wave from E at G. Similarly, it can be proved that all the secondary waves coming from points between A and B are touched by this tangent plane, that is, BGF is a section of the *plane* refracted wave front. Therefore a plane wave is refracted as a *plane* wave from a plane refracting surface.

Draw NAN′ perpendicular to the refracting surface at A. Angle DAN is the angle of incidence, and is equal to the opposite angle N′AH. Angle BAH is its complementary angle. Angle N′AF is the angle of refraction and ∠ BAF is its complementary angle.

$$AH = AB \cos \angle BAH = AB \sin i$$
$$AF = AB \cos \angle BAF = AB \sin r$$
$$\therefore \frac{\sin i}{\sin r} = \frac{AH}{AF} = \frac{V_1}{V_2}, \text{ from (1)}$$
$$= \text{Constant.}$$

Thus the ratio of the sine of the angle of incidence and the sine of the angle of refraction is *constant* and is equal to the velocity of light in the first medium divided by the velocity of light in the second medium. As already shown the refracted ray lies in the *same* plane as the incident ray and the normal at the point of incidence. This constant ratio is called the refractive index of the lower medium with respect to the upper medium and is equal to μ_2/μ_1, or V_1/V_2.

Refraction at a plane surface. Let XY be the section of a plane reflecting surface and AC that of a plane wave front, both being perpendicular to the plane of paper (Fig. 57). When the wave front strikes the reflecting surface at A, a secondary wave is given out from there. Similarly, other points of the reflecting surface become in turn sources of secondary waves when the incident wave strikes them. In Fig. 57 the upper end C of the wave front is just striking B, and a

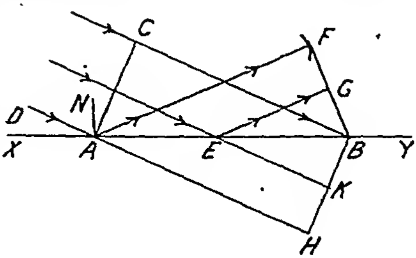


Fig. 57.

secondary wave is just starting from there. Had the reflecting surface been absent, the wave front would have occupied the position HKB. Owing to the presence of the reflecting surface, the secondary wave from A has travelled in the upper medium a distance equal to AH.

With A as centre draw an arc of radius equal to AH, and from B draw a tangent plane to touch this at F. The right-angled triangles ABC, ABF, and ABH are equal, as CB = AH = AF, and AB is common. If EG is drawn perpendicular on FG, in the triangles AFB and AHB, EG is equal to EK, and, therefore, the secondary wave given out from E touches the tangent plane passing through B and F. Similarly, all the secondary waves given out by the reflecting surface touch this tangent plane at this instant. Thus a plane wave is reflected as a *plane* wave from a plane reflecting surface.

$$\angle FAB = \angle CBA = \angle DAX \quad [\because DA \parallel CB$$

Therefore, if AN is the normal at A, the complementary angle DAN of $\angle$ DAX is equal to the complementary angle NAC of $\angle$ CAB. Thus the angles of incidence and reflection are *equal*, and the incident ray DA, the reflected ray AF, and the normal AN at the point of incidence lie in the *same* plane.

Q. 75. Describe and explain clearly how interference fringes are formed and derive a formula connecting their width with the wave length of the light used.

Ans. When two sources give out waves simultaneously, their combined effect at a point depends on their relative phases there, and the distribution of energy due to one is disturbed by the other. At some points the two waves arrive in phase and strengthen each other, while at some other points their effects are opposite. Points at the crests or troughs of two waves have the greatest energy, and points at the crest of one and the trough of the other have the least. This redistribution of energy is called **interference.**

The two sources should emit continuously waves of the same period and same phase or *constant difference of phase*, as what we observe at a point is the result of billions of waves that pass through it. No two parts of the same source give out light in the same phase, and much less can this be expected from two separate sources. Two *coherent* sources

are obtained by using one source and making a part of light travel one path and another part of light some other path. Usually both the sources are images of the same source or one is the image of the other. Light from the original source changes constantly in phase, but as both the parts of light suffer the same change of phase their difference of phase remains the *same*. Different parts of a broad source give positions of maximum reinforcement at different places, and their superposition leads to uniform illumination. To avoid this the two sources should be *very narrow*.

Light coming from a very narrow slit S passes through two parallel and equally narrow slits S_1 and S_2 and spreads out

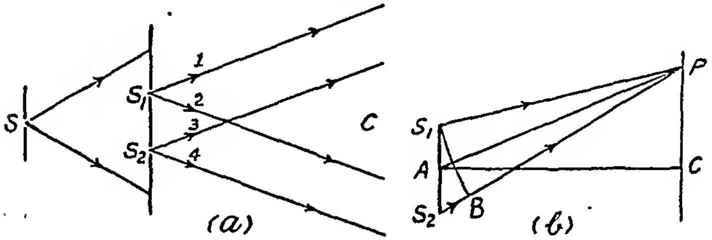


Fig. 58.

[Fig. 58 (*a*)]. The first slit S_1 gives out light between the rays 1 and 2, and light from the second slit S_2 is confined between rays 3 and 4 in the plane of paper. Light from one slit is superposed on the light from the other slit in the *common* region C between rays 2 and 3, after they have crossed, and here interference fringes are formed with alternate bright and dark bands perpendicular to the plane of paper.

A screen CP is placed perpendicular to the plane of paper and parallel to S_1S_2 [Fig. 58 (*b*)]. A is the middle point of S_1S_2 and AC is drawn perpendicular to S_1S_2 and CP. As C is equidistant from S_1 and S_2, the waves from the two sources arrive there in the same phase, and reinforce each other, so that C is the central bright band. As we move outward from C, the distances from S_1 and S_2 become unequal; the waves get out of step, and arrive there with some phase difference. At a certain point the path difference is equal to half the wave-length λ of the monochromatic light used. Here the

waves arrive in *opposite* phases and darkness is produced. This is the first dark band. At an outer point the path difference is equal to λ so that the waves arrive in phase. They completely reinforce each other and the first bright band is formed here.

To find the condition of some other point P, join it with S_1, S_2 and A, and with P as centre draw arc S_1B of radius equal to PS_1. To obtain the fringes appreciably apart, the sources are taken *very near* each other, and, therefore, S_1B is practically a straight line. Thus S_1BS_2 is a right-angled triangle and similar to the right-angled triangle ACP, as the angle between S_1S_2 and S_1B is equal to the angle between their respective normals AC and AP.

$$\therefore \quad \frac{CP}{AP}=\frac{S_2B}{S_1S_2}$$

or

$$CP=\frac{AP}{S_1S_2}\times S_2B$$

$$=\frac{AC}{S_1S_2}\times S_2B,$$

as CP is very small as compared with AC and, therefore, AC is practically equal to AP. P is on a bright band or a dark band according as the path difference BS_2 is equal to an *even* multiple or *odd* multiple of $\frac{\lambda}{2}$, so that the corresponding phase difference is 0 or π. For nth bright or dark band BS_2 is equal to $n\lambda$ or $(2n-1)\frac{\lambda}{2}$.

$\therefore$

For nth dark band $CP=\frac{AC}{S_1S_2}(2n-1)\frac{\lambda}{2}$

" " bright " " $=\frac{AC}{S_1S_2}\times n\lambda$

" $(n+1)$st " " " $=\frac{AC}{S_1S_2}(n+1)\lambda$

$\therefore$ Width of a fringe $=\frac{AC}{S_1S_2}\{(n+1)\lambda-n\lambda\}$

$$=\frac{AC}{S_1S_2}\times\lambda=\frac{D\lambda}{d},$$

where D and d are equal to AC and S_1S_2 respectively. Thus the width of all the fringes is the same and is directly proportional to the wave-length λ and the distance of the screen from the sources, and inversely proportional to the distance S_1S_2 between the sources.

Q. 76. Describe and explain how interference fringes are formed with Fresnel's mirrors.

Fresnel's Mirrors. Two plane glass mirrors OM_1 and OM_2, either silvered on the front surface or blackened at the back to avoid multiple reflection, are inclined to one another at an angle *slightly* smaller than 180° (Fig. 59). A narrow slit S illuminated by monochromatic light is placed perpendicular to the plane of paper and parallel to the common edge O of the mirrors. Light reflected from the two inclined mirrors appears to come from two different virtual sources, S_1 and S_2, and interference bands are seen in the region C *common* to the two reflected beams. The bands are hyperbolas which are practically straight at a great distance from the sources. By adjusting the angle of inclination of the mirrors, the distance between the sources and, therefore, that between the bands can be adjusted.

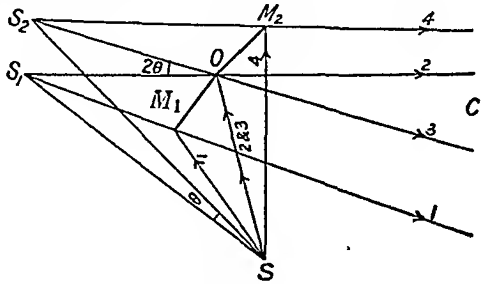


Fig. 59.

To find the positions of the two virtual sources, draw SS_1 and SS_2 perpendiculars on OM_1 and OM_2 respectively, and take S_1 and S_2 as much behind the corresponding mirrors as S is in front of them. Rays 1 and 2 are incident at the extremities of OM_1, and after reflection appear to come from the virtual source S_1. Similarly, rays 3 and 4 after reflection from the second mirror appear to diverge from the second virtual source S_2. The two reflected beams overlap in the common region C between the reflected rays 2 and 3, and interference bands are produced. That the bands are due to

interference is proved conclusively by the fact that they *disappear* when one of the mirrors is covered with some opaque substance.

If the *very small* angle between OM_1 and M_2O produced, and that between their normals SS_1 and SS_2 is equal to θ *radian*, the angle between the rays 2 and 3 reflected at O from the two mirrors is equal to 2θ, and, therefore, S_1S_2 is equal to $OS \times 2\theta$ ($\because OS_1 = OS_2 = OS$). For further treatment see Q. 75.

Q. 77. In an experiment with Fresnel's mirrors the fringes were observed with an eye-piece fitted with crosswires and found to be 0·25 mm. apart. With a convex lens of 18·75 cm. focal length, placed between the mirrors and the eyepiece, real images of the virtual sources were formed on the crosswires, and their distance apart was 0·723 cm. when the lens was 75 cm. from the crosswires. Find the wave-length of light used.

Ans. Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, where v and u are respectively the distances of an object and its image from a convex lens of focal length f, and putting $f = -18·75$ cm. and $v = -75$ cm., we get

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= -\frac{1}{75} + \frac{1}{18·75}$$

$$= \frac{-1+4}{75} = \frac{1}{25}$$

$$\therefore \quad u = 25 \text{ cm.}$$

$$\frac{\text{Distance between virtual sources}}{\text{Distance between their images}} = \frac{25}{75} = \frac{1}{3}$$

or
$$\text{Distance between virtual sources} = \frac{0·723}{3}$$

$$= 0·241 \text{ cm.}$$

$$\text{Distance between sources and crosswires} = 25 + 75$$

$$= 100 \text{ cm.}$$

$$\text{Width or interference fringes} = 0·025 \text{ cm.}$$

∴ Wave-length of light

$$= \frac{\text{Width of fringes} \times \text{distance between sources}}{\text{Distance between sources and crosswires}}$$

$$= \frac{0{\cdot}025 \times 0{\cdot}241}{100}$$

$$= 0{\cdot}00006025 \text{ cm.}$$

Q. 78. Explain the displacement of interference fringes when a thin sheet of glass is placed in the path of light from one of the two interfering sources, and show how this may be used for measuring the refractive index of glass. How does this result support the wave theory of light?

Ans. See Q. 75 and Fig. 58 for the formation of interference fringes and the relation

$$x = CP = \frac{D}{d} \times S_2B,$$

or $$S_2B = \frac{dx}{D}, \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

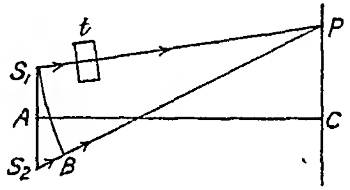


Fig. 60.

where C is the position of the central bright band, S_2B is the path difference at P for monochromatic light of wave-length λ coming from S_2 and S_1, and D and d are equal to AC and S_1S_2 respectively. The refractive index μ of a medium is equal to the ratio of the velocity of light in air (better vacuum) to its velocity in that medium, that is, in travelling any thickness of that medium light takes μ times as much time as it takes in travelling the *same* distance in air. Therefore, a thickness t of the medium is *equivalent* to a thickness μt of air.

On placing a glass sheet of thickness t in the path S_1P, the actual air path is reduced to $S_1P - t$, and that in glass being equivalent to μt in air, the total equivalent air path from S_1 to P becomes equal to $S_1P - t + \mu t$, which is greater or smaller than BP according as μ is greater or smaller than 1. Thus the path S_1P is changed by $\mu t - t = (\mu - 1)t$, and the interference fringes are displaced. The central bright band is no longer at C. If μ is greater than 1, the path of light from S_1 is increased, and it is equal to that from S_2 at a point on the P

side of C. Otherwise the central band is shifted to the other side.

$$\text{Path difference at } P = S_2P - \{S_1P + (\mu - 1)t\}$$
$$= S_2B + BP - S_1P - (\mu - 1)t$$
$$= S_2B - (\mu - 1)t$$
$$= n\lambda,$$

if the nth bright band is at P in the final *displaced* position of the bands.

Or $$(\mu - 1)t + n\lambda = S_2B = \frac{dx}{D}, \quad \cdots\cdots \quad \text{(from (1)}$$

and $$x = \frac{D}{d}\{(\mu - 1)t + n\lambda\} \quad \cdots\cdots \quad (2)$$

Here x is the distance of P from C, where the central band is formed when there is *no* glass lamina placed in the path S_1P. By putting n equal to O in (2), the displacement of the central band, that is, its distance x_0 from C is given by

$$x_0 = \frac{D}{d}(\mu - 1)t \quad \cdots\cdots\cdots \quad (3)$$

It is found that the central band is displaced *towards* P. Thus $(\mu - 1)$ is positive, μ is greater than 1, and, therefore, the velocity of light in glass is *smaller* than in air. This is in accordance with, and supports, the wave theory of light and disproves the corpuscular theory, which predicts the velocity of light in any material medium to be *greater* than in air.

By this method t or μ can be calculated from the measurement of other terms in (3). With monochromatic light all the fringes are *exactly similar*; the central band cannot be distinguished from the rest, and its displacement cannot be determined. As with white light the central band is white and others are coloured, first white light is used to find the approximate position of the cental band and then monochromatic light is employed for the accurate determination of its position.

Q. 79. Light from two point sources falls on a screen. What are the conditions that interference may be produced on the screen?

How will you make use of this principle for finding the refractive index of air? (*Punjab, 1929*)

Ans. See Q. 75 for the conditions for interference.

Monochromatic light from a very narrow slit is passed through a convex lens so that the emergent rays run parallel to its principal axis. Part of this light passes through a chamber A with glass sides and slit S_1 and the rest through a similar chamber B and slit S_2. These slits are very narrow and are placed parallel to S. They are the two coherent sources which give rise to interference fringes on the right, and then are observed with a micrometer eyepiece.

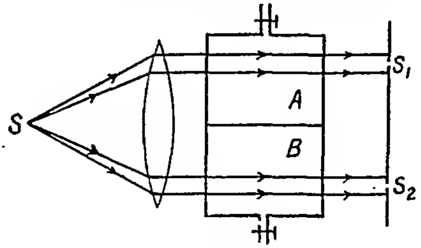


Fig. 61.

First both the chambers are evacuated and the position of the interference fringes is observed. Then *one* of the chambers is filled with the gas, whose refractive index is to be measured, at a known pressure, and the displacement of the fringes is measured. See Q. 78 for the explanation of displacement and calculation of the value of refractive index.

Q. 80. Describe an arrangement for producing interference fringes, and explain the method for measuring wave-length with it.

Discuss briefly the phenomenon of interference in relation to the law of conservation of energy.

(*Calcutta, 1936*)

Ans. **Fresnel's Biprism.** An obtuse-angled prism P, which may be supposed to consist of two prisms of *very small* refracting angles placed with their bases together, is placed with the two refracting edges parallel to a very narrow slit S, from which monochromatic light is coming (Fig. 62). Rays 1 and 2 are deflected by the upper prism towards its base and after passing through it appear to come from S_1. Similarly, rays 3 and 4 on emerging from the lower prism appear to come from S_2. Thus S_1 and S_2 are the two coherent

sources, and interference fringes are formed in the region *common* to the two pencils of rays emerging from the two prisms.

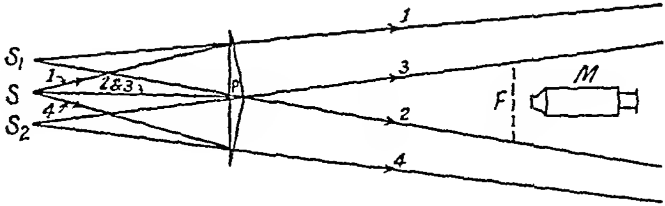


Fig. 62.

As the fringes are *very narrow*, a reading microscope M, fitted with a micrometer eyepiece, or having crosswires and capable of lateral motion parallel to S_1S_2, is used and its position is adjusted so that the fringes are formed in its focal plane F. The distance between a *large* number of bands is measured and from this the distance between two consecutive bright bands is found. This distance is equal to $\frac{D\lambda}{d}$, where λ is the wave-length of monochromatic light used, D the distance of F from S, and d the distance between the two coherent sources S_1 and S_2. See Q. 75 for finding this relation.

To find the position of F, an upright needle is adjusted in front of M so that there is no parallax between it and the interference bands there. For finding d, a convex lens, whose focal length is *less* than one-fourth the distance SF, is placed between S and F and adjusted, *without* disturbing the biprism, to form real images of S_1 and S_2 in the focal plane F of the microscope. The distance I_1 between these images is measured. Then the lens is moved and adjusted in a second position to form real images of S_1 and S_2 in the plane F. If the distance between the images in the second case is I_2, then $d=\sqrt{I_1 \times I_2}$. In one case the convex lens forms a magnified image and in the other it is diminished, and as the distances of object and image from the lens in one case are respectively equal to the distances of image and object in the second case,

$$\frac{d}{I_1}=\frac{I_2}{d}, \qquad \text{or} \qquad d=\sqrt{I_1 \times I_2}.$$

Thus knowing D, *d*, and the breadth of bands, the wave-length of light used is calculated.

Interference and Law of Conservation of Energy. Light energy is transferred from one place to another in the form of transverse waves in ether, As a wave passes a point, the ether particle there is disturbed, and its intensity is proportional to the *square* of its displacement. When two coherent sources of light send waves over a common region, the distribution of energy due to each is distributed by the other, and this *redistribution* of energy is called interference. Where the waves from the two sources arrive together in the *same phase* (crest and crest, or trough and trough), the resultant displacement is equal to the sum of the two component displacements due to the two sources separately. or twice the displacement due to either source, and the intensity of light is *four* $(=2^2)$ times that which would be produced by either source in the absence of the other. These are the positions of bright bands. At other places where the crests (or troughs) of waves from one source arrive at the same time as the troughs (or crests) of the other, they are in *opposite* phase, and the resultant displacement and intensity are equal to *zero*. Here dark bands are produced.

As over any area the number of the bright bands is equal to the number of dark bands, the *average* intensity (light energy received per unit area per unit time) is equal to *half* the intensity of bright bands, or *twice* the intensity produced by either source in the absence of the other. Therefore, when interference is produced between two sources, the amount of light energy received by any area in any time is equal to the *sum* of the energies sent to that area by the two sources separately in the same time, and this is in accordance with the law of conservation of energy. The light energy missing at the dark bands is transferred to the bright bands, and there is no increase or decrease in the total amount of light energy.

Q. 81. What do you mean by " interference of light"? Explain why we should not expect to get interference between two neighbouring independent sources or two separate portions of the same source.

Fringes are produced by a Fresnel's biprism in the focal plane of a reading microscope which is 100 cm. from the slit. A lens inserted between biprism and microscope gives two images of the slit in the two positions. In one case the two images of the slit are 4·05 mm. and in the other case 2·90 mm. apart. If sodium light ($\lambda = 5893 \times 10^{-8}$ cm.) is used, find the distance between the interference bands. (*Bombay, 1935*)

Ans. See Q. 75 for interference and the conditions for coherent sources.

Problem. The distance between two consecutive bright bands is equal to $\frac{D\,\lambda}{d}$, where D is the distance of the focal plane of the microscope from the slit, d the distance between the two coherent sources, and λ the wave-length of light used. The distance between the sources is equal to the square root of the product of the distance between their real images formed by the convex lens in the two positions.

Distance between slit and microscope D

$$= 100 \text{ cm.}$$

Distances between images of slits

$$= 4{\cdot}05 \text{ and } 2{\cdot}90 \text{ mm.}$$

$\therefore$ Distance between slits d

$$= \sqrt{0{\cdot}405 \times {\cdot}290} \text{ cm.}$$

Wave-length of light used $\lambda = 5893 \times 10^{-8}$ cm.

$\therefore$ Distance between consecutive bands

$$= \frac{100 \times 5893 \times 10^{-8}}{\sqrt{0{\cdot}405 \times 0{\cdot}290}} = 0{\cdot}0172 \text{ cm.}$$

Q. 82. Explain the formation of colours in a thin film. Show why the colours of a film when viewed by reflected and transmitted light are complementary.
(*Punjab, 1931*)

Ans. **Colours of Thin Films.** Consider a very *thin* transparent film bounded by opposite sides AB and CD and

of an optically denser medium of refractive index μ, placed in air (Fig. 63). A wave of light coming from a point in an extended source and moving along EF is incident on AB at angle θ. It is partly reflected along FG and the rest is refracted along FH making the angle of refraction ϕ. The refracted wave is partly reflected along HI and the rest transmitted along HL. At I the wave is partly reflected along IM and partly transmitted along IK. Again the wave suffers reflection and refraction at M. Light is reflected internally many times, but each time it becomes fainter, and therefore the first few reflections only are of any importance.

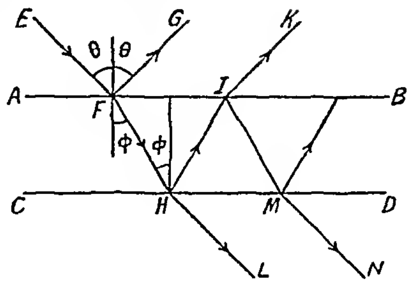


Fig. 63.

Colours by Reflected Light. When the thickness of the film is very small as compared with the wave-length λ of light used, rays FG and IK are practically coincident and the additional distance travelled by IK in the film is negligible. Phase of IK is the same as that of the *incident* light, but FG has suffered a *sudden reversal* of phase on being reflected from an optically denser medium, therefore FG and IK differ in phase by π and the film appears dark, that is, no light is reflected.

As the thickness of the film is increased, the path traversed by IK in the film becomes appreciable. A part of the incident wave reaches I *directly* and is partly reflected from there. These two parts of the *same* wave proceed from I in slightly different directions and are brought to focus by a convex lens on a screen or by the eye of the observer. Their phase difference there is the *same* as at I. When the incident wave reaches I directly, the position of its wave front in the film is obtained by drawing a perpendicular from I on FH. This perpendicular makes angle ϕ with IF, and the distance that the wave has travelled in the film is equal to IF sin ϕ or $2t \tan \phi \sin \phi$, where t is the thickness of the film. The path retardation suffered by IK is the *further* distance that it

traverses *in* the film before emerging at I and its equivalent path retardation *in air* is obtained by multiplying it by μ.

$$\text{Path retardation} = \mu(\text{FH}+\text{HI}-2t\tan\phi\sin\phi)$$
$$= \mu\left(\frac{2t}{\cos\phi}-\frac{2t\sin^2\phi}{\cos\phi}\right).$$
$$= \frac{2\mu t\,(1-\sin^2\phi)}{\cos\phi} = \frac{2\,\mu t\cos^2\phi}{\cos\phi}$$
$$= 2\,\mu t\cos\phi$$

When this path retardation is equal to $\frac{\lambda}{2}$, the phase difference due to it is equal to π, directly reflected light is in phase with the light that has suffered one internal reflection, and the film appears bright. Similarly, when the retardation is equal to λ, or an *even* multiple of $\frac{\lambda}{2}$, the film appears dark, while for retardation equal to an odd multiple of $\frac{\lambda}{2}$ it appears bright. For light of different colours (different wave-lengths), different thicknesses produce this retardation. Other waves coming from different points of the extended source of light and partly reflected directly from I and partly emerging from it after undergoing refraction and reflection in the film reach the lens, or the eye of the observer, by slightly different paths, but they are all brought to focus at the same point as the parts of the first wave considered. If the lens L has a *small* aperture and subtends a *very small* angle at I, the angle of refraction ϕ for all the waves received by it and the path retardation for all the pairs are practically the same, as the film is very thin. If the aperture of the lens is not small or the film is not very thin, path retardation is not the same for all pairs, and, due to overlapping of maxima and minima, uniform illumination is produced.

If the thickness of the film changes uniformly from one end to the other and monochromatic light is incident on it, alternate bright and dark bands are seen, bands for blue light being shorter than for red right. When white light is used, the thin end of the film appears dark, next to it

violet light is reflected, then blue, green, yellow, orange and red. Further on there is overlapping of bands of different colours, as their breadths are different, and beyond it there is uniform illumination.

Transmitted Light. The transmitted rays HL and MN have experienced no sudden reversal of phase, and, therefore, when the film is very thin as compared with λ, these rays reinforce each other, and the film appears bright. Whole of the incident light is transmitted and none is reflected. If the thickness of the film is gradually increased and the path retardation of MN becomes $\frac{\lambda}{2}$ or an odd multiple of $\frac{\lambda}{2}$, the film appears dark by transmitted light. In this case the whole of the incident light is reflected and none is transmitted. Similarly, for path retardation equal to an even multiple of $\frac{\lambda}{2}$, the film appears bright by transmitted light. For light of different wave-lengths this will occour at different thicknesses of the films. Thus, *when the film appears bright by transmitted light, it appears dark by reflected light and vice versa.*

If the thickness of the film changes uniformly from one end to the other and monochromatic light is used, alternate bright and dark bands are seen, blue bands being of shorter width than red bands. With white light coloured bands are seen, and for a given thickness, light of certain wave-lengths is reflected, while the rest of the wave-lengths, which are suppressed by interference in the reflected light, are transmitted. Colours which are present in the reflected light are absent in the transmitted light, and vice versa, and therefore, the colours of a film when viewed by reflected and transmitted light are complementary.

Q. 83. Explain the formation of colours in thin plates.

A parallel beam of sodium light ($\lambda = 5890 \times 10^{-8}$ cm.) is incident on a thin glass plate ($\mu = 1{\cdot}5$), such that the angle or refraction into the plate is 60°. Calculate

the smallest thickness of the plate which will make it appear dark by reflection. (*Punjab 1933*)

Ans. See Q. 82. For the formation of colours in thin plates.

Probelm. The path retardation in a plate of thickness t and refractive index μ by one internal reflection is equal to $2\,t \cos \phi$, when ϕ is the angle of refraction in it, and its equivalent air path is equal to $\mu \times 2\,t \cos \phi$. The plate appears dark by reflected light if this path retardation is equal to an *even* multiple of λ, where λ is the wave-length of light used in *air*. For the least thickness of plate, the path difference should be $2 \times \frac{\lambda}{2} = \lambda$.

$$2\,\mu\,t \cos \phi = \lambda$$

$$2 \times 1{\cdot}5 \times t \times \tfrac{1}{2} = 589 \times 10^{-8}\text{cm.}$$

$$\therefore\ t = \frac{589 \times 10^{-8}}{1{\cdot}5}$$

$$= 3933 \times 10^{-8}\text{ cm.}$$

Q. 84. A vertical soap-film is gradually drained away. Explain the Phenomenon when film is seen by reflected light. When will it cease to reflect light? (*Bombay, 1934*)

Ans. As the film is drained away, it becomes very thin at the top, and its thickness increases *uniformly* downward. When light is incident on it, a part of it is reflected and the rest is refracted. The second part is partly transmitted and partly reflected at the back surface of the film and then emerges at the front face. See Q. 82 for the formation of interference bands with monochromatic light and white light. These bands are *horizontal* as the film is vertical and its thickness along a horizontal line is the same. The upper edge is very thin as compared with the wave-length of light used and appears dark by reflected light.

When the film becomes very thin throughout as compared with the wave-length of light, the path retardation in the film is practically equal to zero, the directly reflected and internally

reflected waves, being in opposite phase, completely and interfere with each other, and no light is reflected, *whatever* be the wave-length of light used.

Q. 85. What are Newton's rings, and how are they produced ? How will you determine the wave-length of light by measuring the diameters of Newton's rings ? Prove the relation used. (*Punjab, 1935*)

Ans. **Newton's Rings.** A source of monochromatic light is placed at the principal focus of a vertical lens L_1, and the rays of light after passing through it become parallel to

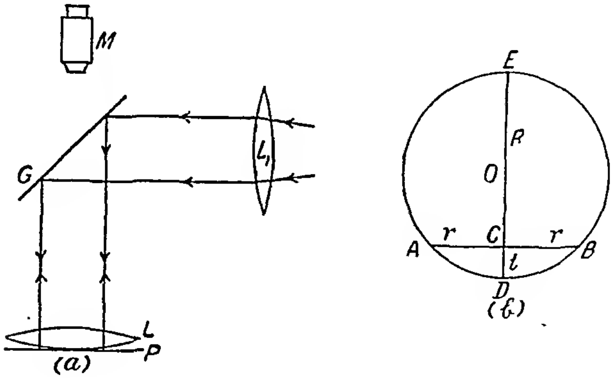


Fig. 64.

its principal axis [Fig. 64 (*a*)]. They are then reflected downward by a glass plate G, held at 45° to the vertical, and fall normally on a convex lens L, of large radius of curvature of lower face, placed in contact with a horizontal plane glass plate P. Light is reflected upward partly from the lower face of the lens and partly from the glass plate, that is, at the *upper* and *lower* faces of the thin air film formed between lens L and plate P, and interference occurs. A ray of light incident on the air film is reflected back and forth many times. At each reflection some light escapes and the reflected light becomes weaker and weaker, so that the first few reflections are of any importance. Similarly, some of the light is transmitted through P and interferes with the light reflected downward

from the lower face of the lens. Thus interference fringes are formed both by reflected and transmitted light.

To examine the fringes formed by reflected light, a reading microscope M is placed above G and is focussed on the air film between L and P. As the lower surface of lens L is a part of a sphere, the thickness of the air film is the *same* at all points equidistant from the point of contact of L and P, and, therefore, the fringes are *circular* rings. Fig. 64 (*b*) is a vertical section of this sphere, of radius of curvature R, through its centre O, and at a distance r from the point of contact D the thickness of the air film is equal to t=CD. From the geometry of a circle,

$$CA \times CB = CD \times CE$$
$$r \times r = t\,(2R - t)$$
$$= 2tR,$$

as t is very small as compared with R, and $(2\,R - t)$ is practically equal to 2R,

$$\therefore\ t = \frac{r^2}{2R}$$

As the interference fringes are viewed *normally*, the path difference for light reflected from the *lower* face of the lens and the upper face of the plate is equal to $2t$. The reflection from the plate (optically denser) is accompanied by a sudden reversal of phase, and as this coresponds to a path difference of $\frac{\lambda}{2}$, a bright or dark fringe is formed according as the actual path difference $2t$ is equal to an odd or even multiple of $\frac{\lambda}{2}$.

At the point of contact there is practically no actual path difference, but due to the sudden reversal of phase of light reflected by the plate P, it appears *dark* by reflected light.

For first bright ring $2t = \frac{\lambda}{2}$

„ nth „ „ $„ = (2n-1)\frac{\lambda}{2}$

$$= \frac{r_n^2}{R} = \frac{D_n^2}{4R},$$

where r_n and D_n are respectively the radius and diameter of the nth bright ring.

$$\therefore \text{ Diameter } D_n \text{ of } n\text{th bright ring} = \sqrt{4R(2n-1)\frac{\lambda}{2}}$$
$$= \sqrt{2R\,(2n-1)\lambda}$$

and Diameter $D_{(m+n)}$ of $(m+n)$th bright ring

$$= \sqrt{2R\,(2m+2n-1)\lambda}$$

$$\therefore \text{ Wave-length } \lambda = \frac{D^2_{(m+n)} - D_n^2}{4Rm}$$

The diameters of $(m+n)$th and nth bright rings are measured with the travelling microscope and the radius of curvature R of the *lower* face of lens L is measured with a spherometer.

Q. 86. An opaque obstacle is held with its sharp edge parallel to a narrow illuminated slit and a screen is placed on the other side of the obstacle. Explain the intensity of illumination of the screen at, within, and without the geometrical shadow.

Ans. **Diffraction at an Edge.** In Fig. 65 slit S, obstacle PQ, and screen CB are all perpendicular to the plane of paper, and edge P is parallel to the slit. The slit gives out cylindrical waves and their section in the plane of

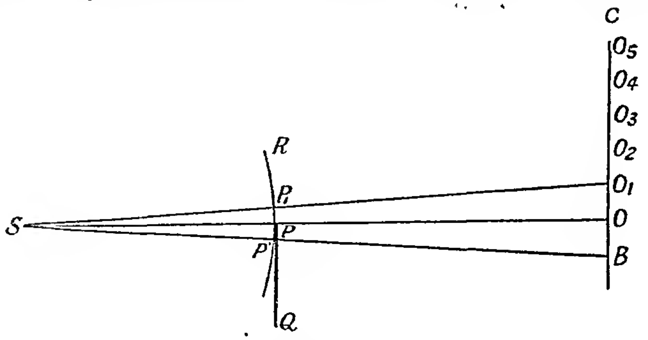


Fig. 65.

paper is circular. RPP′ is a part of the wave-front at P. Produce line SP to meet the screen at O. Then according to geometrical optics, A should be the edge of the shadow

cast by the obstacle: the part of the screen below O should be dark, and the part above O should be illuminated, so that at O there should be an *abrupt* change from darkness to uniform illumination.

If λ is the wave-length of light used, with O as centre draw spheres of radii $OP+\frac{\lambda}{2}$, $OP+\lambda$, $OP+\frac{3\lambda}{2}$, ..., to divide the wave-front into half-period zones. See Q. 73 for the propagation of light. For O, P is the pole of the wave-front, and it receives light energy from only the upper half of the wave-front. If $2a_1$, $2a_2$, $2a_3$, ..., are the amplitudes of disturbances received from the corresponding half-period zones, amplitude A of the resultant disturbance at O, as shown in Q. 73, is given by

$$A=a_1-a_2+a_3-a_4+\ldots\ldots\ldots+a_n$$
$$=\frac{a_1}{2}+\frac{a_n}{2}$$
$$=\frac{a_1}{2}.$$

In the absence of the obstacle, the disturbance at O would have been due to the whole of the wave-front, and the amplitude of the resultant disturbance would have been equal to $2\times\frac{a_1}{2}=a_1$. As the intensity of illumination is proportional to the square of the amplitude of disturbance, the intensity of illumination at O is *one quarter* of the intensity produced there in the absence of the obstacle.

For a point B below O, its pole P' is below P. The obstacle intercepts not only the lower half of the wave-front but *also* some zones of the upper half. B receives light energy from only the remaining *exposed* part of the upper half, and the amplitude of the resultant disturbance is practically equal to half the amplitude of the disturbance from the first exposed zone. As we go below O, more of the zones of the upper half of the wave-front are intercepted, and as the disturbance from any zone is smaller than that from an inner zone, the intensity of illumination decreases *gradually*. When the first few powerful inner zones are cut off, the intensity is

reduced to zero, as the disturbances from the remaining exposed zone cancel out each other. Thus inside the geometrical shadow the intensity of illumination decreases *rapidly* but *gradually*.

A point outside the geometrical shadow has its pole *above* P. It receives light energy not only from the whole of the upper half of the wave-front but also from a part of the lower half. While the amplitude of the resultant disturbance from the upper half is fixed and is equal to $\frac{a_1}{2}$, the amplitude of the resultant disturbance from the exposed part of the lower half, and, therefore, due to the whole of the exposed wave-front, is *maximum* or *minimum* according as the number of half-period zones in the exposed part of the lower half is *odd* or *even*. At a point O_1, for which the pole of the wave-front is P_1, the first half-period zone of the lower half is exposed. Amplitude A_1 of the resultant disturbance is given by

$$A_1 = \frac{a_1}{2} + a_1 = \frac{3a_1}{2},$$

and intensity of illumination I_1 here, is proportional to $\left(\frac{3a_1}{2}\right)^2$ This is *nine* times the intensity I at O.

At O_2, two zones of the lower half are exposed, and

$$A_2 = \frac{a_1}{2} + a_1 - a_2.$$

As a_2 is slightly less than a_1, A_2 and I_2 are slightly *greater* than their corresponding values A and I at O, but *much smaller* than their values at O_1.

At O_3, three zones of the lower half are exposed, and

$$\begin{aligned} A_3 &= \frac{a_1}{2} + a_1 - a_2 + a_3 \\ &= a_1 + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \frac{a_3}{2} \\ &= a_1 + \frac{a_3}{2}. \end{aligned}$$

As $\frac{a_3}{2}$ is slightly *smaller* than $\frac{a_1}{2}$, A_3 is smaller than A_1, but much greater than A_2 and A, and I_3 is proportional to $\left(a_1 + \frac{a_3}{2}\right)^2$

At O_4, four zones of the lower half are exposed.

$$\begin{aligned}A_4 &= \frac{a_1}{2} + a_1 - a_2 + a_3 - a_4 \\ &= \frac{3a_1}{2} - \frac{a_2}{2} - \left(\frac{a_2}{2} - a_3 + \frac{a_4}{2}\right) - \frac{a_4}{2} \\ &= \frac{3a_1}{2} - \frac{a_2}{2} - \frac{a_4}{2}.\end{aligned}$$

As a_2 is slightly greater than a_4, A_4 and I_4 are slightly *greater* than A_2 and I_2 respectively but smaller than their corresponding values at O_3.

Similarly, at O_5, when five zones of the lower half are exposed,

$$\begin{aligned}A_5 &= \frac{a_1}{2} + a_1 - a_2 + a_3 - a_4 + a_5 \\ &= a_1 + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2}\right) + \frac{a_5}{2} \\ &= a_1 + \frac{a_5}{2}\end{aligned}$$

Again A_5 and I_5 are *smaller* than A_3 and I_3 respectively, as a_5 is smaller than a_3, but *greater* than their corresponding values A_4 and I_4 at O_4.

Thus outside the geometrical shadow there are maxima and minima of light intensity, that is, alternate bright and dark (comparatively) bands are produced parallel to the edge of the obstacle. As we go out of O, the intensity of the maxima *decreases* and that of the minima *increases* and after a certain stage uniform illumination is produced. These changes of intensity are shown in Fig. 66, where ordinates represent the intensity of illumination and abscissae the position on the screen. Within O intensity decreases rapidly but gradually without any changes of maxima and minima. The first band at O_1 is the brightest. The line through N gives the intensity of illumination in the absence of the obstacle. The width of the bands goes on decreasing outward, the first being the broadest.

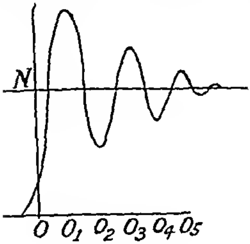


Fig. 66.

With white light coloured bands are obtained, and blue bands are less wide than red bands.

Q. 87. In Q. 86 explain the formation of fringes when the obstacle is (*a*) broad, (*b*) very narrow.

Ans. (*a*) See Q. 86 for the changes of intensity of light at, within, and without the geometrical shadows of the two edges. In the Fig. 67, AB is the geometrical shadow cast by the obstacle PQ. The bands on the outside of A are due to edge P *alone*, that is, due to the part of the wave-front above P, as due to *obliquity* the effect of the wave-front below Q is negligible. Similarly, the bands on the outside of B are due to edge Q alone. Intensity decreases rapidly and regularly within A and B, and the rest of the geometrical shadow is *dark*. The breadth of the diffraction bands is *independent* of the breadth of the obstacle.

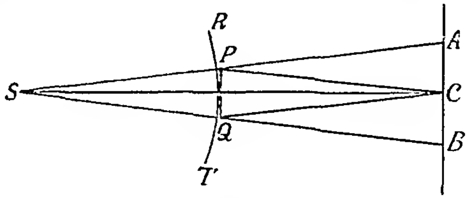


Fig. 67.

(*b*) **Very Narrow Obstacle.** When the breadth of the obstacle is extremely small (fine wire) as compared with its distance from the screen, light bending into the geometrical shadow from the two sides is superposed on the screen, and bands are produced *inside* the geometrical shadow due to *interference* between the wavelets coming from the immediate neighbourhood of the edges P and Q. A point C is *bright* or *dark* according as the difference between the distances PC and QC is an *even* or *odd* multiple of *half* the wave-length of light used. These bands, unlike the diffraction bands, are narrow, of equal width, and almost equidistant, and their width is inversely proportional to the width of the obstacle. (See Q. 75.) With an extremely narrow obstacle, these interference bands may spread out of the geometrical shadow.

Q. 88. What is diffraction of light ? Explain the formation of diffraction pattern when a monochromatic beam of light passes through a narrow rectangular aperture. (*Punjab, 1936*)

Ans. **Diffraction.** When waves pass through an opening, sideways spreading takes place, and its magnitude depends on the size of the opening and the wave-length of the waves. It occurs only if the opening is very small. The smaller the opening, for a given wave-length, the greater is the sideways spreading, and *vice versa*, but the size of the opening is to be taken relative to the wave-length of the waves. For light waves the opening has to be very small and then the propagation of light is not according to the laws of geometrical optics, that is, its propagation is not strictly rectilinear. This sideways spreading of light at an edge, obstacle, or opening is called **diffraction** and the phenomenon is observed when part of a wave is intercepted by an obstacle.

Narrow Rectangular Aperture. A very fine slit S, giving out monochromatic light, a rectangular aperture QR, and screen AB are perpendicular to the plane of paper (Fig. 68). The aperture is parallel to the slit, and a line passing through S and the middle point P of the aperture is perpendicular to the screen and meets it at C. The waves given out by the slit are cylindrical and the circular section of one of them at the aperture is shown in Fig. 68. The variation of the intensity of illumination of the screen *depends* on the size of the aperture and various cases arise according to its width.

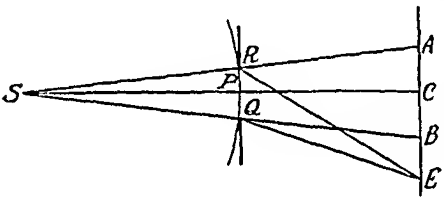


Fig. 68.

1. When the aperture is not very narrow and contains *many* half-period zones with respect to a point on the screen, its edges R and Q behave like two separate edges independent of one another. See Q. 86. The intensity of illumination decreases gradually and rapidly within each shadow, while just outside A and B diffraction bands are formed parallel to the aperture. They are unequal in width and the broadest are next to A and B. There is *no variation* of intensity near C.

2. When the aperture RQ comprises *few* half-period zones, point C, equidistant from R and Q, has *maximum* or

minimum intensity according as the number of zones with respect to *it* is odd or even. If the screen is brought *forward*, the number of zones in RQ *increases*, and the intensity at C undergoes these changes of maxima and minima.

(a) If the number of zones with respect to C is odd, say 3, it has maximum intensity, and if the displacements at C due to these zones are $2a_1$, $2a_2$, $2a_3$, the resultant displacement is equal to

$$2(a_1 - a_2 + a_3) = 2\left(\frac{a_1}{2} + \frac{a_3}{2}\right) = a_1 + a_3 \quad \ldots \quad \text{(maximum)}$$

For a point above C the pole of the wave-front is above P and there may be 2 exposed zones in the upper part of the wave-front and 4 in the lower, and the resultant displacement is given by

$$a_1 - a_2 + a_1 - a_2 + a_3 - a_4 = a_1 - a_4 \quad \ldots \quad \ldots \quad \text{(minimum)}$$

At a higher point 1 zone from the upper and 5 zones from the lower part of the wave-front are exposed.

$$\text{Displacement } = a_1 + a_1 - a_2 + a_3 - a_4 + a_5$$
$$= a_1 + \frac{a_1}{2} + \frac{a_5}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(maximum)}$$

For A the pole of the wave-front is at R and 6 zones of its lower part alone are exposed.

$$\text{Displacement } = a_1 - a_2 + a_3 - a_4 + a_5 - a_6$$
$$= \frac{a_1}{2} - \frac{a_6}{2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(minimum)}$$

Similarly, part CB is crossed by alternate bright and dark bands.

The pole of the wave-front with respect to a point E, within the geometrical shadow, is *below* Q. It receives light energy from the zones of only the upper half of the wave, and the intensity of illumination at it is *maximum* or *minimum* according as the aperture comprises *odd* or *even* number of zones with respect to *it*, that is, ER—EQ is equal to $(n + \frac{1}{2})\lambda$ or $n\lambda$, where n is an integer and λ is the wave-length of light used. These bands are *narrower* than those in AB, and their

intensity decreases as their distance from the edge of the geometrical shadow increases until there is uniform darkness.

(*b*) If the number of half-period zones with respect to C is *even*, its intensity is *minimum*. As in the last case, it is surrounded by alternate light and dark bands.

3. If the aperture comprises *one*, or less than one, half-period zone with respect to C, it is *always bright*, and there are *no* bands in AB. But for a point in the geometrical shadow within A or B the number of zones in the slit may be more than one, and it is on a maximum or minimum according as the number of these zones is odd or even.

Q. 89. Explain the rectilinear propagation of light from the point of view of the wave theory.

Describe and explain the shadows formed on a screen by an opaque circular disc when placed in the path of rays of a point source of monochromatic light.
(*Bombay, 1926*)

Ans. **Rectilinear Propagation.** See Q. 73.

Shadow of a Disc. Let S be a point source of monochromatic light, of wave-length λ, in the plane of paper, RPQ a section of a very *small* opaque disc and ACB of a screen, both held perpendicular to the plane of paper, so that line SP is perpendicular to them, passes through the centre of the disc, and meets the screen at C (Fig. 69). Then P is

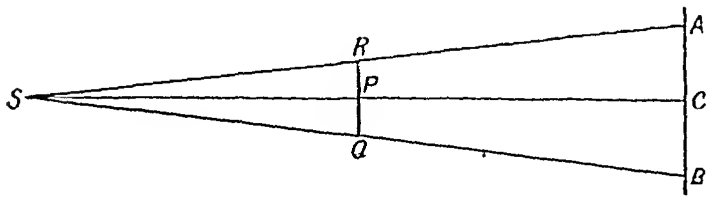


Fig. 69.

the pole of the spherical wave-front at P with respect to C. With C as centre and radii equal to $CP+\frac{\lambda}{2}$, $CP+\lambda$, $CP+\frac{3\lambda}{2}$, $CP+2\lambda$, ..., draw spheres to divide the wave-front into

half-period zones. According to geometrical optics, there should be complete darkness in the shadow AB cast by the disc, but the wave theory of light leads to a different result.

The illumination at C is due to the resultant effect of all these half-period zones. As proved in Q. 73, when *many* zones are exposed, the resultant displacement at any point is equal to *half* the displacement produced there by the wavelets from the *first exposed* zone. If the disc covers the first zone, the resultant displacement at C is equal to half the displacement from the second zone, and this is practically the same as that from the first zone, because the displacements from successive outer zones are in only *slightly decreasing* order. Similarly, if the first two zones are hidden by the disc, the displacement at C is due to the rest of the exposed zones and is equal to half the displacement from the third zone. This again is only slightly smaller than when the first zone is intercepted. Thus when the disc is very small and covers the first *few* half-period zones, the centre of its geometrical shadow is *always bright* and its intensity of illumination is approximately the same as if the disc were absent.

If the screen is brought near the disc, distance CP decreases, and the number of half-period zones, with respect to C, hidden by the disc *increases*. As the displacement from any zone is *weaker* than that from an inner zone, the intensity of illumination of C decreases, but it does not undergo changes of maxima and minima.

While for a point on the axis of the disc, the wavelets from *all* the points on the boundary of the disc are *always* in phase and, therefore, reinforce each other, this is not the case for other points on the screen. To find the condition of any point it is considered as the centre for dividing the wave front into half-period zones, and the resultant effect of all the exposed zones is found. The bright spot at C is surrounded by alternate bright and dark *circular* bands with their centre at C.

Q. 90. Explain the diffraction pattern produced when plane waves of light after passing through an aperture are focussed by a convex lens on a screen.

Ans. A rectangular aperture AB, convex lens L and screen QR are held perpendicular to the plane of paper, and the principal axis LP of the lens is perpendicular to the aperture and screen (Fig. 70). Plane waves (parallel rays) coming

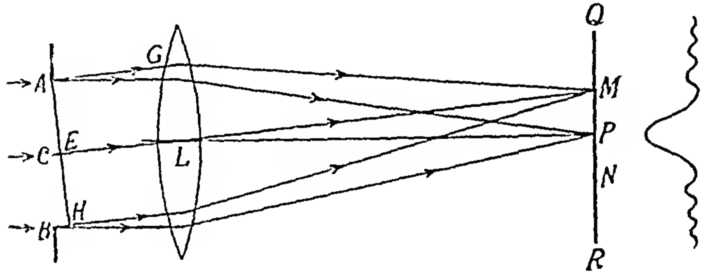


Fig. 70.

from a point of distant object are incident normally on the aperture, and are focussed on the screen placed in the focal plane of the lens.

According to geometrical optics, if the lens is free from spherical and chromatic aberrations, a bright image with sharp edges should be formed on the screen. Its size should depend on the size of the object and the relative positions of the screen and object with respect to the lens but should be *independent* of the size of the aperture or lens. This follows from the strict rectilinear propagation of light, but as according to the wave theory there should be a slight bending, a sharply defined image is not obtained.

A light wave is propagated by reinforcement.and interference of wavelets from all the points of its wave front. A convex lens changes the paths of these wavelets, and its focus is the space where *all the wavelets reinforce each other and beyond it interfere.* All the points in the aperture AB give out wavelets in different directions, and *all* the rays in the *same* direction are focussed on the screen at the same point. All the rays parallel to LP (principal axis) are brought together by L at P (principal focus) in the *same* phase. They reinforce

each other at P, and, therefore, the intensity of light there is *maximum*.

Similarly, all the rays parallel to A'G are brought to focus at M, but here the disturbances from different points of AB are *not* in the same phase. A line AEH drawn perpendicular to AG is a section of the wave-front in *this* direction, and the phase difference at M between disturbances from any two points of AB is the *same* as that in the wave-front AEH, because all the disturbances from AEH take the *same* time to reach M.

This phase difference is due to the different distances that the disturbances from AB travel before reaching AEH. If BH is equal to wave-length λ of light used, phase difference between the disturbances from A and B is equal to 2π. Again, if C is the middle of AB, CE is equal to $\frac{\lambda}{2}$, phase difference between the disturbances from A and C is equal to π, and these disturbances cancel out each other. As C divides the aperture into two halves, the disturbances from all the points in one half are in *opposite* phase to the disturbances from the *corresponding* points in the other half, and, therefore, the resultant effect at M is *minimum*. If CEL makes angle θ radian with LP, the angle between AB and AH is also equal to θ, and the first minimum occurs at M, so that

$$BH = AB \sin \theta = \lambda \tag{1}$$

or

$$\theta = \frac{\lambda}{AB} = \frac{PM}{LM}, \tag{2}$$

as θ is very small.

Again, if CE is equal to λ, the disturbances from all the points of one-half are *reinforced* by those from the corresponding points of the second-half and maximum intensity is obtained. In general, a point has minimum intensity when BH is equal to an *integral* multiple of λ, and its intensity is maximum when this path difference is equal to an odd multiple of $\frac{\lambda}{2}$. Thus, *instead of a sharp image, we get* a bright central band at P surrounded on each side by narrow alternate maxima and minima. The central band is very bright and *most* prominent, and the intensity of the succeeding maxima falls off *rapidly*. N is the first

minimum on the other side of P, and MN is the width of the central band.

$$\text{Width of central band} = 2 \times \text{PM} = \frac{2 \times \text{LM} \times \lambda}{\text{AB}} \qquad (3)$$

These changes of intensity are represented by the intensity curve on the right of Fig. 70.

With a circular aperture, advanced mathematical analysis shows that

$$\theta = \frac{1{\cdot}22\ \lambda}{\text{AB}}$$

A bright disc at the centre is surrounded by alternate circular maxima and minima, and the diameter of the central disc is equal to $\frac{2 \times 1{\cdot}21 \times \lambda \times \text{LP}}{\text{AB}}$. Not only a *point* object gives rise to a central disc of *finite* size, but the size of this diffraction disc depends on the size of the aperture, or the aperture of the lens used, and is *inversely* proportional to it.

Q. 91. Explain clearly the distinction between the resolving and magnifying powers of a telescope, and find the conditions that the images of two very near points of an object may be seen as separate.

Ans. **Resolving Power.** See Q. 90 for diffraction through a circular aperture. The waves of light received by

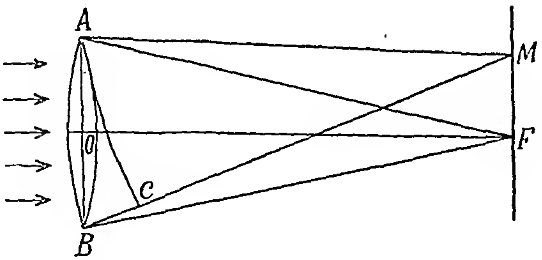


Fig. 71.

a telescope objective from a very distant object are plane. The objective O of a telescope forms the image of a distant object in its principal focal plane at F (Fig. 71). According to

geometrical optics, when the objective is free from spherical and chromatic aberrations, a point object gives rise to a point image; the image of an extended object has sharp edges, and its size does *not* depend on the aperture of the objective.

The ring supporting the lens (the lens itself) serves as a circular aperture, and each point of an object gives rise to a diffraction pattern in the focal plane of the objective, with a central bright disc surrounded by alternate circular maxima and minima, that is, the image of a point of the object is *not* a point but of *finite* size. These different diffraction patterns overlap, and two of them can be *just* distinguished, that is, the corresponding points in the object can be just observed as separate, when the *central* maximum of one falls on the *first* minimum of the other.

The central maximum of the diffraction pattern of a point of the object on the principal axis of the object is formed at its principal focus F. Maxima and minima occur in the focal plane FM, and any point is on a maximum or minimum according as the difference of its distances from B and A is an odd or even multiple of half the wave-length λ of light used, for then the aperture contains for that point odd or even number of half-period elements respectively.

With M as centre draw an arc of radius MA, cutting MB in C. For M to be the *first* minimum, BC should be equal to $2\times\frac{\lambda}{2}=\lambda$. (M should be very close to F in Fig. 71.) Let the aperture and focal length of the objective be equal to D and f respectively, and let the distance of the first minimum at M be x from F. Then,

$$\frac{MF}{OF}=\frac{BC}{AB}$$

or

$$\frac{x}{f}=\frac{\lambda}{D}$$

or

$$x=\frac{\lambda f}{D}$$

In Fig. 72, the intensity of the diffraction pattern of the image of point P is shown. If the central maximum of the diffraction pattern of the image of point Q falls at M, that is, on the first minimum of the diffraction pattern of P, the two diffraction patterns can be *just* distinguished from each other. The angle $\delta\theta$ subtended by the two points P and Q on the objective is equal to the angle subtended by their images (central maxima) on it. This is the angular limit of resolution of the telescope, and its value is equal to $\frac{x}{f}$ or $\frac{\lambda}{D}$. More rigorous treatment shows that the *minimum* angular separation which two point objects must have so that they can be just distinguished from each other is equal to $\frac{1{\cdot}22\,\lambda}{D}$. The *resolving power* of a telescope is equal to the reciprocal of this expression.

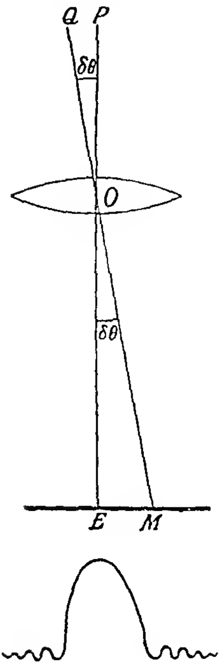


Fig. 72.

Distinction between Resolving and Magnifying Powers. The resolving power of a telescope is its ability to form *separate* images of two *neighbouring* point objects. As shown above, the images of two point objects are *just* seen as separate if the *central* maximum of the diffraction pattern of one falls on the *first* minimum of the other. The greater the aperture of the objective and the smaller its focal length and wave-length of light used, the smaller is the diameter of the central diffraction disc, that is, the sharper are the diffraction bands and more rapidly do they fade off, and, therefore, smaller is the angular separation $\delta\theta$ at which the two point objects can be seen as distinct from each other.

The magnifying power of a telescope is the ratio of the angle subtended by the image of an object, as seen with it, on the observer to the angle subtended by the object directly, and is equal to the focal length of its objective divided by the focal length of its eyepiece. For seeing distinctly two neighbouring

points of the image formed by the objective they should subtend a certain *minimum* angle on the eye of the observer, or for seeing them distinct at a given distance, there should be a certain minimum distance between them.

By increasing the focal length of the objective, the distance between the images of two neighbouring points of the object is increased, and the angle subtended by them on the observer becomes greater. But the central diffraction discs of the two images are also increased, and unless the aperture of the objective is increased to make those discs narrow, the two points may not be seen as separate. Therefore this increase in magnification is of no use.

Thus the resolving power of a telescope, as distinct from its magnifying power, refers to its ability to reveal the details of an object. It tells us not only how far the centres of the

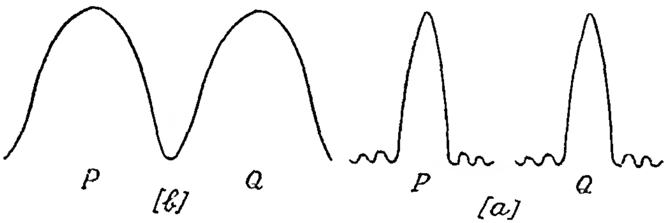


Fig. 73.

diffraction discs of two points are from each other, but also how narrow they are and thus can be seen as separate. The resolving power of one telescope may be greater than that of another though the magnifying power of the first may be equal to, or even smaller than, that of the second. In Fig. 73, the distance between the centres of the two diffraction discs, P and Q, is the same in the two cases, but in (*a*) the discs are narrower than in (*b*). Thus while the magnifying power is the *same* in the two cases, the resolving power in the first case is greater than in the second case.

Q. 92. Explain the action of a diffraction grating. Deduce the formula which connects the wave-length of the diffracted light, its deviation, and the grating constant. (*Punjab, 1935*)

Ans. Plane Transmission Diffraction Grating. It consists of a glass plate having thousands of *equidistant parallel* lines ruled on it with a diamond point, the width of spacings

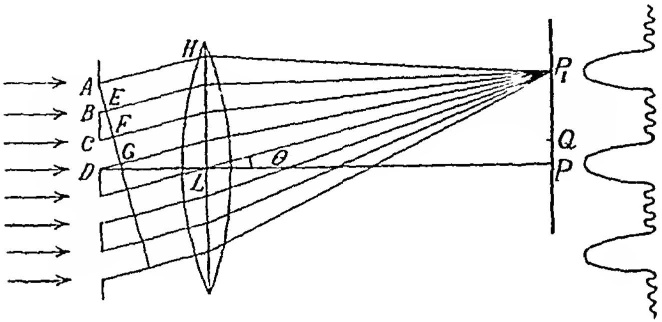


Fig. 74.

being a and that of the rulings equal to b (Fig. 74). Plane light waves (parallel rays) of wave-length λ fall normally on it from the left. The rulings reflect back the incident light and do not transmit it, while the spacings, which serve as slits, transmit light, and it is brought to focus in the focal plane of a convex lens L, whose principal axis LP is along the *normal* to the grating.

Each slit would produce its own diffraction pattern, but interference occurs between the diffracted waves from the different slits, and *very sharp* and *bright* maxima are observed at points where *all* of them reinforce each other. By increasing the number of slits, the maxima become *sharper*, more *intense*, and *wider* apart. All the rays going parallel to the principal axis of the lens arrive in the *same* phase at P and reinforce each other, so that the central bright band is formed there.

Other rays moving parallel to AH are focussed by the lens at P_1. If AEFG is drawn perpendicular to AH, it represents a section of the wave-front at A in this direction, and the phase difference between different rays at P_1 is the *same* as in this wave-front because all the rays reach P_1 from this wave-front in the *same* time. This phase difference is due to, and can be calculated from, the path difference between the rays reaching AEFG from the grating ABCD.

If AH makes angle θ with LP, angle between ABCD and AEFG is also equal to θ. A and C are the *corresponding* points of two consecutive slits, and so are B and D.

$$BE = a \sin \theta$$
$$CF = (a+b) \sin \theta$$
$$DG = (2a+b) \sin \theta$$
$$\therefore \quad DG - BF = a \sin \theta = CF$$

Similarly, the path difference for rays from *any other* pair of corresponding points in consecutive slits is the *same*. Therefore all the rays *completely reinforce* each other if $(a+b) \sin \theta = n\lambda$, where n is an integer, and $(a+b)$ is called *grating constant*. For the central band n is equal to zero. As $(a+b)$ is very small, θ is large and increases with n. With white light coloured spectra are obtained, red being more deviated than blue, but the central band is white as here *no* colour suffers any deviation. The *order* of a spectrum is given by n, the central being of zero order.

In a direction making a *very small* angle with LP, the path difference for the rays from the *extreme* (first and last) slits may be λ, so that the path difference for rays from the corresponding points of two consecutive slits is equal to $\frac{\lambda}{N}$ where N is the *total* number of slits. If the grating is considered to be divided into two halves, the wavelets from the *corresponding* slits of the two halves are in *opposite* phase $\left(\text{path difference } \frac{\lambda}{2}\right)$ and cancel out each other, so that the maxima are very *sharp* and are separated by comparatively *large* dark spaces.

At an outer point the path difference for rays from the extreme slits is equal to $\frac{3\lambda}{2}$. If the grating is considered to be divided into three equal parts, the wavelets from the corresponding slits of two consecutive parts cancel out each other. The wavelets from the slits of the third parts also are not in phase, but they do not cancel out *completely* and a slight intensity is produced. Thus, like the lateral alternate maxima and minima in the case of a single aperture, there

are many small *subsidiary* maxima and minima between the *principal* maxima of a grating. The first subsidiary minimum after the central maximum occurs at Q. On the right a curve shows the relation between intensity at a point and its position on the screen.

Q. 93. Mention the different methods for determining the wave-length of light, and give an account of the one you consider the most accurate.

(*P. U. 1938*)

Ans. The wave-length of light can be measured with (*a*) spectrometer and prism, (*b*) Newton's rings, (*c*) Fresnel's biprism, and (*d*) diffraction grating. Of these the *last* method is the most accurate.

Diffraction Grating. The slit of a spectrometer is made narrow and vertical and placed in front of a source of monochromatic light. The collimator and telescope of the spectrometer are adjusted for parallel light and the position of the telescope for viewing the slit directly is noted. A plane diffraction grating is mounted with its ruled surface over the centre of the table and rulings up and down, and perpendicular to the lines joining two of the levelling screws. The levelling screws are adjusted until the height of the image of the slit formed by reflection, and not diffraction, from either of the two faces is the *same* as when seen directly in the telescope. In this way the faces of the grating are made *vertical.*

Next the telescope is fixed at *right angles* to the collimator, and the spectrometer table is turned until the image of the slit is seen reflected from the grating surface. In this position the plane of the grating makes an angle of 45° with the incident and reflected rays. By turning the table through 45° in the right direction, the grating is set *perpendicular* to the incident rays.

Next the ruling of the grating are to be set *parallel* to the vertical axis about which the telescope turns. For this the third screw *in* the plane of the grating is adjusted so that the top of *all* the images is at the *same* height above the horizontal cross-wire of the telescope. Finally, the slit is made *very narrow* and turned until a diffracted image is as

sharp as possible. In this position the slit is *parallel* to the lines of the grating.

Then the source of light whose wave-length is to be measured is placed in front of the slit. The telescope is set to get the positions of a line in the first order spectra on either side of the central band. Half the difference between these two positions gives the angle of diffraction θ for that line. Observations are repeated with 2nd order spectra, and so on. If g is the grating constant, that is, the distance between the corresponding points of two consecutive slits, and n the order of spectrum,

$$g \sin \theta = n\lambda$$

or

$$\lambda = \frac{g \sin \theta}{n}$$

The value of g is given by the manufacturer, or is found by counting the number of lines per unit length with the help of a powerful microscope.

Q. 94. Show that, when using a diffraction grating. the visible spectra of order higher than the first overlap (4×10^{-5} cm. $< \lambda > 76 \times 10^{-6}$cm.) and that a grating with 14000 lines to the inch cannot give a spectrum of the 4th order. (*Bombay, 1933*)

Ans. In a diffraction grating the relation between the grating constant $(a+b)$, wave-length λ, angle of diffraction θ, and order of spectrum n is given by

$$(a+b) \sin \theta = n\lambda \quad \ldots \ldots \ldots \quad (1)$$

In this problem the shortest (violet) and longest (red) wavelength of light used is equal to 4×10^{-5}cm. and $7{\cdot}6 \times 10^{-5}$ cm. respectively. If θ_1 and θ_1', θ_2 and θ_2', θ_3 and θ_3', and θ_4 and θ_4' are the corresponding angles of diffraction in the first, second, third, and fourth order of spectra respectively, then putting the values of λ in (1),

$$(a+b) \sin \theta_1 = 1 \times 4 \times 10^{-5}$$

$$(a+b) \sin \theta_1' = 1 \times 7{\cdot}6 \times 10^{-5}$$

or

$$\sin \theta_1 = \frac{4 \times 10^{-5}}{(a+b)} \quad \ldots \ldots \quad (2)$$

and

$$\sin \theta_1' = \frac{7{\cdot}6 \times 10^{-5}}{(a+b)} \quad \ldots \ldots \quad (3)$$

The sine of the angle of diffraction in any higher order spectrum is equal to the product of the order of the spectrum and the sine of the corresponding angle of diffraction in the first order of spectrum. Therefore.

$$\sin\theta_2=\frac{8\times10^{-5}}{(a+b)},\qquad \sin\theta_3=\frac{12\times10^{-5}}{(a+b)},\qquad \sin\theta_4=\frac{16\times10^{-5}}{(a+b)}$$

$$\sin\theta_2'=\frac{15{\cdot}2\times10^{-5}}{(a+b)},\qquad \sin\theta_3'=\frac{22{\cdot}8\times10^{-5}}{(a+b)},\qquad \sin\theta_4'=\frac{30{\cdot}4\times10^{-5}}{(a+b)}$$

As all these angles are *acute* and $\sin\theta_2'$ is greater than $\sin\theta_3$, therefore the angle of diffraction θ_2' for the red part of the second order spectrum is *greater* than the angle of diffraction θ_3 for the violet part of the third spectrum, that is, the second and third order spectra partly *overlap*. Again θ_3' is greater than θ_4. Thus only the first order spectrum is free from overlapping.

$$\text{Number of lines}=14{,}000 \text{ per inch.}$$

$$=\frac{14000}{2{\cdot}54}\text{ per cm.}$$

$$\therefore\ \text{Grating constant}=\frac{2{\cdot}54}{14000}\text{ cm}$$

$$\sin\theta_4=\frac{16\times10^{-5}\times14000}{2{\cdot}54}$$

$$=0{\cdot}8819$$

and

$$\sin\theta_4'=\frac{30{\cdot}4\times10^{-5}\times14000}{2{\cdot}54}$$

$$=1{\cdot}676$$

From the sine tables θ_4 is found to be equal to $61°\ 54'$. As the sine of an angle cannot be greater than 1, angle θ'_4 can not exist, that is, excepting a small part of the spectrum on the violet side, no light is diffracted in the fourth order spectrum.

Q. 95. Execute a detailed comparison between the spectra produced by a prism and a diffraction grating. Show that it is only the latter that can be used for the determination of the absolute value of the wave-length of monochromatic light. (*Bombay, 1928*)

Ans. **Comparison of Prismatic and Grating Spectra.**

1. A prism gives only *one* spectrum, while a grating gives one or more spectra on each side of the central band.

2. The deviation produced by a prism in light of any wave-length depends on its nature and refracting angle and the wave-length of light used, whereas that produced by a grating, while depending on the wave-length and the grating constant g, is independent of the grating material. If θ is the angle of diffraction for the *nth* order spectrum,

$$g \sin \theta = n\lambda$$

or

$$g \cos \theta . \frac{d\theta}{d\lambda} = n$$

or

$$\frac{d\theta}{d\lambda} = \frac{n}{g \cos \theta}$$

This term $\frac{d\theta}{d\lambda}$ is called **dispersive power** and indicates the change of angle of diffraction with wave-length. Its value shows that it increases with the order of spectrum, and by making g *half*, that is, by doubling the number of lines per unit length, the width of any part of the spectrum is *doubled*.

For this reason the spectrum, of a given source of light, produced by one grating is exactly *similar* to that obtained with any other grating, the only difference being the length of the spectrum, which depends on the grating constant. The relative dispersion (separation) between two wave-lengths produced by one prism in any part of the spectrum is not the same as that produced by a prism of some other material; while in some cases even the *order* of colours is not the same. Thus the prismatic spectra are not necessarily similar and regular.

3. In a grating the sine of the angle of deviation (diffraction) for any kind of light is proportional to its wave-length, but in a prism light of greater wave-length is usually *less* deviated than light of shorter wave-length.

The diffraction grating gives us the *most accurate* method for measuring the absolute wave-length of any part of the spectrum, the relation being

$$g \sin \theta = n\lambda$$

This requires the determination of θ, which can be measured very accurately, and the value of g which is given by the manufacturer. The interference method involves the measurement of *very small* quantities, such as, width of bands and the distance between the images of the coherent sources. As these measurements cannot be carried out with extreme accuracy, the interference method is *inferior* to the grating method.

Q. 96. What is meant by the resolving power of a diffraction grating? Derive the expression for it.

(*Punjab, 1933*)

Ans. The **resolving power** of a diffraction grating is its capacity to separate two *neighbouring* lines in the spectrum. The light of each wave-length forms a separate diffraction pattern of the slit. If these diffraction patterns are close together, they overlap, and the closer they are, the more difficult it is to see them as separate. They can be *just* distinguished as separate, in any order of the spectrum, if the principal maximum of one falls on the *first* subsidiary minimum of the other. If two wave-lengths λ and $\lambda+\delta\lambda$ can be *just* distinguished in a given order of the spectrum, then $\frac{\lambda}{\delta\lambda}$ is called the resolving power of the grating for that order of the spectrum.

Consider a diffraction grating of N *total* number of spaces,

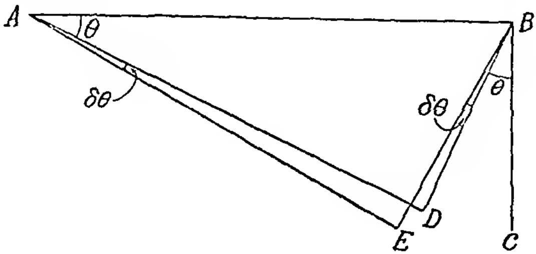


Fig. 75.

between A and B, and grating constant g producing nth order principal maximum for wave-length λ in the direction BD, at angle θ to the normal BC to the grating (Fig. 75). From A draw AD perpendicular on BD. Then BD is the path difference for the rays coming from the *first* and *last* slits in

this direction, and its length l is given by

$$l=(N-1)\ g \sin\theta=(N-1)\ n\lambda, \quad \ldots\ldots (1)$$

as the path difference for rays from the corresponding points of two consecutive slits is equal to $n\lambda$.

In the direction BE, making a *slightly* greater angle $\theta+\delta\theta$ with BC, the path difference BE between the rays coming in this direction from the first and last slits is increased by λ, and is equal to $l+\lambda$.

If the grating is considered to be divided into two halves, the path difference for the rays coming in the direction BE from the *corresponding* slits of the two halves is greater than their path difference in the direction BD by $\frac{\lambda}{2}$. As they reinforce each other in the direction BD, they are in *opposite* phase in the direction BE and, therefore, completely cancel out each other. Thus the *first* subsidiary minimum for the nth order occurs in the direction BE, making angle $(\theta+\delta\theta)$ with the normal to the grating.

If light of wave-length $\lambda+\delta\lambda$ has its principal maximum of the nth order in the direction BE of the first subsidiary minimum for λ, the two wave-lengths can be just distinguished from each other in *this* nth order spectrum. Evidently for this the path difference for rays of wave-length $\lambda+\delta\lambda$ coming from the first and last slits is equal to BE, and its value $l+\lambda$ is given by

$$l+\lambda=(N-1)g\ \sin(\theta+\delta\theta)=(N-1)n(\lambda+\delta\lambda) \quad \ldots\ldots \quad (2)$$

Subtracting (1) from the corresponding sides of (2), we get

$$\lambda=(N-1)n.\delta\lambda$$

or

$$\delta\lambda=\frac{\lambda}{(N-1)n}$$

$$=\frac{\lambda}{Nn},$$

as N is *very large* and $(N-1)\ n$ is practically equal to Nn. Therefore, if two wave-lengths of nearly the same value λ are to be distinguished in the nth order spectrum of a diffraction grating of N total number of lines, the difference between them should be at least equal to $\frac{\lambda}{Nn}$.

$$\text{Resolving power} = \frac{\lambda}{\delta\lambda} = Nn$$

This shows that, for a given order of spectrum, the resolving power is proportional to the *total* number of grating elements used and not their closeness or grating constant g. With increase in N, the bands become sharper and fade off more rapidly.

Q 97. The waves of light are said to be transverse. What is the evidence for it? (*Punjab, 1931*)

Ans. **Longitudinal and Transverse waves.** In a longitudinal wave the vibrations of the particles are *along* the direction of its propagation, and its properties in all planes passing through its line of propagation are the same On the other hand, a transverse wave consists of vibrations *perpendicular* to its line of propagation. These vibrations may take place in *any* plane passing through the line of propagation, and some properties of the wave may depend on the direction of this plane. If the vibrations are confined to a single plane, its properties in a perpendicular plane passing through the line of propagation may be quite different from those in the plane in which vibrations take place. That light consists of transverse waves is proved by the following experiment.

Tourmaline Plate. When a beam of light falls on a *thin* tourmaline crystal, *two* beams emerge from it in different directions. If the thickness of the crystal is more than 1 mm., one of these beams is absorbed by it and only *one* emergent beam is formed.

Two tourmaline plates are cut with their faces containing the optic axis, and are held parallel to one another. A light beam incident on one, and transmitted by it, is transmitted *fully* by the second plate *only* if the axes of the two plates are *parallel* to one another. On rotating the two plates *together* about the beam of light, the intensity of the beam transmitted by the second plate remains *unchanged*.

When only *one* of the plates is rotated, the intensity of the beam transmitted by the second is *reduced*, and *no* light is transmitted when the axes of the two plates are *perpendicular* to each. If in this position the two plates are rotated *together*, with their axes remaining perpendicular, *no* final beam of light is formed. On further rotating one of the

plates, so that the axis of one is inclined to that of the other at an angle other than 90°, some light is transmitted from the second plate. Its intensity increases as this angle is increased, and becomes *maximum* when the angle is 180°, that is, the two axes again become *parallel* to one another. Any further rotation of the plate results in the decrease of the intensity of the transmitted beam.

This shows that light transmitted by the first plate differs from ordinary light, because it is *fully* transmitted by the second plate *only* if the axis of the second plate is in a *particular direction* perpendicular to the direction of propagation of light. This is not possible in the case of longitudinal waves, and, therefore, light consists of transverse waves. Ordinary light consists of vibrations in all directions perpendicular to its line of propagation, and a tourmaline plate transmits vibrations in one particular direction only, related to the direction of its axis, and the components of other vibrations in that direction. When the axis of the second plate is perpendicular to that of the first, the vibrations transmitted by the first plate have *no* component in this direction of the second plate, and, therefore, no light is transmitted by it.

Q. 98. Distinguish clearly between ordinary and polarised light. Explain how light is polarised by reflection and how it can be detected. State Brewster's law and find a relation between the polarising angle and the refractive index of the reflecting plate.

Ans. **Ordinary and Polarised Light.** Light consists of *transverse* waves in the ether. The vibrations of the ether particles in a transverse wave are *perpendicular* to its direction of propagation, and, unlike a longitudinal wave, whose properties with respect to *any* plane passing through its line of propagation are the *same*, its properties may be *different* in different planes, because the vibrations may take place in *any* one of these planes. Transverse waves in which vibrations are confined to any *particular* direction are not *symmetrical*, and exhibit properties related to, and depending on, that direction. This depending of certain properties on direction is called *polarity*, and such waves are said to be *polarised*.

The vibrations are of a *fixed* type, and light waves exhibit different properties in different directions at right angles to

their direction of propagation. When the vibrations are confined to a *single direction*, the waves are said to be *plane polarised*. If the paths of the vibrating particles are annular and circular or elliptical, light is called *circularly* or *elliptically* polarised.

In a beam of ordinary light the vibrations of the ether particles are *not* confined to a particular direction. These vibrations take place successively in different directions in a plane perpendicular to the direction of propagation, but they follow each other so rapidly that they appear to be in all directions, and its properties are the same in all directions in that plane.

Polarisation by Reflection. When light is reflected from a surface, it is found to be partially plane polarised. The degree of polarisation in the reflected light varies with the angle of incidence, and it is *completely* polarised at a certain angle of incidence called **polarising angle** for the given reflecting surface. For glass, the value of the angle is about [illegible]. If the angle of incidence is greater than this, the reflected light is again not completely polarised.

A ray AB is partly reflected along BC and partly refracted along BD at a point B of the reflecting surface EF (Fig. 76). In the incident ray AB, vibrations are taking place successively in all directions, but they follow each other so rapidly that they appear to be in *all* directions perpendicular to AB, and all of them may be considered to be resolved into two parts: (*i*) parallel to the plane of incidence, and (*ii*) perpendicular to this plane.

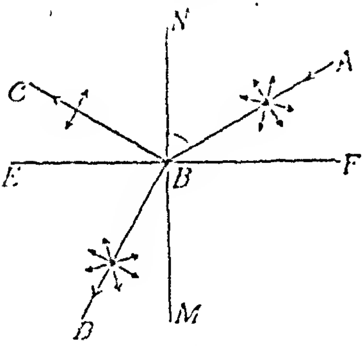


Fig. 76

The vibrations perpendicular to the plane of incidence are always parallel to the reflecting surface, and are reflected for *all* angles of incidence. The vibrations in the plane of incidence are inclined to the reflecting surface, and their degree of inclination and the amount reflected change with

the angle of incidence. At the polarising angle, *all* vibrations which are not parallel to the reflecting surface are transmitted together with *some* vibrations parallel to the surface, while the rest of the vibrations parallel to the reflecting surface are reflected.

If a tourmaline crystal is placed with its axis in the direction of the reflected ray and rotated, light is extinguished in a certain position of the crystal. If now this crystal is placed with its axis parallel to BD, it has to be turned through 90° to decrease the intensity of the transmitted light to a minimum. Thus the reflected light is completely plane polarised, while the transmitted light is partly plane polarised. The reflected light is said to be polarised *in* the plane of incidence, that is, its vibrations are perpendicular to this plane.

The polarisation of the reflected light can also be tested with another plate of the same material. If this second plate is placed to receive reflected light, and its plane of incidence is set *perpendicular* to that of the first, the vibrations in the incident ray on the second plate are *only* in its plane of incidence, and, therefore, no light is reflected from its surface at the polarising angle. In other positions of its plane of incidence or for a different angle of incidence, some light is reflected.

Brewster's Law. Brewster found from experiment that at the polarising angle the reflected and refracted rays are perpendicular to one another. In Fig. 76, according to this experimental law ∠CBD is equal to 90°. Therefore, if NBM is the normal at B, the angle of incidence i (=∠NBA=∠NBC) is the complementary of the angle of refraction r (=∠MBD).

$$i=(90^\circ-r)$$

$$\cos i=\cos(90^\circ-r)$$

$$=\sin r$$

But $$\sin i=\mu \sin r,$$

where μ is the refractive index of the plate.

$$\therefore\ \sin i=\mu \sin r=\mu \cos i$$

or $$\mu=\frac{\sin i}{\cos i}=\tan i$$

Thus the tangent of the polarising angle is equal to the refractive index of the plate.

Q. 99. (*a*) Describe the phenomenon of double refraction, and discuss its connection with the polarisation of light.

Explain the ordinary and the extraordinary rays in a crystal and the construction of a Nicol's prism.
(*Punjab, 1934*)

(*b*) A thin tourmaline crystal allows only a single polarised beam to pass through it and so does a Nicol. Is the action in the two cases similar? (*Punjab, 1931*)

Ans. (*a*) **Double Refraction.** Iceland spar is a form of crystallised calcium carbonate. Its crystals are rhombs, the six faces being parallelograms of angles 102° and 78° nearly (Fig. 77). At *two opposite* corners, A and H, only the angles of *all* the three faces meeting there are obtuse, whereas at other corners the angle of one face is obtuse and of the other two acute. A line *equally* inclined to the three edges meeting at A or H is called the *optic axis* of the crystal. It is a *direction* and not a particular line, and the crystal is *symmetrial* about it. A plane containing the optic axis and perpendicular to two opposite faces of the crystal is called a *principal plane* of these faces.

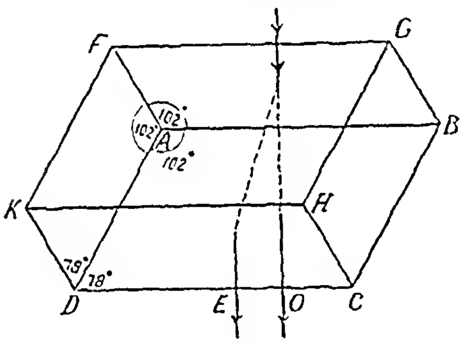


Fig. 77.

When a luminous point is looked at through such a crystal, its *two* images are seen. One of these is called *ordinary* image and is formed in the normal expected position, while the other is called *extraordinary*, as it is formed by rays which do not necessarily follow the ordinary laws of refraction. It is generally formed away from the expected position *even* for normal incidence. If the crystal is rotated about the incident beam of light, the ordinary image remains stationary, but the extraordinary image revolves round the

ordinary image, the line joining the two remaining parallel to the shorter diagonal of the emerging face.

If another crystal is placed, with its principal plane parallel to that of the first, to receive the two emergent beams of light, the two images are *more* widely separated, the ordinary and extraordinary rays from the first giving rise to ordinary and extraordinary rays respectively. On rotating the second crystal about the incident light, *four* images are seen. The ordinary ray O from the first gives rise to an ordinary ray OO′ and an extraordinary ray OE′. Similarly, the extraordinary ray E from the first crystal is divided into an ordinary ray EO′ and an extraordinary ray EE′. Not only OE′ and EE′ rotate about OO′ and EO′ respectively, but all the four images *change* in intensity. The intensity of OO′ and EE′ decreases while that of OE′ and EO′ increases.

When one crystal has been turned through 45°, all the four images are *equally* bright. At 90°, OE′ and EO′ alone are visible. These changes in the intensity of the four images are shown in the following table :—

...	0°	90°	180°	270°	360°
OO′	Maximum	0	Maximum	0	Maximum
OE′	0	Maximum	0	Maximum	0
EO′	0	Maximum	0	Maximum	0
EE′	Maximum	0	Maximum	0	Maximum

When only one crystal is used and rotated, the two images O and E are, and remain throughout, of the *same* intensity.

Nicol's Prism. See Q. 100.

(*b*) A **Tourmaline** crystal also produces double refraction, and splits a beam of ordinary light into two separate ordinary and extraordinary beams. If its thickness is more than 1 mm., the ordinary beam is *absorbed* by it and the extraordinary beam *alone* emerges. Thus there is no need of any special device, as in the case of Nicol, for getting rid of the ordinary beam.

Q. 100. Describe the function and the principle of construction of a Nicol Prism. If a similar prism were prepared from quartz, would it serve a similar purpose? If not, discuss what modifications would be necessary. (*Bombay, 1929*)

Ans. Nicol's Prism. A rhomb of Iceland spar is cut into two halves by a plane AC *perpendicular* to the principal plane of faces AB and CD and passing through its opposite corners A and C containing *three* obtuse angles, and the faces AB and CD are cut so that the plane AC makes an angle of 22° with the edges AD and CB (Fig. 78). The cut faces are polished, and the two halves are cemented together by a *thin* layer of Canada balsam.

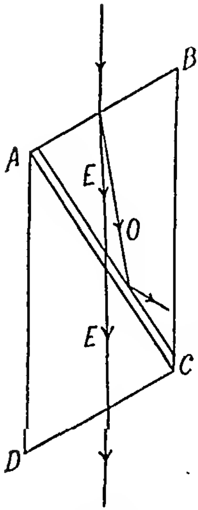


Fig. 78.

A ray of light on entering the face AB is divided into an ordinary ray O and an extraordinary ray E, and they travel with *different* velocities, the velocity of the ordinary ray being *smaller* than that of the extraordinary ray. The velocity of light in Canada balsam lies *between* the velocities of the ordinary and extraordinary waves in calcite, so that Canada balsam is optically *rarer* for the ordinary ray and denser for the extraordinary ray than calcite. Therefore if the plane AC is suitably inclined to make the angle of incidence of the ordinary ray on it greater than the critical angle for the two media, the ordinary ray suffers *total internal* reflection and is absorbed by the tube containing the Nicol. The extraordinary ray proceeds from an optically rarer to a denser medium and its angle of incidence is smaller than that of the ordinary ray, so that it does *not* suffer total internal reflection. It is thus isolated, and plane polarised light is obtained.

When used for obtaining polarised light, the Nicol is called a **polariser.** It may also be used as an **analyser** to find out if light is plane polarised, and if so, in what plane. The transmitted beam (extraordinary) is polarised *perpendicular* to the principal plane of the prism, that is, its vibrations are *in* this plane, and, therefore, a Nicol does *not* transmit vibrations perpendicular to its principal plane. If a second Nicol is held next to the first, with their principal planes *parallel,* light transmitted by the first is *freely transmitted* by the second. On rotating the second Nicol, the intensity of light transmitted by it decreases, and when the principal planes of the two are

perpendicular to each other, *no* light is transmitted, for then the extraordinary of the first becomes *ordinary* for the second and suffers total internal reflection in the second Nicol. Therefore, if light is incident on a Nicol, and on rotating it about the incident light as axis, the transmitted light is completely cut off, the incident light must be plane polarised in the principal plane of the Nicol in *this* position, because then only the ordinary beam alone is formed in it, and this is completely reflected and absorbed by it.

When the incident light is not polarised, its vibrations take place in *all* directions perpendicular to its direction of propagation, and their resultant rectangular components in *any* two directions are *equal*. If a Nicol is rotated about such a beam of light, the intensity of the transmitted beam remains the *same* and about half of that of the incident beam. With partially polarised light, the intensity of the transmitted beam changes, but it is extinguished in *no* position.

Quartz also produces double refraction, but the velocity of the ordinary ray in it, unlike calcite, is *greater* than that of the extraordinary ray. The velocity of light in Canada balsam is *greater* than that of the ordinary and extraordinary rays in quartz, that is, this cement is optically *rarer* than calcite for *both* the ordinary and extraordinary rays. Therefore, if a Nicol of quartz is prepared with Canada balsam, both the rays may suffer total internal reflection or both may be transmitted.

In order that a quartz crystal may be used as a Nicol prism, Canada balsam should be replaced by another transparent cement whose refractive index lies *between* the refractive indices of quartz for the ordinary and extraordinary rays. As in quartz the velocity of the ordinary ray is *greater* than that of the extraordinary ray, the latter alone can suffer total internal reflection, and the transmitted beam is *ordinary*, that is, it is plane polarised in the principal plane of the crystal.

Q. 101. Explain and show by means of Huygen's construction the directions of refracted rays in uniaxal crystals for cases:—

(*a*) Optic axis parallel to the face of the crystal and perpendicular to the plane of incidence,

(*b*) Optic axis parallel to the face of the crystal and also parallel to the plane of incidence,

(*c*) Optic axis inclined to the face of the crystal and in the plane of incidence. (*Bombay, 1935*)

Ans. **Huygens' Construction.** In the case of double refraction each point of the refracting surface becomes a centre of *two* secondary wavelets as the incident wave sweeps over it. The ordinary wavelet travels with the *same* velocity in all directions in the refracting medium and is spherical. The extraordinary wavelet travels along the optic axis with the *same* velocity as the ordinary, because a ray of light incident *normally* on a crystal along its optic axis is *not* separated into two rays and the two emerge without any phase difference, but in other directions its velocity is *different*, the difference being the *greatest* for directions perpendicular to the optic axis. This wavelet is an ellipsoid with its major or minor axis along the optic axis of the crystal according as the velocity of the extraordinary ray is smaller or greater than the velocity of the ordinary ray in directions other than the optic axis.

(*a*) Optic axis parallel to the refracting surface and perpendicular to the plane of incidence. In Fig. 79, AB is the trace of the incident plane wave front and AC of the face of a uniaxal negative crystal, both being perpendicular to the plane of the paper. Here the velocity of the extraordinary ray is *greater* than that of the ordinary. The extraordinary ellipsoid may be considered to be generated by revolving an ellipse about the optic axis as its minor axis, and its section in a plane *perpendicular* to this axis is a *circle*. As in this case the optic axis is parallel to the face of the crystal and perpendicular to the plane of incidence, the section of the extraordinary wavelet in this plane is a circle, that is, the extraordinary ray has the *same* velocity in all directions in the plane of incidence.

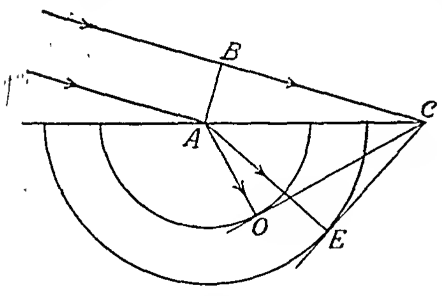


Fig. 79.

When the incident wave front reaches A, it becomes a source of two wavelets. To find the position of these wavelets

in the plane of paper when the incident wave-front reaches C, draw two circles of radii AO and AE equal to $\frac{BC}{\mu_0}$ and $\frac{BC}{\mu_e}$ respectively, where μ_0 and μ_e are the refractive indices of the crystal for the corresponding rays.

From C draw tangent planes to meet these wavelets in O and E. Then CO and CE are the traces of the ordinary and extraordinary wave-fronts, and AO and AE are the corresponding rays. In this case the extraordinary ray obeys both the laws of refraction.

(*b*) **Optic axis parallel to the refracting surface and the plane of incidence.** In this case the optic axis is parallel to AC, and, therefore, the ordinary spherical wavelet and the extraordinary ellipsoidal wavelet touch each other in the refracting surface (Fig. 80). The section of the spherical wavelet is circular, while that of the ellipsoid is part of an ellipse in the plane of paper. Both the ordinary and extraordinary rays, AO and AE, lie in the plane of incidence and their position is found as in the last case.

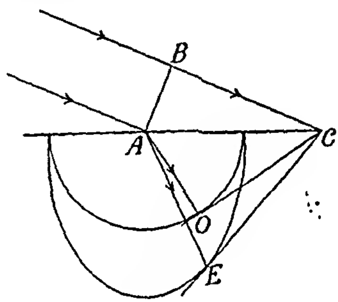


Fig. 80.

(*c*) **Optic axis inclined to the refracting face and in the plane of incidence.** In this case the optic axis at A is along AX and the ordinary spherical wavelet and the extraordinary ellipsoidal wavelet touch each other in the plane of paper at X (Fig. 81). The section of the spherical wavelet is a semi-circle while that of the ellipsoid is a part of an ellipse in the plane of paper. As in the first two cases, both the ordinary and extraordinary rays, AO and AE, lie *in* the plane of paper, and their position is found by drawing tangents CO and CE to the circle and ellipse respectively.

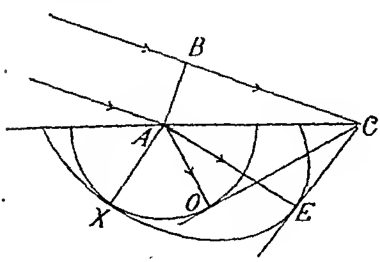


Fig. 81.

Q. 102. A thin pencil of plane polarised light is allowed to be incident normally on a calcite plate cut parallel to the optic axis. Explain what is observed when the plate is rotated about the pencil of light as axis. (*Calcutta, 1935*)

Ans. Let ABCD be the section of the calcite plate held perpendicular to the plane of paper, FG be the thin beam of light incident normally on AB, and let this light be polarised in the plane of paper, that is, its vibrations are perpendicular to the plane of paper. The optic axis of the plate is parallel to the refracting faces. On rotating the plate about FG, its optic axis remains throughout perpendicular to FG but becomes inclined at *different* angles to line AB and the plane of incidence.

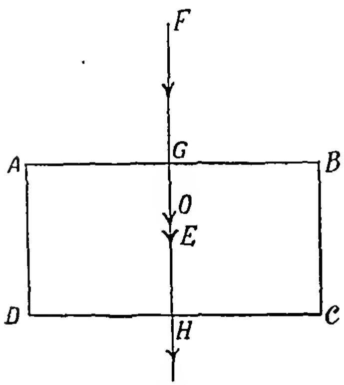


Fig. 82.

When the optic axis is along AB, the principal plane of the plate coincides with the plane of incidence. The incident light is polarised *in* the principal plane of the plate and passes out of it as an *ordinary* beam, the intensity of the transmitted beam being the same as that of the incident beam, if no light is observed by the plate.

On rotating the plate about the incident beam FG as axis, the optic axis becomes inclined to line AB, say, at an angle θ. The principal plane of the plate now makes angle θ with the plane of polarisation of the incident light and $(90°-\theta)$ with the vibrations in the incident beam. If a is the amplitude of the incident vibrations, its rectangular components in and perpendicular to the principal plane are equal to $a \cos (90°-\theta)$ and $a \sin (90°-\theta)$ respectively.

The calcite plate is capable of transmitting vibrations either as ordinary or as extraordinary rays. The ordinary ray is polarised in the principal plane and its vibrations are perpendicular to this plane, while the extraordinary ray is polarised

perpendicular to the principal plane and its vibrations take place in this plane.

$$\text{Amplitude of ordinary vibrations} = a \sin (90^\circ - \theta)$$
$$= a \cos \theta$$
$$\text{,, ,, extraordinary ,,} = a \cos (90^\circ - \theta)$$
$$= a \sin \theta$$
$$\therefore \text{ Intensity of ordinary beam} \propto a^2 \cos^2\theta$$
$$\text{,, ,, extraordinary ,,} \propto a^2 \sin^2\theta.$$

Thus on rotating the plate from its initial position, the single incident beam is divided into two refracted beams, ordinary O and extraordinary E. As the incident light falls normally on the plate and *perpendicular* to its optic axis, the two beams travel along the *same* line GH. The second refracting face of the plate being parallel to the first, no deviation is produced at H, and the emergent beam is in line with FGH. But the two refracted beams travel with *different* velocities, in the plate; the extraordinary beam travels faster than the ordinary, and this introduces *phase difference*, which depends on its thickness and the velocities of the two rays in it. On emerging from the plate, the two rectangular vibrations have different amplitudes and some phase difference, and they recombine into a single vibration.

On increasing angle θ, the intensity of the ordinary beam decreases while that of the extraordinary beam increases, but the sum of their intensities remains the *same* and equal to that of the incident beam ($a^2\sin^2\theta + a^2\cos^2\theta = a^2$). Thus the transmitted beam remains equally bright throughout.

When the principal plane of the plate is inclined at 45° to the plane of polarisation of the incident light, the amplitudes of the ordinary and extraordinary beam are *equal*. On emerging they combine to give elliptically polarised light, the axes of the ellipse being inclined at 45° to the directions of the component vibrations. Two particular cases of this are circularly polarised and plane polarised light when the phase difference is equal to an integral multiple of $\frac{\pi}{2}$ and 0 respectively.

When θ is equal to 90°, the vibrations of the incident beam lie in the principal plane of the calcite plate and have *no* component perpendicular to this plane. In this case the refracted ray in the plate is extraordinary only and emerges as plane polarised with its plane of polarisation *parallel* to that of the incident beam.

For θ between 90° and 180°, 180° and 270°, and 270° and 360°, ordinary and extraordinary vibrations are formed in the plate. At 180° and 360° only ordinary beam is formed, while at 270° extraordinary beam alone is produced, and in all these three cases the transmitted light is plane polarised.

Q. 103. Describe a method of obtaining elliptically polarised light. How will you distinguish experimentally between elliptically polarised light and a mixture of plane polarised and unpolarised light?

(*Punjab, 1938*)

Ans. **Elliptically Polarised Light.** A narrow beam of plane polarised light is made to fall *normally* on a plate of calcite cut with its optic axis *parallel* to its refracting faces. When the principal plane of the plate is inclined at an angle θ to the plane of polarisation of the incident light and a is the amplitude of the incident vibrations, the incident beam is divided into *rectangular* vibrations of ordinary and extraordinary beams with amplitudes equal to $a \cos \theta$ and $a \sin \theta$ respectively.

When θ is equal to 45°, the amplitudes of the ordinary and extraordinary vibrations are *equal*. The two travel along the same line, that is, the normal in the plate at the point of incidence, but with *different* velocities, and, therefore, emerge from the plate with some phase difference, which depends on the thickness of the plate and its refractive indices for the two beams. On emerging from the plate, the two rectangular vibrations combine into a single elliptical vibration, whose major and minor axes are inclined at 45° to the directions of the component vibrations.

In Fig. 83, the two rectangular simple harmonic motions are taking place along ACE and FCI and the positions of the executing particles are denoted for one-eight of the time-period of each. If the vibration along ACE is ahead of the other

vibration by $\frac{\pi}{4}$, the position of the first at B from left to right occurs at the moment when the second is passing through C and is going towards F, and the resultant position is given by B. Similarly, K and L are the resultant positions obtained from A and G upward, and F and B from right to left. Other resultant positions are obtained in the same manner, and we get the ellipse. This is called **elliptically** polarised light. It may also be supposed to be due to two rectangular vibrations of *unequal* amplitudes along its major and minor axes with a phase difference of $\pi/2$.

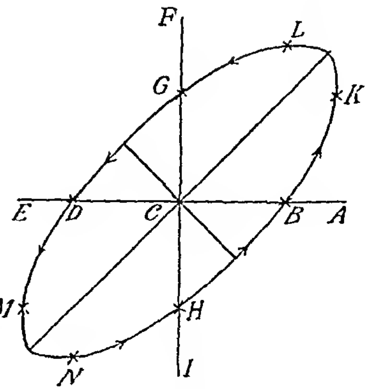


Fig. 83.

Experimental Detection. When elliptically polarised light is passed through a Nicol prism, its intensity changes from a minimum to a maximum when the Nicol is rotated. It is never reduced to zero, and thus it resembles a mixture of plane polarised and ordinary unpolarised light.

A plate of a doubly refracting material cut with its refracting faces containing its optic axis is called a **quarter wave plate**, if its thickness is such that it produces a phase difference of $\pi/2$ (or an odd multiple of $\pi/2$) between the ordinary and extraordinary rays passing through it along a normal to its faces. If the elliptically polarised light is passed through such a plate with its optic axis *parallel* to the major or minor axis of the incident elliptical vibrations, the phase difference between its rectangular components along its two axes is *further changed* by $\pi/2$ and made equal to an integral multiple of π. With this *new* phase difference, the two rectangular components on emerging from the quarter wave plate combine together to give *rectilinear* vibrations, that is, the transmitted light is *plane* polarised. When this light is examined with a Nicol, its intensity changes from maximum to zero in its crossed positions. A mixture of plane polarised and unpolarised light after being

passed through a quarter wave plate is *not* completely stopped in *any* position of the Nicol.

Q. 104. Distinguish between plane polarised and circularly polarised light. Describe how circularly, polarised light is produced and how it can be experimentally distinguished from plane polarised light.

Ans. Light consists of transverse waves in the ether whose particles vibrate perpendicular to the direction of propagation of the waves. In the ordinary light these vibrations take place successively in *all* directions, but they follow each other so quickly that they appear to be executed simultaneously. When the vibrations take place in any *particular* direction, light is said to be *polarised*. It is called plane polarised or circularly polarised according as the vibrations take place parallel to a straight line or in circular orbits.

Production and Detection. See Q. 103. When the thickness of the calcite plate is such that the phase difference between the two equal rectangular components (ordinary and extraordinary) is equal to $\pi/2$ (or odd multiple of $\pi/2$), the resultant elliptical motion becomes *circular*. Conversely a circular vibration may be resolved into two linear simple harmonic vibrations of *equal* amplitudes and phase difference $\pi/2$ in *any* two perpendicular directions.

If circularly polarised light is passed through a Nicol prism and the latter is rotated, *no* change is produced in the intensity of the transmitted light. On rotating the Nicol, the intensity of one rectangular component in any direction decreases while that of the other increases and their sum remains the *same*. In this way circularly polarised light behaves like unpolarised light, and the two cannot be distinguished with a Nicol alone.

If circularly polarised light is passed through a quarter wave plate, a further phase difference of $\pi/2$ (or odd multiple of $\pi/2$) is produced between the two rectangular components, and their resultant phase difference becomes equal to an integral multiple of π. With this *new* phase difference the two rectangular components give plane polarised light. Therefore if circularly polarised light is first passed through a quarter wave plate and then through a Nicol, the intensity of the transmitted light changes from maximum to *zero* in two crossed positions of the

Nicol. No such change is produced by the quarter wave plate in the case of unpolarised light.

Q. 105. Describe the phenomenon of rotatory polarization. Explain in full the main parts of some form of instrument for measuring the strength of sugar solution by means of this property. (*Punjab, 1935*)

Ans. **Rotatory Polarisation.** In Fig. 84, a beam of plane polarised light obtained from a polarising Nicol PN is

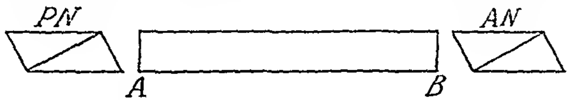


Fig. 84.

passed through a glass tube AB containing water and then falls on another Nicol AN which serves the purpose of an analyser. Light is transmitted completely or stopped altogether according as the principal plane of AN is parallel or perpendicular to the principal plane of PN.

Placing the two Nicols in *crossed* positions, tube AB is emptied and then filled with a sugar solution. It is found that now some light is transmitted by AN although its principal plane is perpendicular to that of PN, and for stopping light completely, AN has to be rotated through a certain definite angle. This shows that plane polorised light on being passed through a sugar solution still remains plane polarised but its plane of polarisation on leaving AB is *different* from that on entering it.

Thus the sugar solution has *rotated* the plane of polarisation of the light incident on it through the same angle as that through which AN has to be rotated to stop light. Such substances are called *optically active.* Some produce clockwise rotation and some anti-clockwise rotation when looking towards the source of light. The amount of rotation that they produce depends on their nature and concentration, length of the tube, wave-length of light used, and temperature. For a given wave-length of light and at a given temperature, the *specific rotation* S of an optically active substance is given by

$$S = \frac{\theta}{lD},$$

where θ is the angle of rotation, l the length of its column

traversed in *decimetre* and D its density. For a solution, D is its concentration in gm. per 100 c.c.

Laurent's Half-Shade Polarimeter. Knowing the specific rotation of a sugar solution, its concentration can be calculated from the observed values of θ and l. An instrument used for this purpose is called a *polarimeter* or *saccharimeter*. The above simple arrangement cannot be used for *accurate* results, because it is very difficult to know the *exact* position of AN when it completely stops light. The intensity of light decreasesg radually with the rotation of the analysing Nicol, and near the position of complete extinction it is very faint.

Sodium light passing through a narrow slit is made parallel by a convex lens L and is then passed through the polarising Nicol PN (Fig. 85). The plane polarised light is then first

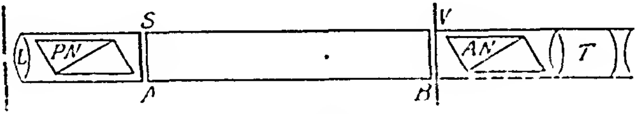


Fig. 85.

passed through a half-shade plate S before passing through the tube AB filled with distilled water. A telescope T is focussed on S, and the analysing nicol AN is adjusted so that the two halves of the half-shade plate appear *eqully* bright as seen through it and its position is noted on the circular vernier scale V.

Then tube AB is filled with the given sugar solution, and once more the position of AN is adjusted by rotating it and the vernier is read. The difference between the two positions of the vernier gives the rotation of the plane of polarisation produced by the sugar solution. Thus knowing S, l, and θ, the concentration of the sugar solution is found out. The experiment is repeated with another tube of half the length. This also removes any ambiguity about the true angle of rotation.

The half-shade plate consists of two semi circular plates one of quartz (shaded) and the other of glass (Fig. 86 *a*). The quartz plate is cut with its refracting faces containing its optic axis, and its thickness is such that a phase difference of π is produced between the ordinary and extraordinary rays of sodium light on passing through it. The glass plate is made

of such a thickness that it absorbs and reflects the *same*

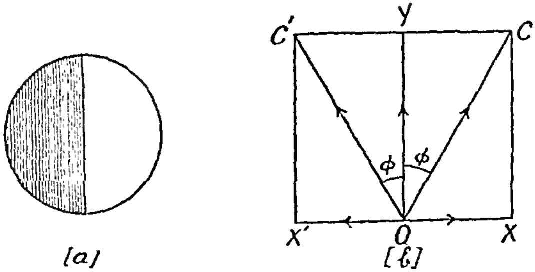


Fig. 86.

amount of light as the quartz plate, that is, the intensity of light transmitted by the two is the *same.*

Plane polarised light from PN on falling normally on the quartz plate is split up into ordinary and extra-ordinary beams. If the optic axis of the quartz plate is parallel to OY and the incident vibrations on it are parallel to OC and their amplitude is represented by this line, then the amplitudes of the ordinary and the extra-ordinary vibrations on entering it are represented by OX and OY respectively, if OXCY is a rectangle, Fig. 86 (*b*). On passing through it a phase difference of π is produced between the two, and on emerging from it if the amplitude of the extra-ordinary vibrations is represented by OY that of the ordinary vibrations is represented by OX', where OX' is *equal* and *opposite* to OX. The two rectangular vibrations again combine, and the amplitude of the resultant vibration is given by OC', which is *equal* to, and makes the *same* angle with OY as, OC.

Thus the quartz plate rotates the plane of polarisation of light coming from PN through 2ϕ, while the plane of polarisation of light passing through the glass plate remains *unchanged.* When the two semi-circular plates are seen through the analyser, the brightness of each will depend on the angle which the plane of polarisation of light coming from it makes with the principal plane of AN, so that generally the two halves do not appear equally bright. When the principal plane of AN is *equally* inclined to the planes of polarisation of light from the two halves, they appear equally bright, and even a *slight* change of the analysing Nicol produces a *large* change between their brightness.

PART IV

SOUND

Q. 106. What produces a simple harmonic motion and what are its characteristic properties?

Two simple harmonic motions of periods 2 : 1 are taking place at right angles. What is the resultant motion?

Consider the effect on the resultant when the ratio of the periods of the component harmonic motions is not an integral number but very nearly so.

(*Bombay, 1930*)

Ans. When a particle is constrained to move in a path so that its acceleration is always directed towards, and is proportional to, its displacement from a fixed point in that path, its motion is called **simple harmonic**.

Characteristic Properties. See Part I, Q. 3 (page 3), for the production of simple harmonic motion and *pp.* 6-7, paras. (1) to (5), for some of its characteristic properties.

(6) *Phase.* The phase of a vibrating particle at any time is its state or condition as regards its position and direction of motion at *that* instant. It is usually measured in terms of the position of the radius OP of the circle of reference with respect to some standard position of it, and is expressed by angle θ described as a fraction of the whole angle 2π, or the time t that has elapsed as a fraction of the time period T (Fig. 87).

(7) *Epoch*. Some time the starting point of the particle in the circle of reference is taken *arbitrarily*, such as D.

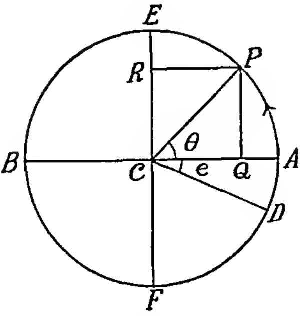


Fig. 87.

$$\angle PCD = \omega t = \theta + e$$
$$\therefore \quad \theta = \omega t - e$$

Angle e, or time $\frac{eT}{2\pi}$, which gives the starting point of the particle P is called its *epoch*. It is not the same as its phase, because its phase *changes* with time but its epoch remains the *same* throughout.

Resultant Motion. Draw two circles of reference of radii CA and C′A′ equal to the amplitudes of the corresponding simple harmonic motions taking place along BA and B′A′ (Fig. 88). Divide the circles into a number of equal parts in the ratio of their corresponding periods (2 : 1) so that each part is travelled in the *same* time in the two cases, and draw lines through these points perpendicular to the paths of simple harmonic motions. As the time periods of the two simple harmonic motions are not the same, the phase difference between them changes from 0 to 2π in one vibration of the *slower* motion, and the form of the resultant curve depends on their phase difference at the *start*.

(1) *Phase difference zero at start*. Starting with the two simple harmonic motions at their extreme positive positions A and A′, the resultant position of the particle is given by M. When the first moves to the left from A to D, the second moves downward from A′ to F, and the resultant position is given by N. Similarly, when the first is at C, E, and B the second is at G, H, and C′ respectively, and the corresponding resultant positions are O, P, and Q. Then the first goes back from B to A and the second from C′ to B′, and the resultant positions are given by R, S, T, and U.

In its next *half* vibration the second goes back from B′ to A′. In the same time the first goes from A to B and back to A, and the resultant position is traced *back* on the curve from U to M. Then this curve is repeated forward and backward over and over again.

(2) If the second motion is ahead of the first by $\pi/2$ at the start, while the first starts from C and goes to the right, the

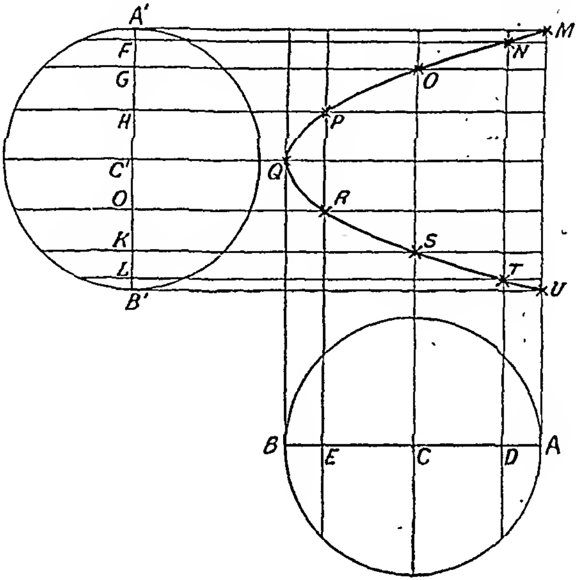


Fig. 88.

second starts from A′ and goes towards C′. Working as before we get the curve in Fig. 89 (a). Proceeding in this way the

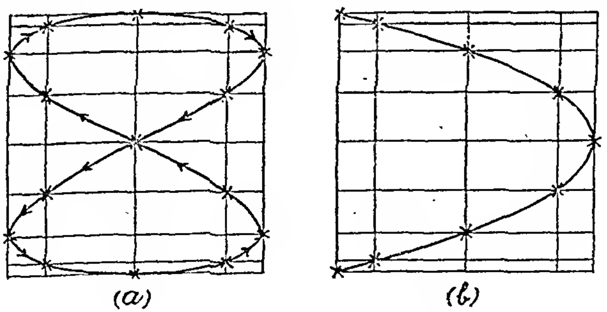


Fig. 89.

resultant curve is obtained for any phase difference at the start. Fig. 89 (b) shows the resultant curve when the second

motion is ahead of the first by π. Here at the start, position A′ in the second motion corresponds to the position B of the first motion.

In all cases where the two periods can be expressed by two simple integral numbers, the particle comes back to the starting point after a time equal to the least common multiple of the two periodic times, that is after the *least* time in which *both* execute an *integral* number of vibrations.

When the two time periods differ very slightly but can be represented by two large integral numbers, the particle describes a large number of curves before coming to the starting point, and then the same series of curves is described again. Let the two time periods instead of being in the ratio of 2 to 1 be 100 to 49. When the first completes half a vibration the second completes not only one vibration but $\frac{1}{49}$ of it more. At the moment when the first completes one vibration, the second completes $2\frac{2}{49}$ vibrations; the second is *ahead* of the first in phase. and their resultant position is *not* the same as at the start. This phase difference goes on increasing and becomes 2π when the first has completed 49 vibrations and the second 100 vibrations. At this moment the resultant position is the same as at the start and the series of curves is again repeated.

If the two periods are *incommensurable*, they can *not* be represented by two integral numbers. In this case the particle executing the resultant vibration *never* comes back to the starting point but goes on describing an *endless* curve.

Q. 107. What are Lissajous' figures? Calculate the resultant of two rectangular simple harmonic vibrations, whose amplitudes as well as periods are in the ratio of 1 : 2, and the phase-difference is 90°.

(*Punjab, 1934*)

Ans. **Lissajous' Figures.** If a particle is impressed simultaneously with two motions, it follows their resultant motion. When the two motions are simple harmonic and perpendicular to each other, the *resultant path traced out by the particle is called a Lissajous' figure.* The nature of this path depends on the *amplitudes* and *frequencies* of the two components and the *phase difference* between them *when* they are impressed on the particle.

Resultant Vibration. As the period of the second simple harmonic motion is twice that of the first, the angular velocity ω of the particle in the circle of reference in the second case is half of the corresponding angular velocity (2ω) in the first case. Let the first simple harmonic motion along the x-axis be of amplitude a, and the second along the y-axis be of amplitude $2a$. If the *second* is ahead of the first by $\pi/2$ at the start the component displacements x and y are given by

$$y=2a\cos\left(\omega t+\frac{\pi}{2}\right)=-2a\sin\omega t \quad . \quad . \quad . \quad (1)$$

$$x=a\cos 2\omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$=a(2\sin^2\omega t+1)$$

$$=2a\sin^2\omega t+a$$

Putting $\sin\omega t$ equal to $\dfrac{-y}{2a}$ from (1), we get

$$x=2a\frac{y^2}{4a^2}+a=\frac{y^2}{2a}+a$$

or $\qquad y^2=2a(x-a)$

This represents a parabola whose axis is parallel to the x-axis.

If the *first* simple harmonic motion is ahead of the second by $\frac{\pi}{2}$, quite a different curve is obtained.

$$y=2\,a\cos\omega t.$$

$$\therefore\ \cos\omega t=\frac{y}{2a}$$

and $\qquad \sin\omega t=\sqrt{1-\dfrac{y^2}{4a^2}}$

$$x=a\cos\left(2\omega t+\frac{\pi}{2}\right)$$

$$=-a\sin 2\omega t$$

$$=-a\,2\sin\omega t\cos\omega t.$$

Putting the values of $\sin\omega t$ and $\cos\omega t$, we get

$$x=-2a\sqrt{1-\frac{y^2}{4a^2}}\times\frac{y}{2a}$$

or
$$x^2 = 4a^2 \frac{(4a^2 - y^2)}{4a^2} \times \frac{y^2}{4a^2}$$
$$= \frac{y^2}{4a^2}(4a^2 - y^2).$$

This represents a figure of the form 8.

Q. 108. Give a general explanation of the manner in which Lissajous' figures may be observed and produced and how they may be practically utilised in acoustical determinations. (*Calcutta, 1921*)

Ans. Lissajous' Figures. See Q. 107.

Two tuning forks, F_1 and F_2, have small mirrors M_1 and M_2 attached to their prongs by *thin* mica sheets, and vibrate in *perpendicular* planes (Fig. 90). The mica sheets are added to increase the effective length of the prongs and thereby increase the extent of vibration of the mirror strips. A *narrow* and strong beam of light coming from S is passed through a convex lens L and falls on M_1. It is reflected from there, falls on M_2, and is then reflected towards the screen O. The lens is adjusted to focus light at O.

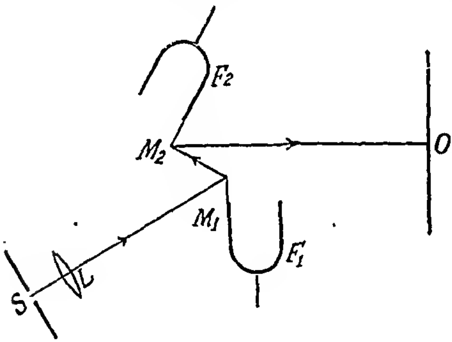


Fig. 90.

The first tuning fork vibrates *in* the plane of paper. While vibrating, its mirror receive light at points at *different* distances from its shank, and if the second fork mirror is stationary, a *straight* line, parallel to the prongs of the fork, is traced by the beam of light on the screen. The second tuning force vibrates in a plane perpendicular to the plane of paper, and when it *alone* vibrates, the beam of light traces a line on the screen *perpendicular* to the first line. When both the tuning forks are set into vibration, the beam of light traces out the *resultant* path, whose shape depends on the

frequencies of the two forks and their phase difference at the *start*.

These figures enable us to find with great accuracy the ratio of the two frequencies if that can be expressed by *small* integers. When the two frequencies are equal, the same figure is traced out over and over again. When they are not equal, the two vibrations starting in phase get out of step ; the phase difference between them goes on increasing, and the shape of the figure traced on the screen goes on, changing, until the fork of shorter period has executed *one* vibration more than the other, for then the two forks are once more in phase. Then the *original* figure is obtained, and once more it goes through the previous changes.

This method may also be used for detecting unison between two forks. If the frequency of one is 200 per sec. and one complete cycle of change of the Lissajous' figure takes 5 seconds, then while the first fork makes 1000 vibrations the second fork makes 1001 or 999 vibrations, and the frequency of the latter is 200·2 or 199·8 per second. The *direction* of change of figure shows which fork is vibrating more quickly and has greater frequency.

Q. 109. Derive a general expression for a displacement of a vibrating particle in a sound wave. Draw displace ment, velocity, and pressure curves, and find their mutual phase relations.

Ans. **Equation of a Harmonic Wave.** Sound consists of longitudinal waves in which the particles of the medium execute simple harmonic vibrations about their *mean* positions and *along* the direction of its propagation. There is a gradual fall of phase *in* the direction of propagation of the wave. If sound is going from left to right, each particle is *ahead* of the next particle on its right and *behind* the last particle on its left in phase. This phase difference between two particles depends on the distance between them, and changes by 2π for a distance equal to the wave-length λ of the waves.

In a simple harmonic motion, usually the angle is measured from the radius CA while the displacement is taken along a perpendicular diameter EF (Fig. 87). If y denotes the displacement CR and θ the angle PCA described in time t with

angular velocity ω in the circle of reference of radius a,

$$y=a \sin \theta=a \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a particle which is at a distance x to the *right* of the first particle, the angle is smaller by an amount proportional to x or equal to Kx, where K is the constant of proportionality, and its displacement is given by

$$y=a \sin (\theta-\mathrm{K}x)=a \sin (\omega t-\mathrm{K}x) \quad . \quad . \quad . \quad (2)$$

When the distance between two particles is λ, their phase difference is equal to 2π.

$$\mathrm{K}\lambda=2\pi$$

or

$$\mathrm{K}=\frac{2\pi}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In one rotation CP describes angle 2π with angular velocity ω, and in the *same* time the wave moves forward through a distance λ with velocity v.

$$\therefore \quad \frac{2\pi}{\omega}=\frac{\lambda}{v}$$

or

$$\omega=\frac{2\pi v}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Putting the values of K and ω in (2), we get

$$y=a \sin\left(\frac{2\pi vt}{\lambda}-\frac{2\pi}{\lambda}x\right)$$

$$=a \sin \frac{2\pi}{\lambda}\left(vt-x\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the displacement were taken along the diameter AB, it would have been equal to $a \cos \frac{2\pi}{\lambda}\left(vt-x\right)$.

Displacement Curve. Divide the circumference of the circle of reference into a number of equal parts and divide a line FH along BA into the same number of equal parts, so that these equal parts indicate *equal* intervals of time (Fig. 91). From the points on the circle draw perpendiculars on ACB, then their lengths give the displacements of the particle vibrating *along* DE. Erect ordinates on the line FGH equal to the displacements at the corresponding points, and draw a smooth curve through their extremities. This will be a

harmonic curve or a *sine* curve, because if length FH corresponds to 360° and the maximum length of the

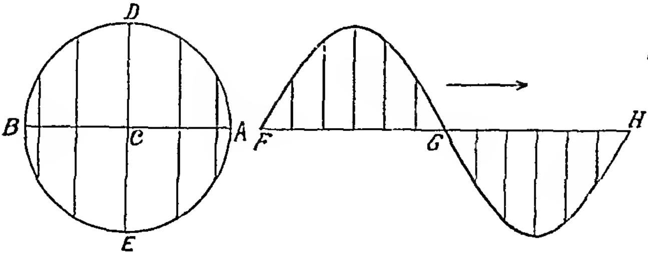


Fig. 91.

ordinate is taken equal to 1 unit, the length of the ordinate at any other point measures the sine of the angle corresponding to that point. Positive and negative ordinates respectively indicate displacement in and opposite to the direction of propagation of the wave.

Velocity Curve. The velocity curve (Fig. 92) is also a harmonic curve, but it lags quarter of a period (or 90°) *behind* the displacement curve, because when the velocity of a vibrating particle is *maximum and positive*, its displacement is *minimum* (zero) and *would* become maximum and positive *after* 90° or a quarter period. In this curve positive and negative ordinates indicate velocity to the right (*in* the direction of propagation of sound) and left respectively.

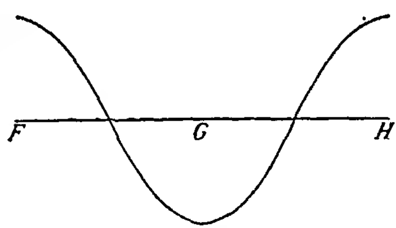


Fig. 92.

Pressure Curve. In sound waves there are *alternate* regions of condensation and rarefaction and they are followed by each other very quickly. In the region of condensation pressure and density are *greater* than their normal values, and *all* the particles are moving *in* the direction of propagation of sound. At the centre of this region there is the *greatest* pressure and density and a particle of the medium here is passing through its *normal* position with *maximum* velocity.

In the region of rarefaction pressure and density are *below* their normal values, and all the particles are moving *opposite* to the direction of propagation of sound. Pressure and density are *greatest* at the centre of this region and a particle there passes through its normal position with *maximum* velocity. At the common boundary of the two regions pressure and density have their *normal* values and a particle here is *stationary*.

Thus the pressure curve is exactly similar to the velocity curve and is in phase with it. The upward ordinates indicate condensation or excess of pressure above the normal, while the downward ordinates represent deficit of pressure or rarefactions.

Q. 110. Obtain an expression for the velocity of transverse vibrations along a stretched string, and thence deduce the frequency of a string vibrating in P segments. (*Punjab, 1937*)

Ans. When a transverse wave moves along a stretched string, its particles vibrate perpendicular to its length, and movement is handed on from particle to particle. A *very small* part AB of it, of length δl, may be considered to be a part of a circle of radius R and centre at C (Fig. 93) and subtend a very small angle $2\delta\theta$ *radian* at the centre, so that

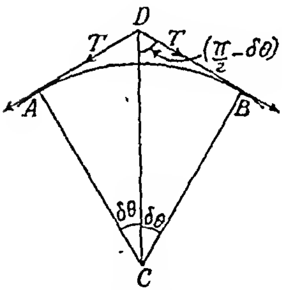


Fig. 93.

$$\delta l = R \times 2\delta\theta \quad . \quad . \quad (1)$$

There is a *uniform* tension (stretching force) T in the string throughout, and acts at A and B along the *tangents* DA and DB respectively. Line DC bisects the angle ACB, and the components of the stretching forces at A and B perpendicular to DC are equal and opposite, and cancel out, while their components along DC are equal and in the *same* direction, each being equal to $T\cos\left(\frac{\pi}{2}-\delta\theta\right) = T\sin\delta\theta = T\delta\theta$.

$\therefore$ Resultant tension along $DC = 2T\delta\theta \quad . \quad . \quad . \quad (2)$

If this string be supposed to be enclosed in a smooth tube of the form of the wave, the reaction of the tube on the string

is in the opposite direction CD. Let the string of mass m per *unit* length move with velocity v from right to left. As its part AB of mass $m\,\delta l$ is moving in a circular path, the centrifugal force on it is equal to $\frac{m\delta l v^2}{R}$. Then the inward force exerted by the string on the tube, and the outward reaction of the tube on the string, are equal to

$$2T\delta\theta-\frac{m\delta l v^2}{R}.$$

If the magnitude of velocity is so adjusted that no force is exerted by the tube, the presence of the tube is not needed to maintain this bend in the string. In this case the string is running to the left or the wave is running along it to the right.

$$\therefore \quad 2T\delta\theta=\frac{m\delta l.v^2}{R}$$

$$=\frac{mR2\delta\theta.v^2}{R} \text{ from (1)}$$

or
$$v^2=\frac{T}{m}$$

and
$$v=\sqrt{\frac{T}{m}} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

This shows that a wave of *any* length will travel with the same velocity, provided T and m are the same.

If the frequency of vibration of the string is equal to n and λ is the wave-length, v is equal to $n\lambda$, and equation (3) becomes

$$n\lambda=\sqrt{\frac{T}{m}}$$

or
$$n=\frac{1}{\lambda}\sqrt{\frac{T}{m}} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

The length of one segment is equal to $\frac{\lambda}{2}$, and that of P segments $\frac{P\lambda}{2}$. If l is the *total* length of the stretched string,

$$l=\frac{P\lambda}{2};$$

or

$$\lambda = \frac{2l}{P}$$

$$\therefore \quad n = \frac{P}{2l}\sqrt{\frac{T}{m}}$$

Q. 111. Explain how stationary waves are produced in a string and state their characteristic properties.

Ans. **Production of Stationary Waves.** When two wavetrains of the *same* period, wave-length, amplitude, and velocity travel in *opposite* directions along a straight line, they interfere and give rise to waves which are *not progressive.* These resultant waves (undulations) are called **stationary waves.** In the case of a string they are produced by the superposition of direct waves and the waves reflected from its fixed ends.

In Fig. 94 (a) a transverse wave is proceeding from left to

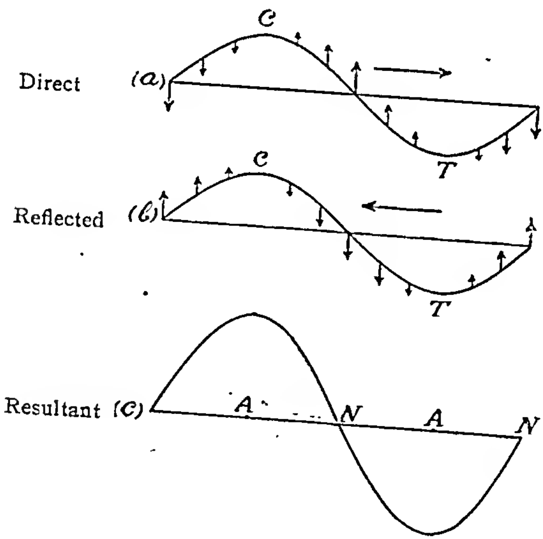


Fig. 94.

right, C being one crest and T the next trough, and the

arrowheads indicate the direction of motion of the particles. The lower curve, Fig. 94(*b*) indicates the reflected wave moving in the opposite direction. At this moment the crests of the two waves are in the *same* position and so are their troughs.

The displacement of any particle due to one wave is in the *same* direction as that due to the other and, therefore, they are added up to give the resultant displacement of any particle, that is, the resultant displacement of any particle is equal to twice its displacement due to either wave, as the amplitudes of the two waves are equal, Fig. 94 (*c*). On the other hand, the velocity of any particle due to one wave is equal and *opposite* to its velocity due to the other wave, and, therefore, not only the particles at crests and troughs but *all* the other particles also are at *rest*. All the particles in the left half are displaced upward by the greatest amount, while those in the right half are displaced downward to their maximum extent. The displacement of the particles at points A is the greatest and gradually reduces to zero at points N.

After a quarter time period each wave has moved a distance equal to a quarter wave length, and their positions are shown in Fig. 95. Here the troughs and crests of one wave train fall over the crests and troughs respectively of the other, and the velocity of any particle due to one wave is equal and opposite to its velocity due to the other. The result is that at this moment all the particles are passing through their *normal* position, each having *its* maximum velocity. All the particles in the left half are moving downward, while those in the right half are moving upward. The velocity of particles at points A is the greatest and gradually reduces to zero at points N.

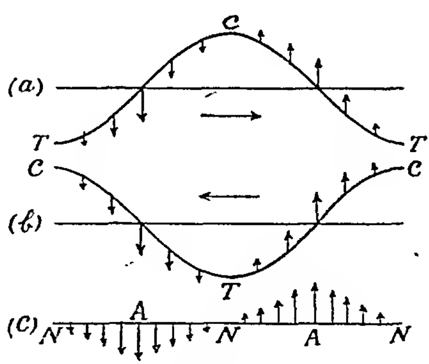


Fig. 95.

After half a time period the condition is represented in Fig. 96. Here all the three curves are *opposite* to the corresponding curves of Fig. 94. As a result of the superposition of the two waves, all the particles are at rest and the displacement of each is maximum, but now all the particles of left-half are displaced downward, while those of the right-half are displaced upward.

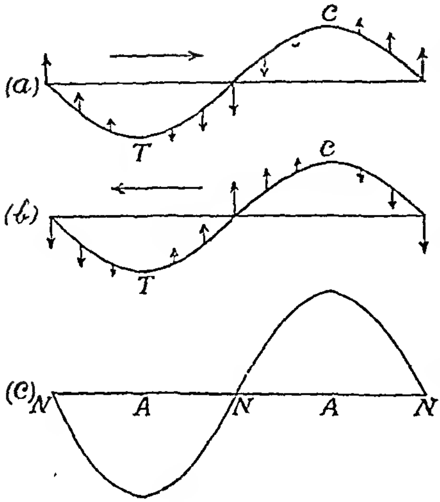


Fig. 96.

After another quarter period, that is, three quarter period from the start, the condition is represented in Fig. 97. Here all the three curves are opposite to the corresponding curves of Fig. 95. In the resultant curve, *all* the particles are passing through their *normal* positions, each moving with its *maximum* velocity but in the opposite direction to that in Fig. 95. Then this condition is gradually changed to the condition given in Fig. 94, and when one period has elapsed, these motions are repeated over and over again.

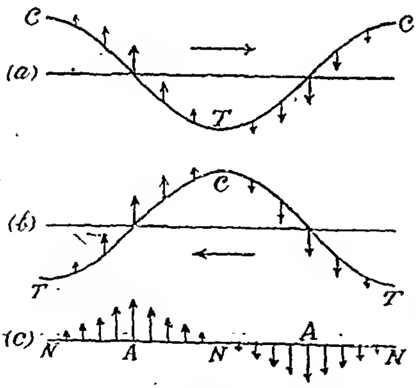


Fig. 97.

Characteristic Properties. 1. These stationary waves are not progressive, that is, the condition of vibration of a particle is *not* passed on to the next particle, and there is *no*

forward or backward motion, simply the amplitude of particles changes from one side to the other.

2. Particles at the points N are *never* displaced, though they suffer the *greatest* change of strain, and are called **nodes.** All other particles are displaced, and their displacement at any moment increases gradually with their distance on either side of a node. Midway between the nodes are points A where at any instant velocity and displacement are *greatest* than at other points, and there is *no* strain. These points are called **antinodea.**

3. All the changes are gone through in one time period of the component waves, and the wave-length of the stationary waves is the *same* as that of the components, the distance between two consecutive nodes or antinodes being equal to half the wave-length.

4. Twice in one vibration all the particles are stationary, and the displacement of each particle is maximum. After half a time period this maximum displacement is on the other side.

5. Twice in one vibration all the particles are in their normal positions, and each has its maximum velocity. After half a time-period all of them pass through their normal positions with maximum velocity, but in the opposite direction.

6. For half a period all the particles in a segment between any two consecutive nodes move in one direction and in the next half in the opposite direction. At any instant the direction of motion of particles in any segment is opposite to that of the particles in the next or last segment.

[**Analytical Treatment.** In a harmonic wave of wave-length λ and moving with velocity v the displacement y of a particle at a distance x from some fixed point as origin at time t is given by

$$y=a \sin \frac{2\pi}{\lambda}(vt-x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a is its amplitude of vibration. At the fixed end of a string a crest is reflected as a trough, and a trough as a crest, and, therefore, for the reflected wave a is negative as compared with its positive value for the incident wave. The value of

x is also negative, as the reflected wave travels in the opposite direction, and, therefore, the displacement due to it is given by

$$y = -a \sin\frac{2\pi}{\lambda}(vt+x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If y_1 and y_2 represent the displacements at a point due to the incident and reflected waves respectively, its resultant displacement y is equal to their algebraic sum.

$$y = y_1 + y_2$$

$$= a \sin\frac{2\pi}{\lambda}(vt-x) - a \sin\frac{2\pi}{\lambda}(vt+x)$$

$$= 2a \cos\left[\frac{\frac{2\pi}{\lambda}(vt-x)+\frac{2\pi}{\lambda}(vt+x)}{2}\right] \sin\left[\frac{\frac{2\pi}{\lambda}(vt-x)-\frac{2\pi}{\lambda}(vt+x)}{2}\right]$$

$$= 2a \cos\frac{2\pi}{\lambda}vt . \sin\frac{2\pi}{\lambda}(-x)$$

$$= -2a \cos\frac{2\pi}{\lambda}vt . \sin\frac{2\pi}{\lambda}x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This shows that the displacement at a given point changes with time and also that at a given time the displacement at different points is different. At points (*nodes*) at distances equal to 0, $\lambda/2$, and even multiple of $\lambda/4$ the displacement is *always* zero. At other points (*antinodes*) at distances $\lambda/4$ and *odd* multiple of $\lambda/4$ the displacement at a given time is maximum, that is, it is *greater* than at any other point and is positive and negative at alternate points. The displacements at points at the same distance on the two sides of a node are equal and opposite.

If the period of vibration of a particle due to either wave is equal to T, the wave travels a distance λ in this time, or λ/v is equal to T, and the above equation may be put in the form

$$y = -2a \cos 2\pi\frac{t}{T} \sin\frac{2\pi}{\lambda}x \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The displacement at a given point changes with time. It is maximum for t equal to an *even* multiple of T/4 and is positive and negative for these alternate values of t, that is, it changes from its maximum value on one side to its maximum value on the other side in T/2. Similarly, the displacement is zero

for t equal to an *odd* multiple of T/4, so that the interval between its consecutive passages through its normal position is equal to T/2. Further, the displacement at *all* points is *zero at the same time*. Therefore the period of vibration of a particle due to the resultant disturbance is the same as that due to the component waves.

By differentiating y with respect to t (x constant) and x (t constant) velocity of the particle and strain at it respectively are obtained.

$$\text{Velocity} = \frac{dy}{dt} = \frac{4a\pi v}{\lambda} \sin \frac{2\pi}{\lambda} vt \sin \frac{2\pi}{\lambda} x \quad . \quad . \quad (5)$$

This shows that the velocity of particles at x equal to zero or *even* multiple of $\frac{\lambda}{4}$ (nodes) is *always* zero, and at any instant particles at x equal to *odd* multiple of $\frac{\lambda}{4}$(antinodes) have *maximum* velocity, while other particles have velocities lying between these extremes and decreasing gradually from a maximum at an antinode to zero at the next node. Further when the displacement ($\propto \cos \frac{2\pi}{\lambda} vt$) of a given particle is *maximum*, its velocity ($\propto \sin \frac{2\pi}{\lambda} vt$) is *minimum*, and *vice versa*, and the period of variation of the two is the *same*.

$$\text{Strain} = \frac{dy}{dx} = -\frac{4a\pi}{\lambda} \cos \frac{2\pi}{\lambda} vt \cos \frac{2\pi}{\lambda} x \quad . \; . \; . \; . \quad (6)$$

This indicates that at points at x equal to zero or *even* multiple of $\frac{\lambda}{4}$ (nodes) strain is maximum, while at points at x equal to an *odd* multiple of $\frac{\lambda}{4}$ (antinodes) it is always zero. At other points it lies between these extremes and decreases gradually from its maximum value at a node to its minimum value at the next antinode on either side of it. The strain at

a point changes with time and is maximum when its displacement is minimum, and *vice versa*.]

Q. 112. Describe and explain Melde's experiment. In an experiment it was found that the string vibrated in 5 loops when 10 grammes were placed in the scale pan. What mass must be placed in the pan to make the string vibrate in 7 loops? (Neglect the weight of the scale pan.) (*Punjab, 1933*)

Ans. **Melde's Experiment.** A light string AB, lying in the plane of paper, is fixed at one end to one prong of a tuning fork F (Fig 98). It passes over a frictionless pulley, and at the other end it carries a scale-pan P in which weights are placed to keep it stretched. Set the fork vibrating *in* the plane of paper and adjust the weight in the scale pan until the string just begins to vibrate in a *single* segment *in* the plane of paper.

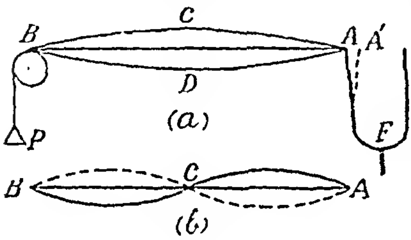


Fig 98

The prongs of the fork move to left and right. When the left prong is in the left extreme position A, the distance between it and B is the smallest; the string is curved and is, say, in the position ACB. Then this prong moves to the right and the string is straightened out. When it is in the right extreme position A′, the distance between it and B is the greatest, and the string in in its normal position along a straight line midway between ACB and ADB.

In its next half vibration this prong of the fork goes to the left, and the string moves on downward. When the prong is once more in its left extreme position A, the string is in its lowermost position ADB. Thus while the tuning fork has executed one vibration the string has completed half a vibration, and, therefore, the frequency of vibration of the string is *half* of that of the fork. By adjusting the stretching force and the length of the string, it can be made to vibrate in any

number of segments, but its frequency is always half of that of the tuning fork.

Next the tuning fork is turned round to vibrate in a plane *perpendicular* to the plane of paper, when it is found that, with the same stretching force, the string vibrates in *two* segments *perpendicular* to the plane of paper [Fig. 98 (*b*)]. In this case the prong of the fork draws the end of the string along with it, and when the prong completes one vibration the string also executes *one* vibration, that is, the frequency of vibration of the string is the *same* as that of the fork.

When the prong is passing through its normal position, the string is also in its normal condition and lies along a straight line. In one extreme position of the prong, the string has the form given by the full line, while in the second extreme position of the prong, after half a time period, the condition of the string is given by the dotted line. At any time the two segments are moving in *opposite* directions. By suitably adjusting the stretching force and the length of the string, it may be made to vibrate in any number of segments, but in all cases its frequency is equal to that of the fork. If the number of segments in the first position of the tuning fork is P, it is always 2P in the second position.

Problem. The frequency n of the transverse vibrations of a string of length l, and mass m per unit length, stretched by force T, and vibrating in P segments is given by

$$n=\frac{P}{2l}\sqrt{\frac{T}{m}}$$

With the same tuning fork connected to the string in the same way, the frequency of vibration of the string remains the *same*, and, therefore, for the same value of n, l, and m, P is inversely proportional to the square root of T. If the stretching force in the second case is x gm. wt., then

$$\sqrt{\frac{x}{10}}=\frac{5}{7}$$

or
$$x=\frac{10\times 25}{49}=5{\cdot}101 \text{ gm. wt.}$$

Q. 113. Derive an expression for the velocity of sound in a gas, and hence show that the change in the velocity of sound in air is about 0·6 metre per degree (contigrade) change in temperature. [Velocity in air at 0° C=330 metres per sec.] *(Punjab, 1938)*

Ans. **Velocity of Sound.** In the following discussion it is assumed that the displacements of the particles of the gas is *very small* as compared with the wave-length of sound waves and at any moment the state of displacement is the *same* at all points in a plane perpendicular to the line of propagation of sound.

First Method. Let sound waves be proceeding from right to left with a velocity V, and wind be blowing with an *equal* velocity in the *opposite* direction, so that for any particular point above the earth the values of pressure and density remain the *same* (Fig. 99). Suppose plane A, perpendicular to the direction of propagation of sound, is in a *normal* condition, while plane B is in a region of *rarefaction*, and P, V, D, and P_1, V_1, D_1, are the corresponding values of pressure, resultant velocity of *air* and density respectively.

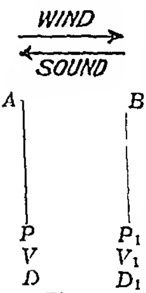


Fig. 99.

Amount of air passing in a unit time through a unit area of A is equal to VD, and is *equal* to the corresponding amount V_1D_1 passing out of B, because there is *no change* of density at any point.

$$\therefore \quad V_1D_1 = VD$$

or

$$V_1 = \frac{VD}{D_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As D_1 is *less* than D, V_1 must be greater than V, and gain of momentum per unit time is equal to $(D_1V_1 \times V_1 - DV \times V)$ or $D_1V_1^2 - DV^2$. According to the second law of motion, this rate of change of momentum per unit area is equal to the difference of forces per unit area at A and B.

$$\therefore \quad P - P_1 = V_1^2D_1 - V^2D$$

Putting the value of V_1 from (1)

$$P - P_1 = \frac{V^2D^2D_1}{D_1^2} - V^2D = \frac{V^2D(D - D_1)}{D_1} \quad . \quad . \quad (2)$$

A unit of mass air occupies volume $\frac{1}{D}$ at A, and it becomes $\frac{1}{D_1}$ at B. The change of volume is equal to $\frac{1}{D_1}-\frac{1}{D}$, and the volume strain is equal to $\dfrac{\frac{1}{D_1}-\frac{1}{D}}{\frac{1}{D}}$ or $\frac{D-D_1}{D_1}$

$$\therefore \quad E=\frac{P-P_1}{\frac{D-D_1}{D_1}}$$

or
$$(P-P_1)=\frac{E(D-D_1)}{D_1}, \quad \ldots\ldots \quad (3)$$

where E is the volume elasticity of air and $P-P_1$ is the stress.

Equating the two values of $P-P_1$ in (2) and (3), we get

$$\frac{V^2D(D-D_1)}{D_1}=\frac{E(D-D_1)}{D_1}$$

or
$$V^2=\frac{E}{D}$$

and
$$V=\sqrt{\frac{E}{D}} \quad \ldots\ldots\ldots \quad (4)$$

As at A the density is normal and the air particles have got no velocity due to sound waves, the velocity V of wind at this point is equal to the velocity of sound.

Problem. Let V metres per second be the velocity of sound at t°C and D and D_0 the densities of air at t° C (273°+t absolute) and 0° C (273° absolute) respectively.

$$330=\sqrt{\frac{E}{D_0}} \quad \ldots\ldots\ldots \quad (5)$$

Dividing (4) by the corresponding sides of (5), we get

$$\frac{V}{330}=\sqrt{\frac{D_0}{D}}$$
$$=\left(\frac{273+t}{273}\right)^{\frac{1}{2}},$$

as the density of a gas varies *inversely* as its *absolute temperature.*

or
$$V=330\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$
$$=330\left(1+\tfrac{1}{2}\times\frac{t}{273}+\text{negligible terms}\right)$$

if t is *very small* compared with 273, so that further terms containing the higher powers of $\frac{t}{273}$ are negligible.

or
$$V=330+\frac{330\,t}{546}$$

$\therefore$ Change of velocity for $t°C = V-330$
$$=\frac{330\,t}{546} \text{ metre per sec.}$$

and „ „ „ „ $1°C = \frac{330}{546}=0{\cdot}6044$ metre per sec.

Second Method. Let sound waves be travelling from left to right along the X-axis OX, and let the displacement at B at a distance x from the origin O be y (Fig. 100). Consider two planes of area A at B and C perpendicular to OX and a very small distance δx apart. Let the force per unit area on plane B be equal to F.

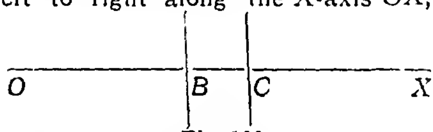


Fig. 100

$$\text{Force on plane B}=\text{A}\times\text{F}$$
$$\text{Rate of change of force}=\frac{d(\text{AF})}{dx}=\text{A}\frac{d\text{F}}{dx}$$
$$\therefore \text{ Change of force for } \delta x=\text{A}\frac{d\text{F}}{dx}\delta x$$
$$\text{Mass of air between A and B}=\text{A}\delta x\times\text{D},$$
where D is the density of air.
$$\text{Displacement at B}=y$$
$$\text{Acceleration of air particles}=\frac{d^2y}{dt^2}$$

Therefore according to the second law of motion,
$$\text{A}.\frac{d\text{F}}{dx}.\,\delta x=\text{A}.\delta x.\text{D}\frac{d^2y}{dt^2}$$

or
$$\frac{dF}{dx} = D\frac{d^2y}{dt^2} \quad \ldots\ldots\ldots \quad (1)$$

Rate of change of displacement $= \frac{dy}{dx}$

Displacement at $C = y + \frac{dy}{dx}\delta x$

$\therefore$ Change of volume between C and $B = A\left(y + \frac{dy}{dx}\delta x - y\right)$

$$= A\frac{dy}{dx}\delta x$$

$$\text{Volume strain} = \frac{A\frac{dy}{dx}\delta x}{A\delta x} = \frac{dy}{dx}$$

Stress (force per unit area) $= F$

$$\therefore \quad \text{Elasticity } E = \frac{\text{stress}}{\text{strain}} = \frac{F}{\frac{dy}{dx}}$$

$$\text{or} \quad F = E\frac{dy}{dx}$$

and
$$\frac{dF}{dx} = E\frac{d^2y}{dx^2}$$

Putting this value in (1), we get

$$E\frac{d^2y}{dx^2} = D\frac{d^2y}{dt^2} \quad \ldots\ldots\ldots \quad (2)$$

In a sound wave of length λ and moving with velocity V, the displacement y of a particle at a distance x from the origin at time t is given by

$$y = a \sin\frac{2\pi}{\lambda}(Vt - x),$$

where a is its maximum displacement.

$$\therefore \quad \frac{dy}{dx} = -\frac{2\pi}{\lambda}a \cos\frac{2\pi}{\lambda}(Vt - x)$$

and
$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2}a \sin\frac{2\pi}{\lambda}(Vt - x) \quad \ldots \quad (3)$$

also
$$\frac{dy}{dt} = \frac{2\pi V}{\lambda}a \cos\frac{2\pi}{\lambda}(Vt - x)$$

and
$$\frac{d^2y}{dt^2} = -\frac{4\pi^2V^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(Vt - x) \quad . \quad . \quad . \quad (4)$$

Putting these values of $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ in (2), we get

$$-E\frac{4\pi^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(Vt - x) = -D\frac{4\pi^2V^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(Vt - x)$$

or
$$E = DV^2$$

$$\therefore \quad V = \sqrt{\frac{E}{D}}$$

Q. 114. Derive a general expression for the velocity of sound in a gas and discuss formulae due to Newton and Laplace.

Calculate the velocity of sound in air at N.T.P. (The ratio of specific heats for air $= 1{\cdot}4$) (*Calcutta, 1933*)

Ans. See Q. 113 for deriving the general expression $U = \sqrt{\frac{E}{D}}$, where U is the velocity of sound in a gas of volume elasticity E and density D.

Newton's Formula. Newton thought that the propogation of sound in a gas takes place under *isothermal* conditions. When the temperature of a gas remains the *same*, the product of the volume V of a given mass of it and pressure P remains constant.

$$PV = \text{constant}$$

$$\therefore \quad P\frac{dV}{dP} + V = 0$$

or
$$P = \frac{dP}{-\frac{dV}{V}} = E,$$

as dP and $-\frac{dV}{V}$ are equal to stress and strain respectively.

$$\therefore \quad U = \sqrt{\frac{P}{D}}$$

The value of U calculated from this equation by putting the values of P and D is found to be *too much below* the value

found by experiment, and this big discrepancy cannot be attributed to any experimental error.

Laplace's Formula. Laplace has pointed out that the propagation of sound in a gas takes place under *adiabatic* conditions and *not* isothermal conditions. Sound travels very rapidly, about 1100 ft. per sec. in air, and there is a sudden increase or decrease of pressure at a point according as it is in a region of condensation or rarefaction. In the first case heat is produced and there is a rise of temperature, while in the second case fall of temperature takes place. As air has a very poor thermal conductivity, there is no time for heat being given out to, or absorbed from, the surrounding region to bring the temperature to its original value. Therefore the propagation of sound takes place under adiabatic conditions, and E represents the *adiabatic elasticity* and not the isothermal elasticity of the gas.

In *both* the cases, the adiabatic elasticity is *greater* than isothermal elasticity. In the region of condensation there is increase of pressure and the gas is squeezed, but due to sudden compression temperature rises and volume increases. Thus the decrease of volume is smaller than it would be if there is no rise of temperature.

In the region of rarefaction, pressure decreases and the gas expands, but due to sudden expansion temperature falls and volume decreases, and the expansion of volume is smaller than it would have been had there been no fall of temperature. Thus in both the cases, for a given stress, the strain is *smaller* and elasticity is *greater* than if there was no change of temperature.

If γ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume, then

$$PV^{\gamma} = \text{constant}$$

$$\therefore\ P\gamma V^{\gamma-1}\frac{dV}{dP} + V^{\gamma} = 0$$

or

$$\gamma P = \frac{dP}{-\frac{dV}{V}} = E$$

Thus the adiabatic elasticity is equal to γP and not P, and the original equation for the velocity of sound becomes

$$U=\sqrt{\frac{\gamma P}{D}}$$

Problem.

Atmospheric Pressure $P=76\times 13{\cdot}59\times 981$ dynes/sq. cm.

Density of air at N.T.P $=0{\cdot}001293$ gm./c.c.

$$\gamma=1{\cdot}4$$

$$\therefore \text{ Velocity of sound}=\sqrt{\frac{1{\cdot}4\times 76\times 13{\cdot}59\times 981}{0{\cdot}001293}}$$

$$=33120 \text{ cm./sec.}$$

Q. 115. Explain the formation of the stationary waves in air in a tube, and state how their properties differ from those of a progressive wave.

Ans. When sound waves are reflected from the closed or open end of a tube, interference occurs between the direct and the reflected waves, and stationary waves are produced. These resultant waves are *not progressive* and may better be called undulations. In a longitudinal wave of wave-length λ, amplitude a, and time period T, the displacement y_1 of a particle at a distance x from some fixed point at time t is given by

$$y_1=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \quad \ldots\ldots \quad (1)$$

(*a*) *Closed End.* When reflection occurs at the closed end of a pipe, or from a medium of smaller freedom of motion, a condensation is reflected as a condensation, and a rarefaction as a rarefaction. As the direction of motion of a particle is *in* or *opposite* to the direction of propagation of sound according as it is in a region of condensation or rarefaction and the reflected wave moves *opposite* to the incident wave, the amplitude of a particle due to the reflected wave is *opposite* to that due to the direct wave, and the value of x is also *negative*. Hence the displacement y_2 of a particle due to the reflected wave is given by

$$y_2=-a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right), \quad \ldots\ldots \quad (2)$$

and its resultant displacement y is given by

$$y = y_1 + y_2$$

$$= a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) - a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$= 2a \cos \frac{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)}{2} \sin \frac{2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) - 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}{2}$$

$$= 2a \cos \frac{2\pi t}{T} \sin \frac{2\pi x}{\lambda} \quad \ldots\ldots\ldots\ldots \quad (3)$$

Thus the displacement of a particle depends on its *position* x and varies with time. At points where x is equal to zero (closed end) or an *even* multiple of $\frac{\lambda}{4}$, $\sin \frac{2\pi x}{\lambda}$ is equal to zero, and there is *never* any displacement. These points are separated by a distance equal to $\lambda/2$ and are called *nodes.*

Midway between nodes lie the points where x is equal to an *odd* multiple of $\frac{\lambda}{4}$. Here $\sin \frac{2\pi x}{\lambda}$ has its *maximum* value, and at any time their displacement is *greater* than at the other points. These points are called *antinodes*, and, like nodes, are separated by a distance equal to $\lambda/2$. The displacements at alternate antinodes are always equal and *opposite.*

At other points displacement lies between these extremes, and decreases gradually from its maximum value at an antinode to zero at the nodes on either side of it. All the particles reach their maximum or minimum displacement at the *same* time, and at any time *all* the particles in a segment, from one node to the next, are displaced in one direction, while those in the last and next segment are displaced in the *opposite* direction.

The displacement at a point at any time is equal and opposite to its displacement after T/2. Starting with maximum displacement of a particle, it becomes zero after T/4, maximum and opposite after T/2, again zero after 3T/4, and reaches its original value after T, so that the period of the stationary waves is the *same* as that of the component waves.

Velocity. The velocity of a particle is obtained by differentiating its displacement with respect to time t (x constant).

$$\text{Velocity} = \frac{dy}{dt} = \frac{-4a\pi}{\text{T}} \sin\frac{2\pi t}{\text{T}} \sin\frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad (4)$$

It is *always* zero at nodes where x is equal to zero or an even multiple of $\frac{\lambda}{4}$, so that $\sin\frac{2\pi x}{\lambda}$ is zero, and at any time it is *maximum* at antinodes where x being equal to an odd multiple of $\frac{\lambda}{4}$, $\sin\frac{2\pi x}{\lambda}$ has its maximum value of $+1$ or -1. At other points at any time it lies between these extremes, and gradually decreases on either side of an antinode.

The velocity of a given particle varies with time. It changes from maximum in one direction to zero in T/4 and takes further time T/4 in reaching its maximum value in the opposite direction. Thus the period of variation of velocity of a particle is the same as that of its displacement. The condition of maximum or zero velocity is reached by all the particles at the *same* time. When the displacement $\left(\propto \cos\frac{2\pi t}{\text{T}}\right)$ of a particle is maximum, its velocity $\left(\propto \sin\frac{2\pi t}{\text{T}}\right)$ is zero, and *vice versa.*

Change of Pressure or Density. The strain at a point is obtained by differentiating its displacement y with respect to x (t constant).

$$\text{Strain} = \frac{dy}{dx} = \frac{4a\pi}{\lambda} \cos\frac{2\pi t}{\text{T}} \cos\frac{2\pi x}{\lambda} \quad . \quad . \quad . \quad . \quad (5)$$

At the nodes, where x is equal to zero or an even multiple of $\lambda/4$, the *change* of pressure is maximum and is positive and negative at alternate nodes, that is, alternate nodes are centres of condensation and rarefaction, and each node is alternately a centre of condensation and rarefaction. At the antinodes, where x is an odd multiple of $\frac{\lambda}{4}$, $\cos\frac{2\pi x}{\lambda}$ is equal to zero, and pressure and density are *always normal.* The change of pressure at other points lies between these extremes and decreases on either side of a node.

The change of pressure and density at a point varies with time. At any instant its value is equal and opposite to its value after T/2, and again becomes the same after T.

(*b*) *Open End*. At the open end of a tube, the air outside the tube has greater freedom of lateral motion, and a condensation is reflected as a rarefaction and a rarefaction as a conednsation. In the region of condensation the displacement of a particle is in the direction of motion of the wave while in the region of rarefaction it is in the opposite direction. Therefore, as the reflected wave moves *opposite* to the incident wave, the displacement of a particle due to the reflected wave, is in the *same* direction as that due to the direct wave, but x has opposite sign. Hence in this case the displacement y_2 due to the reflected wave is given by

$$y_2 = a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the resultant displacement y is given by

$$y = y_1 + y_2$$

$$= a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) + a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

$$= 2a \sin \frac{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) + 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)}{2} \cos \frac{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) - 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)}{2}$$

$$= 2a \sin 2\pi\frac{t}{T} \cos 2\pi\frac{x}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In this case antinodes occur at points for which x is equal to zero (open end) or an *even* multiple of $\frac{\lambda}{4}$, and nodes are midway between these points where x is equal to an odd multiple of $\frac{\lambda}{4}$.

Stationary and Progressive Waves. The differences between the properties of stationary and progressive waves are given below in a tabular form.

Progressive Waves	Stationary Wnves
1. The disturbance moves outward, that is, the vibration characteristic of a particle is *transferred* to the next after some time.	1. The character of the disturbance is *fixed*, and *no* particle transfers its mode of vibration to the next.
2. The amplitude of vibration of every particle is the *same*, but a particle reaches any stage of its displacement at a time *different* from that of the last and the next particles, and not at the same time.	2. The amplitude of vibration of the neighbouring particles is *not* the same, and *all* the particles reach their maximum or minimum displacements at the *same* time. All the particles between two consecutive nodes are in the same phase, while all those in the last or next such segment are in opposite phase.
3. No particle is permanently at rest. Only the particles at the extremities of their displacements are *momentarily* at rest, and this condition is reached by different particles at different times.	3. Particles at the nodes are *always* at rest Other particles reach this condition at the extremities of their displacements, but then all of them are at rest at the *same* time
4. All the particles when they pass through their normal positions, one *after* the other, have the same maximum velocity.	4. All the particles pass through their normal positions at the *same* time, and then each has its maximum velocity, but its magnitude is *different* for different particles.
5. Every region passes successively through the condition of compression, normal pressure, and rarefaction, and these conditions, pass onward.	5. There is no onward motion of these conditions. In any region one of these conditions appears, then disappears, and is followed by the opposite condition.
6. A particle has its maximum velocity and maximum change of pressure at the same time, and reaches its maximum displacement after one-quarter of its period. The velocity and pressure curves agree and are $\frac{T}{4}$ ahead of the displacement curve,	6. No two curves agree. The velocity curve is a head of the displacement curve by $\frac{T}{2}$ and ahead of the pressure curve by $\frac{T}{4}$.

Q. 116. Obtain an expression for the resultant of two wave motions travelling along the same direction, with the same velocity and amplitude, but with slightly different frequencies, and explain its meaning.

Distinguish between beats and combination tones.

(*Punjab, 1936*)

Ans. In a harmonic wave moving with velocity v and of frequency n the displacement y of a partilce at a time t is given by

$$y=a \sin 2\pi nt,$$

where a is the amplitude of its vibrations. Let the two wave motions have slightly different frequencies n and m per second and the displacements of a particle due to them be equal to y_1 and y_2 respectively, then

$$y_1=a \sin 2\pi nt$$
$$y_2=a \sin 2\pi mt$$

and the resultant displacement y of the particle is given by

$$\begin{aligned} y &= y_1+y_2 \\ &= a \sin 2\pi nt + a \sin 2\pi mt \\ &= 2a \sin\left(\frac{2\pi nt+2\pi mt}{2}\right) . \cos\left(\frac{2\pi nt-2\pi mt}{2}\right) \\ &= 2a \cos 2\pi \left(\frac{n-m}{2}\right)t . \sin 2\pi \left(\frac{n+m}{2}\right)t \end{aligned}$$

This equation represents a periodic vibration of amplitude $2a \cos \pi (n-m) t$ and frequency $\left(\frac{n+m}{2}\right)$, which is the arithmetic mean of the frequencies of the two component waves. Its amplitude is not constant but *changes* with time: maximum value is equal to $2a$ or $-2a$ when t is equal to zero or an integral multiple of $\frac{1}{n-m}$, so that $\cos \pi(n-m)t$ is equal to $+1$ or -1, and the intensity of sound, being proportional to the square of amplitude, is maximum. This occurs $(n-m)$ times in one second.

Alternating with maximum amplitude are the positions of minimum (zero) amplitude. This occurs when t is equal to an odd multiple of $\frac{1}{2(n-m)}$, so that $\pi(n-m)t$ is equal to an odd multiple of $\frac{\pi}{2}$ and, therefore, its cosine is equal to zero.

Thus the intensity of the resultant wave rises and falls $(n-m)$ times per second which is equal to the *difference* of the component frequencies. As $(n-m)$ is *very small*, the resultant wave is nearly a simple harmonic wave whose amplitude alternately changes from the sum $(a+a=2a)$ of the amplitude of the components to their difference $(a-a=0)$. These alterations in the intensity of sound when two notes of nearly equal frequencies are produced are called **beats**.

Beats and Combination Tones. When the difference between the frequencies of the two component notes is very small, beats are very rapid and produce a tone, whose frequency is equal to the difference between the frequencies of the component note and is called a **beat tone**.

In addition to the two primary tones sounded together (very strongly) other tones are heard under certain conditions. These tones are called **combination tones**, and their frequency is obtained by different combinations of the frequencies, m and n, of the primary notes. Tones whose frequency is obtained by adding the component frequencies are called **summation** tones. The simplest of these is called *first* summation tone and its frequency is equal to $m+n$. Summation tones of higher orders have frequencies given by $2m+n$, $m+2n$, and so on.

A tone whose frequency is equal to the difference $(n-m)$ between the two component frequencies is called **first difference tone** and is the strongest of all the combination tones Difference tones of higher order can also be obtained and have frequencies such as $2n-m$, $2m-n$, and so on. In addition to these combination tones, **self-combination tones** of frequencies $2m$ and $2n$ are also formed under certain conditions.

A beat tone has the same frequency as the first difference tone, and Koeing thought that the first difference tone is really a beat tone formed by the rapid production of beats. But the experiments of Rucker and Edser have conclusively proved that a difference tone has an *objective* existence and is not a beat tone.

A note is due to a succession of waves. In one half of each wave air is displaced forward, while in the other half it is

displaced backward. On the other hand, each half of a beat consists of decreasing or increasing alternations of displacement; the average displacement of any particle is zero and the average pressure of air is *normal.* As a mere loudness or faintness cannot move the drum of the ear unless there is *change of pressure*, beats cannot produce in the ear a note of their own frequency. Moreover, beats do not explain the formation of summation tones.

Q. 117. Give a short account of the combination tones that would be produced when notes of frequencies 512 and 768 are sounded together. (*Punjab 1929*)

Ans. See Q. 116 for the production of combination tones. The following are their frequencies :—

Primary tones	512,768
Self-combination tones	2×512, 2×768 $=1024,1536$
First summation tone	$512+768=1280$
Second ,, ,,	$2\times512+768=1792$
	and $2\times768+512=2048$
First difference tone	$768-512=256$
Second ,, ,,	$2\times512-768=256$
	and $2\times768-512=1024$

Q. 118. Describe Doppler's principle.

Derive an expression for the change in the frequency of a note due to the relative motions of the observer, source, and medium.

Two aeroplanes are approaching each other and their velocities are 100 and 150 miles per hour. The frequency of a note emitted by the first as heard by the passengers in the other is 1000. Calculate the true frequency of the note as heard by its own passengers. Take the velocity of sound as 750 miles per hour. (*Punjab, 1939*)

Ans. **Doppler's Principle.** When a source of sound gives out a note of frequency n and wave-length λ, and they travel with a velocity V, then all the waves given out in a unit time occupy a distance V, where V is equal to $n\lambda$.

If there is no relative motion between the source and an observer, he receives in any interval the *same* number of waves as are produced in the same time. If there is any relative motion between them, due to the motion of the source, or the observer or both, *the rate at which the waves are received is not equal to the rate at which they are emitted.* The pitch of the note heard rises or falls according as the two approach or recede away from each other, and its change depends on the magnitude of their relative velocity. This change of pitch is called **Doppler's effect,** after the name of the man who first explained it, and the principle by which it is explained is called **Doppler's principle.**

Change of Frequency of Reception. 1. **Observer in motion** Let the observer O move in the direction of sound, that is, *away* from the source S, with a velocity V_1, [Fig. 101 (*a*)]. If in a time t he moves from O to O' and sound travels the distance OA, he receives in this interval all the waves which lie between O' and A at the end of the interval and whose length is equal to O'A. The relative velocity of sound with respect to the observer is equal to $(V-V_1)$ so that O'A is equal to $(V-V_1)\ t_1$ and

$$\text{No. of waves received in time } t=\frac{(V-V_1)t}{\lambda}$$

$$=\frac{(V-V_1)tn}{V}$$

$$\therefore \text{ Frequency of reception } n_1=\frac{(V-V_1)n}{V} \quad . \quad . \quad (1)$$

or $\dfrac{n_1}{n}=\dfrac{V-V_1}{V}=\dfrac{\text{Velocity of sound with respect to the observer}}{\text{Velocity of sound with repect to the source}}$

2. **Source in Motion.** Let the source S move with velocity V_2 towards the observer O and go from S to S' in time t, [Fig. 101 (*b*)]. In this time the source has given out nt waves, and if the front end of the first reaches B, these

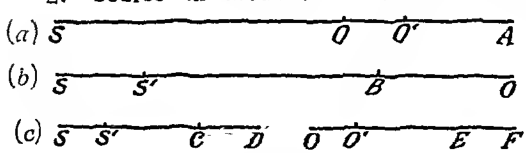


Fig. 101.

waves would have occupied the length SB if the source were at rest, but now their total length is equal to S'B, so that their wave length is *changed* and is *shorter* than λ. The relative velocity of sound with respect to the source is equal to $(V-V_2)$, and

$$S'B=(V-V_2)t$$

$$\therefore \quad \text{Changed wave-length}=\frac{(V-V_2)t}{nt}$$

$$=\frac{V-V_2}{n}$$

The velocity of sound with respect to the observer remains unchanged, and in time t he receives waves whose total length is equal to Vt.

$$\text{No. of waves received in time } t=\frac{Vt}{\dfrac{V-V_2}{n}}=\frac{Vtn}{V-V_2}$$

$$\therefore \quad \text{Frequency of reception } n_2=\frac{Vn}{V-V_2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2)$$

or $$\frac{n_2}{n}=\frac{V}{V-V_2}$$

$$=\frac{\text{Velocity of sound with respect to the observer}}{\text{Velocity of sound with respect to the source}}.$$

3. **Medium in Motion.** If the medium is moving with velocity W in the direction of sound and towards the observer, the velocity of sound with respect to the source and observer becomes $(V+W)$ instead of V, and the frequency of reception in the above two cases is obtained by replacing V by $(V+W)$.

4. **Source, Observer, and Medium in Motion.** Let all the three be moving in the *same* direction to the right. If in time t the source moves from S to S′ and the medium moves through a distance CD equal to Wt, nt waves given out in this time would have occupied distance SC $(=Vt)$ if both the source and medium were at rest, S′C if the source alone were moving, and SD $[=(V+W)t]$ if the medium alone were moving, but now they occupy length S′D equal to $(V+W-V_2)t$ [Fig. 101 (*c*)].

$$\text{Changed wave-length} = \frac{(V+W-V_2)t}{nt}$$
$$= \frac{(V+W-V_2)}{n}.$$

In this interval t, the observer moves from O to O′ and the medium through a distance EF equal to Wt. Sound would have travelled distance OE equal to Vt if the medium were at rest, but due to the motion of the medium, it actually travels distance OF equal to $(V+W)t$. As the observer has moved through the distance OO′ equal to V_1t, he has received in time t all the waves which occupy length O′F equal to $(V+W-V_1)t$.

$$\text{No. of waves received in time } t = \frac{(V+W-V_1)t}{\text{New wave-length}}$$
$$= \frac{(V+W-V_1)tn}{(V+W-V_2)}$$

$$\therefore \quad \text{Frequency of reception N} = \frac{(V+W-V_1)n}{(V+W-V_2)}$$

or
$$\frac{N}{n} = \frac{(V+W-V_1)}{(V+W-V_2)}$$
$$= \frac{\text{Velocity of sound with respect to the observer}}{\text{Velocity of sound with respect to the source}}.$$

$$\text{Change in frequency} = N - n = \frac{(V+W-V_1)n}{(V+W-V_2)} - n$$
$$= \frac{(V_2-V_1)n}{V+W-V_2}$$

[If the observer moves towards the source, his velocity V_1 is negative. When the observer and the source approach each other, the signs of their velocities are reversed after they have crossed.]

Q. 119. Explain Doppler effect in Sound,

The frequency of a whistle of a stationary engine is 600. What is the apparent frequency of the whistle to passengers in a train travelling at 60 miles an hour before and after passing the engine? (*Punjab, 1935*)

Ans. See Q. 118 for Doppler effect.

Problem. Frequency of emission $= 600$ per second.

Velocity of sound $= 1100$ ft./sec.

Velocity of train $= 60$ miles/hr. $= 88$ ft /sec.

$$\text{Wave-length} = \frac{1100}{600} = \frac{11}{6} \text{ ft.}$$

(*a*) *Before passing the engine.*

Velocity of sound with respect to the observer

$$= 1100 + 88 = 1188 \text{ ft./sec}$$

$$\therefore \quad \text{Frequency of reception} = \frac{1188}{11/6} = \frac{1188 \times 6}{11}$$

$$= 648/\text{sec.}$$

(*b*) *After passing the engine.*

Velocity of sound with respect to the observer

$$= 1100 - 88 = 1012 \text{ ft./sec.}$$

$$\therefore \quad \text{Frequency of reception} = \frac{1012}{11/6} = \frac{1012 \times 6}{11}$$

$$= 552/\text{sec.}$$

Q. 120. If a vibrating fork is rapidly moved towards a wall, beats may be heard between the direct and reflected sounds. Account for these, and calculate their frequency, if the fork makes 512 vibrations per sec. and approaches the wall with a velocity of 300 cm. per sec. The velocity of sound may be taken as 330 metres per sec. (*Punjab, 1932*)

Ans. See Q. 119 for the change in frequency when the source is in motion.

The tuning fork lies *between* the wall and the observer, and is moved towards the former and away from the latter. Therefore the waves going towards the wall are *shortened*, while those going *direct* towards the observer are *drawn out*. The waves incident on the wall are reflected by it and are then received by the observer. Thus the observer receives two wave trains of *different* frequencies and beats are produced between them.

Frequency of emission $= 512$/sec.

Velocity of sound $= 330$ metres/sec.

Velocity of turning fork $= 3$ metres/sec.

(*a*) *Waves received directly.*

Velocity of sound relative to the fork $= 330 + 3$
$= 333$ metres/sec.

$\therefore$ Changed wave-length $= \dfrac{333}{512}$ metre

Velocity of sound relative to observer $= 330$ metres/sec.

$\therefore$ Frequency of reception $= \dfrac{330 \times 512}{333} = 507{\cdot}4$/sec.

(*b*) *Reflected waves.*

Velocity of the sound relative to fork $= 330 - 3$
$= 327$ metres/sec.

$\therefore$ Wave-length of incident and reflected waves
$= \dfrac{327}{512}$ metre.

Velocity of sound relative to observer $=$ 330 metres/sec.

$\therefore$ Frequency of reception $= \dfrac{330 \times 512}{327} = 516{\cdot}7$/sec.

Frequency of beats $= 516{\cdot}7 - 507{\cdot}4 = 9{\cdot}3$/sec.

Q. 121. The whistle of an engine moving at 30 miles per hour is heard by a motorist driving at 15 miles per hour and estimated to have a pitch of 500. What must be the actual pitch of the whistle, to the nearest whole number, when—

(*a*) the two are moving in opposite directions but approaching each other ;

(*b*) the two are moving in opposite directions but away from each other ;

(*c*) the two are moving in the same direction, the motorist being behind the engine ;

(*d*) the two are moving in the same direction, the motorist being in front of the engine ?

The velocity of sound may be taken to be 1200 ft. per sec. (*Punjab, 1937*)

Ans. Frequency of emission $= 500/\text{sec.}$

Velocity of sound $= 1200$ ft./sec.

,, ,, engine $=$ 30 miles/hr. $= 44$ ft./sec.

,, ,, motorist $=$ 15 miles hr. $= 22$ ft./sec.

(*a*) Velocity of sound with respect to the source

$$= 1200 - 44 = 1156 \text{ ft./sec.}$$

$$\therefore \text{ Wave-length } = \frac{1156}{500} \text{ ft.}$$

Velocity of sound with respect to the observer

$$= 1200 + 22 = 1222 \text{ ft./sec.}$$

$$\therefore \text{ Frequency of reception } = \frac{1222}{1156} \times 500 = 529/\text{sec.}$$

(*b*) Velocity of sound with respect to the source

$$= 1200 + 44 = 1244 \text{ ft./sec.}$$

$$\therefore \text{ Wave-length } = \frac{1244}{500} / \text{sec.}$$

Velocity of sound with respect to the observer

$$= 1200 - 22 = 1178 \text{ ft./sec.}$$

$$\therefore \text{ Frequency of reception } = \frac{1178 \times 500}{1244} = 474/\text{sec.}$$

(*c*) Velocity of sound with respect to the source

$$= 1200 + 44 = 1244 \text{ ft./sec}$$

$$\therefore \text{ Wave-length } = \frac{1244}{500} \text{ ft.}$$

Velocity of sound with respect to the observer

$$= 1200 + 22 = 1222 \text{ ft./sec.}$$

$$\therefore \text{ Frequency of reception } = \frac{1222}{1244} \times 500 = 491/\text{sec.}$$

(*d*) Velocity of sound with respect to the source

$$= 1200 - 44 = 1156 \text{ ft./sec.}$$

$$\therefore \text{ Wave-length } = \frac{1156}{500} \text{ ft.}$$

Velocity of sound with respect to the observer

$$= 1200 - 22 = 1178 \text{ ft./sec.}$$

$$\therefore \text{ Frequency of reception } = \frac{1178 \times 500}{1156} = 510/\text{sec.}$$

Q. 122. Write short notes on the following:—(*a*) interference of sound waves, (*b*) beats, and (*c*) combination tones. How can the existence of these phenomena be shown? (*Calcutta, 1930*)

Ans. (*a*) **Interference.** When two wave-trains of sound of the same wave-length simultaneously pass over a common region, they are superposed over one another, and each *disturbs* the distribution of energy due to the other. This is called *interference.* At some points the two waves arrive in the *same* phase and the resultant displacement is equal to the sum of the separate displacements due to the two waves, while at some other points they arrive in *opposite* phase and the resultant displacement is equal to the difference of the component displacements. If the amplitudes of the two waves are equal, in the first case the intensity of sound is quadrupled while in the second case complete silence is produced. At other points the phase difference lies between 0 and π and the intensity of sound has different values between these extremes.

Experiment. A vibrating tuning fork F is held over the projected end A of a branched tube, the length of whose right

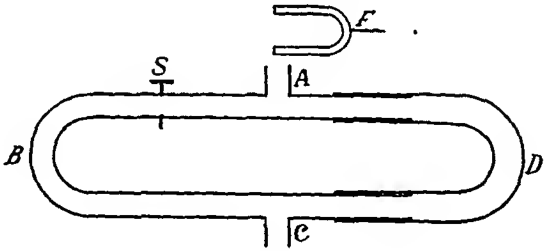


Fig. 102.

arm can be varied and a tube attached to the other projected end C is held in the ear of the observer (Fig. 102). The waves produced by the fork are divided into two parts, travelling along the *different* paths ABC and ABD, and arrive at C in the same phase or opposite phase according as the difference of these two paths is equal to an even or odd multiple of half the wave-length of the waves produced.

By drawing in or out the outer tube of the right arm, its length is decreased or increased and thereby any desired path

difference is produced. It is found that when the path difference is equal to *half* the wave-length, almost no sound is heard at C. That this silence is due to interference is proved by the fact if in this condition the left arm is closed with a stop cock S, sound is heard, and on again opening this arm the intensity of sound is reduced to a minimum.

(*b*) **Beats.** When two sources of sound of slightly *different* frequencies are sounded together and near each other, a periodic fluctuation of the intensity of sound produced is heard. This rise and fall in the intensity of resultant sound is called a beat, and the frequency of the beats is equal to the difference between the component frequencies.

Even if the two notes start in phase, they immediately get out of phase and the resultant displacement at a point becomes less than the sum of the two component displacements. The phase difference increases and becomes equal to π when the note of greater frequency gains half a wave-length more than the other. At this stage the intensity of sound is the least. With further increase in phase difference the resultant displacement of the particle increases, and when the wave of greater frequency gains one wave-length over the other, the two displacements are in phase, and the intensity of sound produced is the greatest. Again the intensity of sound decreases and the previous changes are produced.

Experiment. Beats are heard when two tuning forks of slightly different frequencies and mounted on resonance boxes are sounded together near each other. By loading the prong of one of them with different amounts of wax or with same amount at different places, its frequency is varied and thus the frequency of the beats produced is also changed.

(*c*) **Combination Tones.** See Q. 116. They are produced when two notes are sounded together *very strongly*. According to Helmhottz, when the displacements are large, the restoring force is not proportional to the displacement, and the principle of superposition does *not* apply. In such a case a simple harmonic force produces not only a simple harmonic vibration of its own frequency but gives rise to combination tones also.

Experiment. When two notes of frequencies 512 and 768 are *strongly* sounded together on a harmonium, a note of

frequency 256, which is equal to the difference between the frequencies of the two primaries, is also heard. It becomes distinctly audible if the note of this frequency (256) is first sounded and the ear is prepared for it.

Q. 123. How will you show experimentally the interference of sound waves? Apply this principle to explain the existence of silence zones in the neighbourhood of a powerful source of sound. (*Punjab, 1929*)

Ans. **Interference.** See Q. 122 for showing experimentally the interference of sound waves.

Silence Zones. When a powerful source is sounded, an observer in its neighbourhood receives not only sound waves directly from it but also those reflected from some big surface and the intensity of the two is almost the same. The distances travelled by these two sets of waves are *different*, and when they are superposed, interference is produced. They completely reinforce or cancel out each other according as they arrive in the same phase or opposite phase.

In the second case the path difference is equal to an odd multiple of half wave-length, and the observer hears no sound. If the observer moves towards or away from the source, sound is again heard. Its intensity increases and becomes maximum when the path difference becomes equal to an *even* multiple of half wave-length. This proves conclusively that the silence zone is due to the phenomenon of interference.

Q. 124. Explain :—Free vibrations, forced vibrations, and resonance, giving an example of each.

How is the principle of resonance used to analyse a complex note? (*Punjab, 1936*)

Ans. **Resonance and Free Vibrations.** If a body is displaced from its position of equilibrium and released, it vibrates with a *definite* period, which depends on its dimension, and is called its *natural period* or period of free vibrations. When a periodic force is applied to the body, it is set into vibrations whose amplitude and period depend on the period of the applied force and the natural period of the body. If the two periods are exactly *equal*, the applied force helps to increase the velocity of the body at *each* step of its vibration. *Every* new impulse *adds* to the effect of the previous impulses, and

the body vibrates with a very *large* amplitude. This is called the phenomenon of **resonance,** and as the period of vibration of the body is *equal* to its natural period, its vibrations are called **free vibrations.**

Vibrations of large amplitude are also produced even if the applied force is intermittent but the interval between consecutive impulses is equal to, or is an integral multiple of, the natural period of the body.

A turning fork is excited and held over a tube filled with water. By gradually lowering the level of water in it, the length of the air column is increased, and when its natural period of vibration becomes equal to that of the fork, sound of *very great* intensity is produced. Its intensity is much greater than that of the sound produced by the fork alone.

Forced Vibrations. If the natural period of the body is not the same as that of the applied force, the body at first tends to vibrate with its own period; but its motion is sometimes helped and sometimes opposed by the applied force, and, therefore, large amplitude is not developed. After some struggle, the body begins to vibrate with the period of the applied force, and its amplitude, which depends on the applied force and the two periods of vibration, is *small* as compared with the case of free vibrations. Such vibrations which do *not* agree with the natural period of the body but are produced by the applied periodic force are called **forced vibrations.**

When a tuning fork is excited and held in hand, the intensity of sound produced is low. It gives out energy at a *slow* rate, as very little of air is set into vibrations by its prongs, and even some of that which is disturbed slips round them. If its stem is pressed against a board, the board is set into forced vibrations. As the vibrations are now communicated to a much *greater* amount of air by the *large* surface of the board, a very loud sound is produced.

Analysis of a Complex Note. A complex note is made up of two or more tones of different frequencies, and to analyse such notes Helmholtz used resonators. A resonator consists of a hollow vessel with a large opening at B and a small

projecting end at A (Fig. 103). The air inside a given resonator has a natural period of vibration, and can resound to this particular frequency only.

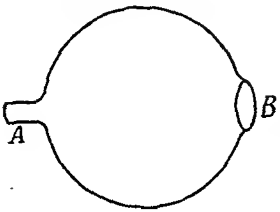


Fig. 103.

End A is held in the ear and the complex note to be analysed is allowed to play on B. If this note contains the particular frequency to which the resonator responds, this component is picked out, and sound of large intensity is produced. A set of many resonators, each responding to a different frequency, is used to find the different components.

Q. 125. Explain the terms forced vibrations and resonance. Why should two tuning forks be very accurately in unison to show response while a tuning fork and an air column require to be only approximately tuned? (*Punjab, 1935*)

Ans. **Resonance and Forced Vibrations.** See Q. 124.

While maximum resonance is obtained when the period of the applied force is exactly equal to that of the body to which it is applied, some effect is produced even when the two periods are not equal, and this depends on the rate at which the vibrations of the body die out. Mistuning has little or much effect according as damping is large or small.

In resonance every new impulse is reinforced by all the previous impulses still effective. The vibrations of a tuning fork, due to its large mass and small frictional forces, die out *very slowly*. If the second tuning fork is not of the same period as the first, a new impulse is found to be in opposite phase to an early impulse, whose effect has not as yet disappeared, and the two neutralise each other. Thus the cumulative effect of the impulses is not obtained, and no resonance is produced unless the two forks are in unison.

On the other hand, the vibrations of an air column die out *quickly*, and the cumulative effect of the impulses is not much. Even if the inducing tuning fork is not of the same period as the air column, some resonance is obtained. When a new impulse arrives, it is not found in opposite phase to any one of the existing impulses, as the early impulses, to one of which it would have been in opposite phase, have already disappeared, and therefore it is *not* cancelled out. In this

case some mistuning is permissible, and the tuning should be so close that the *existing* impulses should be nearly in the same phase.

Q. 126. Describe the action of a mouth-piece of an organ flute pipe, and show that an open end pipe will produce sounds richer in overtones than a closed end one. (*Punjab, 1937*)

Ans. **Flute Pipe.** Air is blown into the organ pipe at A (Fig. 104). It rushes through the narrow slit B and issues from it in a thin sheet. Then it strikes the tapering sharp edge C, and due to some want of symmetry in the direction of the jet, or some other accidental circumstance, it is deflected slightly inward or outward and starts a feeble compression or rarefaction in the pipe.

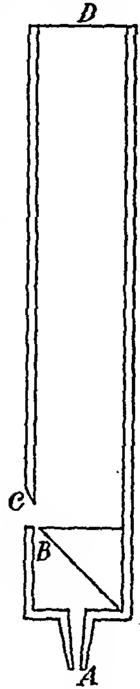


Fig. 104.

If a compression is started, it travels up the pipe and is reflected at the closed end D as a compression. When it comes back, it deflects C outward and is reflected as a rarefaction at this open end. As the air rushes out of the mouth to produce a rarefaction, it drives the jet of air coming from A outward. This causes drop in pressure and starts a rarefaction in the pipe which helps the reflected rarefaction.

Interference occurs between the direct and reflected waves and stationary waves are set up in the pipe with a node at its closed end and an antinode at its mouth. The rarefaction is reflected as a rarefaction at D, and when it arrives at the mouth of the pipe it is reflected as a compression. At the same time the sheet of air is sucked in and a new compression is started. Thus if the vibration is once started, the energy of the air jet increases it until the rate of radiation of energy in the form of waves is equal to its rate of supply. If end D is open, a compression is reflected as a rarefaction and a rarefaction as a compression, and stationary waves are produced as before, but now there is a node at both the mouth and the open end of the pipe.

Overtones of Closed Pipes. When a closed end pipe gives out its fundamental note, it has an antinode at its mouth and the next node at its closed end, that is, its length is equal to

one fourth of the wave-length of the note produced [Fig. 105(a)].

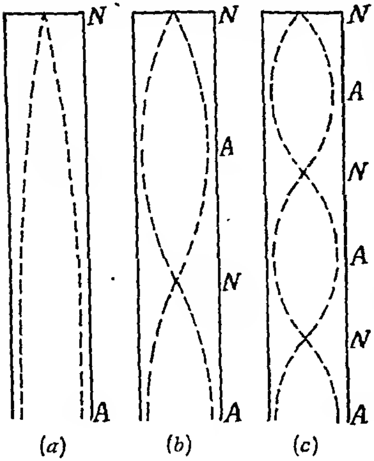


Fig. 105.

On blowing harder, over-tones are produced, in which additional nodes and antinodes are introduced between the node at its closed end and antinode at its mouth.

The next possible mode of vibration is one in which one more node and an antinode are introduced, and this first over-tone is shown in Fig. 105 (b). Here the length of the pipe is equal to three-fourth of the wave-length of the note produced, and, therefore, the frequency of the this overtone is *three* times the frequency of the fundamental note. The mode of vibration of the pipe in the case of the second over-tone is shown in Fig. 105 (c). In this case the length of the pipe is equal to five-fourth of the wave-length of the note given out, that is, the frequency of the note is *five* times that of the fundamental. Thus the frequencies of the overtones are *odd* multiples of the frequency of the fundamental note.

Overtones of Open Pipes. When an open pipe gives out its fundamental note, there is an antinode at its mouth and another at its open end with a node between them, and its length is equal to half the wave-length of the note produced [Fig. 106 (a)]. The next possible mode of vibration of the pipe is shown in [Fig. 106 (b)], and here the first overtone is given out. The length of the pipe is equal to the wave-length of the overtone, and, therefore, the frequency of the first over-tone is *twice* the frequency of the fundamental note,

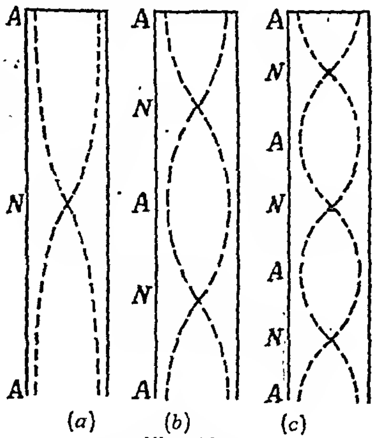


Fig 106.

The wave-length of the second overtone [Fig. 106(c)] is one-third the wave-length of the fundamental note and its frequency is *three* times that of the fundamental. Thus the frequencies of the overtones of an open pipe are an *integral* (*both odd and even*) multiple of that of the fundamental.

As in the case of a closed pipe even overtones are *absent*, and the intensity of an overtone *decreases* with its order, the sound produced by an open pipe is richer in its overtones than that produced by a closed pipe.

Q. 127. Explain with diagrams the nature of the vibrations of a tuning fork. What special features make it a valuable instrument in the scientific study of sound? (*Calcutta, 1931*)

Ans. **Tuning Fork.** A tuning fork may be considered as a bent-rod whose free ends, when struck, execute *transverse* vibrations due to its inertia and *rigidity*. In Fig. 107(*a*), a rod vibrates up and down, as shown by the dotted curves, with two nodes and two antinodes. The overtones are not harmonics, and the frequency of the first overtone is about 6·5 times that of the fundamental. As the rod is bent, the two nodes come *nearer* together, and the different stages are shown in Fig. 107 (*b*), (*c*) and (*d*). In the final condition the two nodes lie very near each other at the bases of the prongs and a stem is fixed between them.

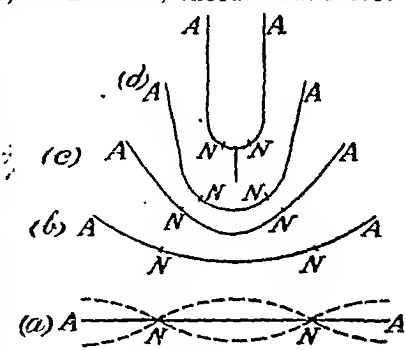


Fig. 107.

In the original rod, Fig. 107 (*a*), the free ends A move up and down together and, therefore, in the final bent condition the two prongs move inward and outward *together*, that is, they alternately approach and recede away from each other, while its base between N and N still vibrates *up* and *down* though to a *very slight* extent. Its pitch is raised or lowered according as its prongs or base are filed. In the first case its moment of inertia is decreased, while in the second case its stiffness is

diminished. With rise of temperature, its size increases and rigidity decreases, and its pitch is lowered by about 0·01 per cent for 1°C rise of temperature.

Special Features. 1. When struck gently, it emits a note which is practically a pure tone and free from overtones. Any overtone produced dies away very quickly. If it is mounted on a resonance box, the tone produced is still more pure. The box has its own overtones, but excepting the fundamental they do not lie near the overtones of the fork, and, therefore, only the fundamental is strengthened.

2. It can be excited electrically and thereby its sound can be produced for any length of time.

3. Its frequency can be determined easily and accurately and can be both raised and lowered by any amount.

Q. 128. Explain the working of the Kundt's tube in determining the relative speeds of sound in two gases, and mention the various other determinations which can be made with the help of Kundt's tube. (*Bombay, 1935*)

Ans. **Kundt's Method.** Two glass tubes, AB and CD, of about 4 cm. diameter and 2 metres length are supported in a horizontal position on wooden V blocks, and a rod EF, which carries at its end discs of slightly smaller diameter than the tubes, is fixed in the corks B and C so that one quarter of it is in each tube (Fig. 108). The outer ends of the tube are closed with pistons whose position can be changed by moving the tubes G and H. A hole is bored in the corks B and C for the exit of the gas that the tubes contain.

Fig. 108.

Before fitting the tubes with the pistons and rod they are *dried* by passing hot air through them. Then a *thin* layer of *dry* lycopodium powder is laid along the *bottom* of each tube and they are fitted with the rod and pistons. The tubes are then filled with the two thoroughly *dried* gases, in which the relative speeds of sound are to be determined, through the handle tubes G and H. A thermometer is laid along each tube to know the temperature of the gas in it.

By stroking the rod EF at the middle with a damp or resined cloth longitudinal vibrations are produced in it with nodes at B and C and antinodes at E and F and the middle of the rod. The discs E and F vibrate forward and backward and produce periodic longitudinal disturbances in the two gases. Starting with the pistons A and D at the extremities of the tubes, they are slowly moved inward until, due to resonance, gases are set into steady vibrations. The longitudinal waves formed in the gases are reflected at the outer pistons and form stationary waves which have a node at the pistons A and D and and an anti-node at the discs E and F.

If the vibrations produced in the gases are sufficiently strong, lycopodium powder is agitated. It is cleared off from the antinodes and collects in parallel ridges at the nodes, and the *middle* of each ridge may be taken as the node. The distance between the *end* ridges is measured in each tube and by dividing it by the number of intervals the mean half wave-length, and therefore the mean wave-length, of the notes produced in the two gases is found.

As the inducing rod is the same, the vibrations produced in the two gases are of the *same* frequencies, and, therefore, the ratio of the velocities of sound in them is equal to the ratio of the respective wave-lengths of the stationary waves formed in them.

$$\frac{\text{Velocity of sound in AB}}{\text{Velocity of sound in CD}}=\frac{\text{Wave-length in AB}}{\text{Wave-length in CD}}$$

[A better arrangement is to have the lycopodium line not at the bottom of the tubes but along its side. In this case when the gas resounds, the powder remains sticking to the tube at the *nodes* only and falls to the bottom at other places.]

Other Uses. 1. The wave-length of the note produced in the rod is equal to its length, and thus the velocity of sound in it may be compared with that in either of the two gases.

2. Knowing the velocity of sound in the rod and its density, the value of its Young's modulus may be calculated from

$$\text{Velocity}=\sqrt{\frac{\text{Young's modulus}}{\text{Density}}}$$

3. The velocity of sound in a gas is equal to $\sqrt{\frac{\gamma\times\text{Pressure}}{\text{Density}}}$, where γ is the ratio of its specific heat at-

constant pressure to its specific heat at constant volume. If the velocity of sound in the other gas is known, the velocity of sound in the first gas can be found, and combining this with the value of its pressure and density, the value of γ for it may be calculated.

4. By enclosing one of the tubes in an outer tube and passing steam through the latter, the temperature of the gas in the inner tube is raised and thus the temperature coefficient of the velocity of sound in the gas may be found.

5. By filling one of the tubes with a transparent liquid, the velocity of sound in it can be found. But in this case instead of lycopodium a heavy powder, such as silica, is used.

Q. 129. Explain the Stroboscopic method of finding the frequency of a tuning fork. In what way would you expect the rise of temperature to affect the frequency, and why? (*Punjab, 1930*)

Ans. **Stroboscopic Method.** *A vibrating body appears to be stationary if it is either illuminated or seen intermittently at regular intervals such that all its positions at such intervals are indistinguishable from one another and the smallest interval at which the body appears to be stationary is equal to its period of vibration.*

The stroboscopic disc used for measuring the frequency of a tuning fork has a number of *equally* spaced *narrow radial* slits cut near its circumference (Fig. 109). It is mounted on the axle of an electric motor, whose speed can be gradually changed and accurately measured with a speedometer or a revolution counter and a stop watch. The tuning fork is placed behind this disc and close to it, so that it vibrates *perpendicular* to one of the slits and in a plane *parallel* to the plane of the disc. A telescope is focussed through *this* slit on a prong of the tuning fork which is strongly illuminated.

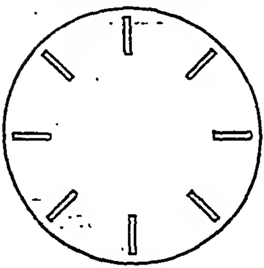


Fig. 109.

The tuning fork is set into vibrations, and then electric motor is started with a very *high* speed so that the time taken by one slit to go into the position of the next slit in front of

it is *smaller* than the time period of the fork. Every time a slit passes in front of the fork, its prong is seen through the telescope, and as it is seen more than 16 times per second, due to *persistence* of vision, it appear to be seen continuously. As the speed of rotation of the disc is gradually decreased, the prong appears to be vibrating more and more *slowly*, and it appears to be *stationary* when the time taken by one slit to take the position of the one in front of it is *exactly equal* to the period of vibration of the fork. In this case every time a slit comes in front of the prong, it (prong) is exactly in the *same* position as last time.

$$\text{No. of slits in the disc} = m$$

$$\text{No. of rotations per sec.} = n$$

$$\text{Time of one rotation} = \frac{1}{n} \text{ sec.}$$

$\therefore$ Time taken by one slit to take the position of the next

$$\text{in front} = \frac{1}{m \times n} \text{ sec.}$$

$$= \text{Time period of fork.}$$

and $$\text{Frequency of tuning fork} = m \times n \text{ per sec.}$$

This is the *highest* speed of the disc for which the prong appears stationary. On further decreasing the speed the prong appears to be vibrating, and once more it appears to be stationary when its speed is half of the previous speed for the stationary appearance of the prong. In this case in the time in which one slit moves into the position of the next slit in front of it, the prong makes *two* complete vibrations and the frequency of the fork is equal to $\frac{m \times n}{2}$ per sec., where n is the number of rotations *now* made by the disc in one sec. Similarly, whenever the speed is an *integral* sub-multiple of the *first* speed, the time taken by a slit to occupy the position of the next slit in front of it is an *integral* multiple of the time period of the fork, and its prong appears to be stationary. Thus the experiment is repeated and the mean value of the result is taken.

Effect of Temperature. The frequency of vibration of a tuning fork depends on its dimensions and elastic properties. It varies directly as the thickness of the prongs and the

square root of its elasticity and inversely as the square of the length of its prongs and the square root of its density. The effect of rise of temperature is to increase the size and decrease the density of the fork and *lower* its elasticity. The last effect is the most important, and, therefore, the frequency of the fork *decreases*.

Q. 130. Describe the application of any two of the following for the determination of the pitch :—

(*a*) The stroboscope.

(*b*) The falling plate.

(*c*) The chronograph. (*Punjab, 1939*)

Ans. (*a*) **Stroboscope.** See Q. 129.

(*b*) **Falling Plate.** A smoked glass plate is suspended by a thread in a vertical position, and the tuning fork whose frequency n is to be found is mounted in front of it. The fork carries a fine aluminium style which touches the plate *lightly*, and vibrates along a *horizontal* line when the fork is excited by drawing a violin bow across it.

The fork is set into vibrations and the thread supporting the plate is burnt so that the plate falls *freely* under the action of gravity. The style traces a wavy curve on the plate, and as its motion is accelerated, the curves become more and more elongated (Fig. 110).

Fig. 110.

Let AB and BC be equal to d_1 and d_2 respectively and u be the velocity of the plate when A is in contact with the style and g be the acceleration due to gravity. If they contain the *same* number of waves, say, m, time t taken by the plate in falling from A to B is the same as that for falling from B to C.

$$\therefore \quad d_1 = ut + \tfrac{1}{2}gt^2$$

or
$$2d_1 = 2ut + gt^2 \quad \ldots \ldots \quad (1)$$

and
$$d_1 + d_2 = u(2t) + \tfrac{1}{2}g(2t)^2$$
$$= 2ut + 2gt^2 \quad \ldots \ldots \quad (2)$$

Subtracting (1) from (2), we get

$$d_2 - d_1 = gt^2 = g\left(\frac{m}{n}\right)^2,$$

as t is equal to $\frac{m}{n}$.

or
$$\left(\frac{n}{m}\right)^2 = \frac{g}{d_2 - d_1}$$

or $$n = m\sqrt{\frac{g}{d_2 - d_1}}$$

(*c*) **Chronograph.** A smoked paper is wrapped round a hollow cylinder D which is mounted on an axle with a screw thread, and, therefore, when the cylinder is rotated it moves forward or backward (Fig. 111). The tuning fork F whose frequency is to be found has a fine aluminium style S attached to its one prong in such a way that when the fork is vibrated

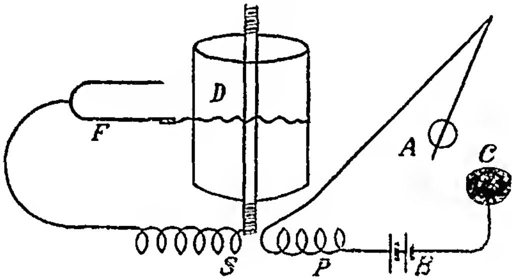


Fig. 111.

and the cylinder is stationary, it traces a line on the cylinder *parallel* to its length.

When the cylinder is rotated, the style, which presses *lightly* on the smoked paper, traces a wavy curve on it. As the cylinder moves forward or backward also when it rotates, the curves for different rotations do not overlap.

In order to measure time, the fork is attached to one end of the secondary S of an induction coil the other end of S being connected to the axle of the cylinder. The primary coil P is connected to the axis of suspension of a pendulum on one side and on the other to a small metallic cup C through a battery B. The metallic cup is full of mercury, and every time the pendulum passes through its normal position, its lower end A touches the mercury surface momentarily. At each break, a heavy induced current is produced in the secondary and a dark spot is formed on the paper.

Thus by counting the number of waves between two *consecutive* spots and dividing it by *half* the time period of the pendulum the frequency of the fork is obtained.

Q. 131. Explain the sensation produced when two notes sounded together are in unison and of frequency 512. per sec., and then the frequency of one of them is gradually increased until it is an octave above the other. Examine the consonance of the important intervals.

Ans. **Consonance and Dissonance.** When two notes of the *same* frequency are sounded together, they produce consonance or an agreeable sensation, because not only their fundamentals but harmonics also are in unison. On increasing the frequency of one of them dissonance is produced, and it is due to the *beats* between the two notes. No unpleasantness is felt if the number of beats per second is upto 4. With greater number the sensation becomes disagreeable, and it becomes most unpleasant for 32 beats per second for *one* of the notes being of frequency 512 per second. This corresponds to an interval of *semitone.*

The jarring effect decreases thereafter and ceases to be disagreeable for 85 beats per second, which corresponds to an interval of *minor third.* For notes of higher frequencies the maximum jarring interval is less than a semitone, and for lower frequencies more than a semitone. Owing to combination tones, a slight unpleasant sensation is produced near the fifth and more near the octave.

Consonance of Various Intervals. In the following discussion it is assumed that (1) harmonics are present, (2) the intensity of an harmonic decreases with its order, and (3) only the first six harmonics, including the fundamental, need be considered.

(*a*) *Major Third.* The two frequencies are in the ratio of $4n$ to $5n$

$4n$	$8n$	$12n$	$16n$	$20n$	$24n$
$5n$	$10n$		$15n$	$20n$	$25n$

There is beating of a semitone between the fourth harmonic of the lower note and the third of the upper. The sixth of the lower and fifth of the upper also produce slightly unpleasant sensation due to beats but the interval is less than a semitone. These two intervals are *weak* and, therefore, the amount of discord is not great.

(*b*) *Fourth.* The two frequencies are in the ratio of $3n$ to $4n$.

$3n$		$6n$		$9n$	$12n$	$15n$		$18n$		
	$4n$		$8n$		$12n$		$16n$			$20n$

The fifth hormonic of the lower note and the fourth of the higher give a semitone interval, but the discord introduced is not much. The third and sixth of the lower give a tone interval with the second, and fourth and fifth of the upper respectively.

(*c*) *Fifth.* The two frequencies are in the ratio of $2n$ to $3n$.

$2n$		$4n$	$6n$	$8n$	$10n$	$12n$
	$3n$		$6n$	$9n$		$12n$

In this there is tone $(\frac{9}{8}, \frac{9}{10})$ beating between the third harmonic of the higher and the fourth and fifth harmonics of the lower. The concord is not perfect, but the discord is not much.

(*d*) *Major Sixth.* The two frequencies are in the ratio of $3n$ to $5n$.

$3n$		$6n$	$9n$		$12n$	$15n$	$18n$	
	$5n$			$10n$		$15n$		$20n$

Here there is a tone beating between the third of the lower and second of the higher, and sixth of the lower and fourth of the higher. This does not introduce much discord.

(*e*) *Octave.* The two frequencies are in the ration of n to $2n$.

n	$2n$	$3n$	$4n$	$5n$	$6n$
	$2n$		$4n$		$6n$

In this case there is *complete* unison, as all the harmonics of the higher note are *coincident* with the even harmonics of the lower note. In fact the octave introduces no new note.

Q. 132. Explain what you understand by the musical interval between two notes. What intervals are used in the diatonic major scale. What is temperament, and why is the tempered scale used in keyed instruments ? (*Calcutta, 1932*)

Ans. **Musical Interval.** The *ratio* of the frequencies of two notes is called the interval between them. If the frequencies of three notes are n_1, n_2, and n_3, $\frac{n_1}{n_2}$, $\frac{n_2}{n_3}$ and $\frac{n_1}{n_3}$

are the intervals between first and second, second and third, and first and third respectively. The interval between first and third is equal to the *product* of the intervals between first and second, and second and third, and not their sum. The special importance of this ratio arises from the fact that when two notes are sounded together, the pleasantness of their combined sensation depends on the ratio of their frequencies and not their difference.

Diatonic Scale. The lowest integers giving the frequencies of the consecutive notes of diatonic scale are 24, 27, 30, 32, 36, 40, 45 and 48, though any particular note may have any frequency.

	C		D		E		F		G		A		B		c
Frequencies	24		27		30		32		36		40		45		48
Intervals		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

The three intervals used are $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$ and they are called *major tone, minor tone* and *semi tone* respectively. The first is greater than the second, and the third $\left(\frac{16}{15}\right)$ is nearly half (*square root*) of the first and second.

Temperament. The intervals between the successive notes of the diatonic scale are neither equal nor do they recur in the same order. In modern music it is necessary, for the sake of variety and to avoid monotony, to use scales having different *tonics* or first keys, but the scale with a definite set of notes can only suit music with a *particular* key as the tonic. If instead of C as tonic some other key, say D, is used as a tonic, the keys of the old scale do not give frequencies of the new scale, as is clear from the following table giving the relative frequencies of the two scales.

Old Scale	24	27	30	32	36	40	45	48
New Scale		27	30'4	33'8,36	40.5	45	50'6	54

Every new tonic requires the addition of more new keys, and if this is followed the instrument becomes cumbersome. On the other hand, if new keys are not added, with a new

tonic the old keys give dissonance. Moreover, the tonic should be of such a frequency as would bring the composition of music within the limited compass of the given instrument. In the case of instruments where no *fixed* keys are used, this difficulty does not arise as the operator can easily change the tonic and adjust other notes.

In order to overcome all these difficulties in the case of keyed instruments, a compromise is effected and intervals of the true scale are **tempered** so that its notes serve equally well all the scales with different tonics. This method of adjusting the notes of the scale is called **temperament.** See Q. 133 for scale of equal temperament, where the major and minor tones are broken into two intervals and all the intervals are made *equal.*

Q. 133. Distinguish between the Diatonic Scale and the scale of Equal Temperament, and show how each has been built up. (*Punjab, 1939*)

Ans. **Diatonic Scale.** The consonant intervals between a note and its octave are *octave* $\left(\frac{2}{1}\right)$, *fifth* $\left(\frac{3}{2}\right)$, *fourth* $\left(\frac{4}{3}\right)$, *major third* $\left(\frac{5}{4}\right)$, and *major sixth* $\left(\frac{5}{3}\right)$, and the integers giving the relative frequencies of the corresponding notes are :—

24 30 32 36 40 48

The interval between the first two and the last two is *too large* and is bridged by adding one more note in each gap. The nearest consonant note to the tonic (24) is the fifth (36), and the upper gap is filled by the third $(36\times\frac{5}{4}=45)$ of this fifth, while the lower gap is filled by the fourth below $(36\times\frac{3}{4}=27)$ this fifth, or one octave below the fifth of this fifth $\left(=\frac{36\times\frac{3}{2}}{2}=27\right)$.

These two notes are not directly related to the keynote, but are closely related to the fifth, which is very closely related to the keynote, and the ratio of the frequencies of

the notes of the full scale are given by

24 27 30 32 36 40 45 48

This scale is composed of notes which stand in simple relation to one another and give the widest possibilities of consonance.

Scale of Equal Temperament. While the diatonic scale comprises most consonant intervals, its scale of definite notes suits *only* music with the first note as the tonic or keynote, and, therefore, does not allow any modulation or change of keynote. In the scale of equal temperament, the big intervals of tones $\frac{9}{8}$ and $\frac{10}{9}$ are broken into two intervals, and the interval of an actave is divided into 12 *equal* intervals. If the frequency of the lowest note is n, the frequencies of the other notes of the scale are :—

$n(2)^{\frac{1}{12}}, n(2)^{\frac{2}{12}}, n(2)^{\frac{3}{12}}, n(2)^{\frac{4}{12}}, n(2)^{\frac{5}{12}}, n(2)^{\frac{6}{12}}, n(2)^{\frac{7}{12}}, n(2)^{\frac{8}{12}}$,

$n(2)^{\frac{9}{12}}, n(2)^{\frac{10}{12}}, n(2)^{\frac{11}{12}}$, and $2n$.

These numbers form a geometrical series, and *any* key may be used as the tonic. The common ratio is equal $2^{\frac{1}{12}}=1{\cdot}0595$ and differs slightly from the semi-tone, which is equal to $\frac{16}{15}=1{\cdot}067$.

The relative frequencies of the notes of the two scales are given below :—

	C	C′	D	D′	E	F	F′	G	G′
Diatonic	1·000	—	1·125	—	1·250	1·333	—	1·500	—
Tempered	1·000	1·059	1·122	1·189	1·260	1·335	1·414	1·498	1·587

	A	A′	B	c
Diatonic	1·667	—	1·875	2·000
Tempered	1·682	1·782	1·888	2·000

Thus this tempered scale gives not only *perfect* freedom of modulation, but as the frequencies of its notes are almost the same as those of the true diatonic scale, they do not give disagreeable beats when they are sounded together. Moreover, the number of notes to an octave is not inconvenient.

PART V

MAGNETISM

Q. 134. Give a short account of the molecular theory of magnetism, and explain in a general way—diamagnetism, paramagnetism, and ferro-magnetism.

How are diamagnetic and paramagnetic substances distinguished experimentally? (*Punjab. 1935*)

Ans. **Molecular Theory.** According to the molecular theory of magnetism, magnetic substances consist of small minute particles which are themselves magnets. These particles are called *magnetic molecules*, and each has a north pole and a south pole. When a magnetic substance does not exhibit magnetic field, its magnetic particles form almost *closed* chains, the effect of polarity of one being neutralised by the opposite polarity of the next particle, and so on. When a magnet is brought near this magnetic substance, its particles re-arrange themselves. Under the action of the magnetising force the chains are opened, and the end particles exhibit free magnetism. As the strength of the magnetising field is increased, magnetic particles are turned more and more in the direction of the magnetising field until the saturation stage is reached. In this condition all the magnetic particles lie with their axes *parallel* to the magnetising field.

Paramagnetic and Diamagnetic Substances. All the magnetic substances are divided into classes, paramagnetics and diamagnetics. The permeability of the paramagnetic substances is *greater* than one, and their susceptibility is *positive*. When they are placed in a magnetic field, they set themselves *along* its lines of force, and tend to go from the weaker part of the field to its stronger part. The near

end of a paramagnetic substance develops *opposite* polarity to that of the inducing magnet, while its farther end becomes a similar pole.

Of all the paramagnetics, iron, cobalt, and nickel exhibit *very strong* magnetic properties, and are given the special name of **ferromagnetic** substances. Other paramagnetics do not even make a near approach to their strong properties. Their permeability and susceptibility are *not* constant, but depend on the strength of the magnetising field and temperaature.

The diamagnetic substances are also magnetised when placed in a magnetic field, but the polarity of their near ends is *similar* to that of the inducing magnet, and magnetism induced in them is feeble as compared with paramagnetic substances. They set themselves *perpendicular* to the lines of force of the magnetic field and tend to move from its stronger part to its weaker part. Their permeability is *less* than one and their susceptibility is *negative*.

Experimental Test. Whether a magnetic substance is a paramagnetic or a diamagnetic is found from its behaviour in a *strong* magnetic field. If a rod of the substance is suspended freely between the pole pieces of a very strong electromagnet and the current is switched on, it sets with its length along its lines of force or perpendicular to them according as it is a paramagnetic or a diamagnetic.

Moreover, a paramagnetic is attracted by a magnet, while a diamagnetic is repelled by it. Here again a very strong magnet has to be used to make the repulsion of the diamagnetic substances appreciable.

Q. 135. Define intensity of magnetisation, and magnetic susceptibility. A bar of steel of length 23 cms. breadth 1·2 cm and thickness 0·5 cm. is placed in a magnetic field of 7·5 gauss and parallel to its length. Find the magnetic moment of the bar, if the permeability is 640. (*Punjab, 1929*)

Ans. **Intensity of Magnetisation.** The intensity of magnetisation of a magnet is its pole strength per unit area of its ends or its magnetic moment per unit volume.

Susceptibility. The susceptibility of a magnetic material is equal to the ratio of the intensity of magnetisation produced in it divided by the strength of the inducing magnetic field.

Problem. The permeability μ and susceptibility κ of a magnetic material and its intensity of magnetisation I, when magnetised by a field of intensity H, are related by

$$\mu = 1 + 4\pi\kappa$$

or
$$\mu H = H + 4\pi I$$

Here $\mu = 640$, and $H = 7{\cdot}5$ gauss

$$\therefore \; 640 \times 7{\cdot}5 = 7{\cdot}5 + 4\pi I$$

or
$$I = \frac{640 \times 7{\cdot}5 - 7{\cdot}5}{4 \times 3{\cdot}142} = \frac{4792}{12{\cdot}57}$$

Breadth of magnet $= 1{\cdot}2$ cm.

Thickness of ,, $= 0{\cdot}5$ cm.

Area of each end $= 1{\cdot}2 \times 0{\cdot}5 = 0{\cdot}6$ sq. cm.

$$\therefore \text{ Pole strength} = \frac{4792}{12{\cdot}57} \times 0{\cdot}6 \text{ C.G.S. units.}$$

Length of magnet $= 23$ cm.

$$\therefore \text{ Moment of the magnet} = \frac{4792}{12{\cdot}57} \times 0{\cdot}6 \times 23$$

$$= 5262 \text{ C.G.S. units}$$

Q. 136. Describe the action of a magnet in (*a*) uniform, (*b*) non-uniform, field. (*P. U. 1937*)

Ans. (*a*) **Uniform Field.** In a uniform magnetic field *both* the magnitude and direction of the magnetic intensity are the *same*, Therefore, as the poles of a magnet are always of *equal* strengths, when it is placed in a uniform magnetic field, its poles experience forces which are equal, opposite and parallel to one another.

In Fig. 112 a magnet NS of pole strength m and length $2l$ is placed with its axis inclined at angle θ to a uniform field of intensity H. Its north pole N is acted on by a force mH

parallel to, and in the direction of, the field, while its south pole S experiences an equal force in the opposite direction. These two forces form an anti-clockwise couple, which tends to rotate the magnet and set it parallel to the field, and its magnitude is equal to $m\text{H} \times 2l \sin\theta = \text{MH} \sin\theta$.

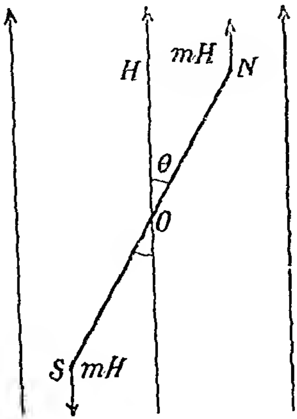


Fig. 112.

The two forces being equal and opposite, their resultant is equal to zero, and as no other force, due to the field, acts on the magnet, it has *no* motion of translation. Thus the magnet simply tends to turn round and set itself with its axis parallel to thefield.

(*b*) **Non-Uniform Field.** In a non-uniform field, the magnetic intensity changes, in magnitude and direction, from point to point. The two poles of a magnet placed in it experience *unequal* forces, and they not only tend to rotate it but also tend to move it bodily.

Suppose the non-uniform field is due to a fixed magnet N_1S_1 and a magnet NS is placed in it (Fig. 113). Pole S experiences forces due to poles N_1 and S_1; they are represented by SA and SB respectively and their resultant is represented by diagonal SC of the parallelogram SBCA. Similarly, pole N is acted on by forces given by NE and ND, and their resultant is represented by the diagonal NF of the parallelogram NEFD. Produce lines FN and CS and from O draw perpendiculars OG and OH on them.

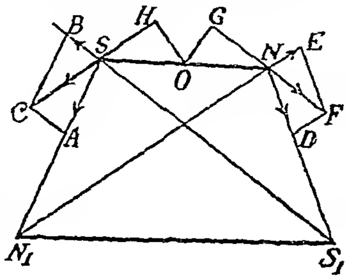


Fig 113.

The forces exerted on N and S are neither equal nor opposite, and they can be resolved into a couple and a translatory force. The couple tends to rotate NS, while due to

the resultant force, it moves bodily towards S_1N_1 (or stronger part of the field) and set itself as near to it as possible.

If NS is pivoted at O, the pivot there resists its translatory motion. It simply rotates and sets itself in a position in which the moments of the forces on its poles about the pivot at O are equal and opposite, that is,

$$NF \times OG = SC \times OH.$$

Q. 137. The intensity of a magnetic field is equal to the rate of change of the magnetic potential with respect to distance. Prove this statement. Find an expression for the potential at a point due to a short magnet and hence derive the intensity of the field at that point. (*Bombay, 1934*)

Ans. **Magnetic Intensity and Potential.** The magnetic intensity, F, at a point is equal to the force experienced by a unit north pole placed there, and work has to be done in moving the north-pole against this force. As the magnetic intensity changes with position, the magnetic force to be overcome does not remain constant, but for a very small distance dx in its direction it remains practically unchanged. The potential difference between two points being equal to the amount of work done in carrying a unit north pole from one point to the other, its value dV for the distance dx is given by

$$dV = -F.\,dx$$

or

$$F = -\frac{dV}{dx}.$$

The negative sign indicates that as x increases V decreases, and *vice versa*. Thus the magnetic intensity at a point in any direction is equal to the rate of change of potential there with distance in *that* direction.

Potential due to a Small Magnet. Consider a *small* magnet NS of pole strength m, length $2l$, and magnetic moment M, and a point P at a great distance x from the middle point O of the magnet, so that OP makes $\angle\theta$ with ON

(Fig. 114). From N draw ND perpendicular on OP, so that OD $= l \cos\theta$. As x is very great as compared with l, NP and SP are practically equal to $(x-l\cos\theta)$ and $(x+l\cos\theta)$ respectively.

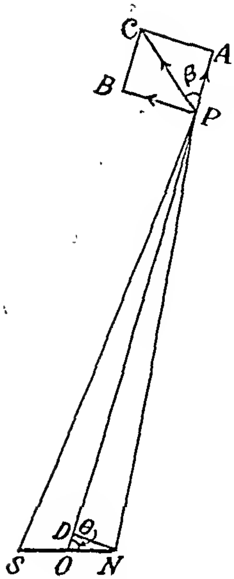


Fig. 114.

Potential at P due to north pole $= \dfrac{m}{\text{NP}}$

$$= \frac{m}{(x-l\cos\theta)}$$

Potential at P due to south pole $= \dfrac{-m}{\text{SP}}$

$$= \frac{-m}{(x+l\cos\theta)}$$

∴ Resultant potential of P

$$= \frac{m}{(x-l\cos\theta)} - \frac{m}{(x+l\cos\theta)}$$

$$= \frac{m(x+l\cos\theta)-x+l\cos\theta)}{(x^2-l^2\cos^2\theta)}$$

$$= \frac{m\times 2l\cos\theta}{x^2-l^2\cos^2\theta}$$

$$= \frac{\text{M}\cos\theta}{x^2}, \quad \ldots\ldots\ldots \quad (1)$$

as l being *very small* as compared with x, $l^2\cos^2\theta$ is negligible as compared with x^2.

Magnetic Intensity. The magnetic intensity at a point in any direction is equal to the rate of change of potential V there with distance x in *that direction* and is given by $-\dfrac{dv}{dx}$.

The magnetic intensity F in the direction OP is obtained by differentiating V with respect to x (θ constant).

$$\text{F}_x = -\frac{d}{dx}\left(\frac{\text{M}\cos\theta}{x^2}\right)$$

$$= \frac{2\text{M}\cos\theta}{x^3} \quad \ldots\ldots\ldots \quad (2)$$

This is represented by PA, a line in continuation with OP.

The magnetic intensity F_y in a direction perpendicular to the first and along the tangent to the circular arc at P and of radius OP is equal to the differential coefficient of V with respect to y in which direction x is constant but θ changes.

$$F_y = -\frac{d}{dy}\left(\frac{M\cos\theta}{x^2}\right)$$

$$= \frac{M\sin\theta}{x^2} \cdot \frac{d\theta}{dy}$$

$$= \frac{M\sin\theta}{x^3}, \quad [\text{as } x.\ d\theta. \text{ is} = dy.$$

This component is represented by PB.

The resultant intensity at P is represented by the diagonal PC of the rectangle PACB and is inclined at angle β to PA or OP such that

$$\tan\beta = \frac{AC}{PA} = \frac{M\sin\theta . x^3}{x^3 . 2M\cos\theta}$$

$$= \frac{\tan\theta}{2}$$

or $$\beta = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

Resultant Intensity at $P = \sqrt{\left(\frac{2M\cos\theta}{x^3}\right)^2 + \left(\frac{M\sin\theta}{x^3}\right)^2}$

$$= \frac{M}{x^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{M}{x^3}\sqrt{1 + 3\cos^2\theta}$$

Q. 138. What is a magnetic shell ? Find the value of potential due to it at a given point. (*Punjab, 1936*)

Ans. **Magnetic Shell.** It is a *thin* sheet of a magnetic

material magnetised at every point in a direction *normal* to the shell there (Fig. 115). Its one face, say upper, is entirely a north pole, and the whole of the other face exhibits south magnetism. As the shell may be curved, the direction of the normal to it *changes* from point to point and so does the direction of its magnetisation. Its intensity of magnetisation I is equal to its pole-strength per unit area, and its *strength* ϕ at any point is measured by the product of the intensity of magnetisation I and thickness t at that point, or by its magnetic moment per unit face area there. This quantity (strength) is independent of the size and shape of the shell.

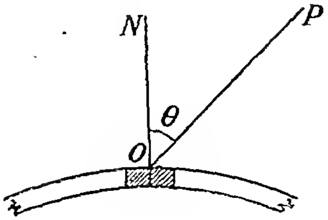


Fig 115.

Potential at any Point. Consider a *very small* element (shaded) of area δA and magnetic moment δM, and join its middle point O with point P where potential is required. If OP makes angle θ with the normal ON, which is along the magnetic axis at O, then as shown in Q. 137 the potential at P due to the shaded element is equal to $\frac{\delta M \cos\theta}{OP^2}$. The shaded element is not perpendicular to OP, but as the angle between two areas is equal to the angle between their normals, the effective part of δA perpendicular to OP is equal to $\delta A \cos\theta$, and this effective area divided by OP^2 is equal to the *solid angle* $\delta\omega$ subtended by the shaded area at P.

$$\text{Potential at P} = \frac{\delta M \cos\theta}{OP^2}$$

$$= \frac{\phi.\ \delta A \cos\theta}{OP^2}$$

$$= \phi.\ \delta\omega$$

If the whole shell is of *uniform* strength ϕ and subtends solid angle ω at P,

$$\text{Potential at P} = \phi\ \omega.$$

Q. 139. Find the potential due to a magnetic shell at a point close to one of its faces. How much work will have to be done in carrying a unit north pole from one face to the other? (*Punjab, 1935*)

Ans. See Q. 138. It is shown there that the potential at a point due to a magnetic shell of uniform strength ϕ is equal to $\phi\omega$, where ω is the solid angle subtended by the shell at that point. When the point is *very close* to one of the faces of the shell, the solid angle ϕ is practically equal to 2π, so that the potential at it is equal to $2\pi\phi$.

The potential difference between two points in a magnetic field is equal to the amount of work done in carrying a unit north pole from one point to the other. As the change of solid angle in going from a point on one face to a point on the other is equal to 4π, the amount of work done in carrying a unit north pole from one face to the other, *outside* the shell, is given by $4\pi\phi$.

Q. 140. Prove that the intensity of the magnetic field due to a small magnet of magnetic moment M at a point situated d cm. from its middle point on a line making an angle θ with the axis of the magnet, is

$$\frac{M}{d^3}\sqrt{3\cos^2\theta+1}$$ (*Punjab, 1934*)

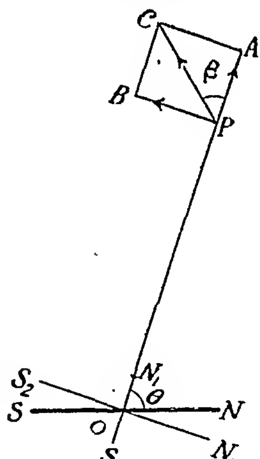


Fig. 116.

Ans. **Magnetic Intensity at any point.** Let P be the point at a great distance d from the middle O of a small magnet NS, so that OP is inclined at angle θ to ON (Fig. 116). The magnetic intensity at a *large* distance d from the middle of a *small* magnet of moment M is equal to $\frac{2M}{d^3}$ or $\frac{M}{d^3}$ according as the point is on the axis or equator of the magnet. (See Q. 141, taking $n=2$). As the moment of a magnet is a *vector* quantity, its component along OP is equal to M cos θ, and the second rectangular component in a perpendicular direction

N_2S_2 is equal to M sin θ. In other words, magnet NS is equivalent to two magnets, N_1S_1 of moment M cos θ and N_2S_2 of moment M sin θ.

Point P is on the axis of N_1S_1 and on the equator of N_2S_2.

Intensity at P due to $N_1S_1 = \dfrac{2M \cos \theta}{d^3}$ along OP . . . (1)

This is represented by a line PA in continuation with OP.

Intensity at P due to $N_2S_2 = \dfrac{M \sin \theta}{d^3}$ parallel to N_2S_2 . (2)

This intensity is perpendicular to the first, and is represented by a line PB drawn parallel to N_2S_2. Their resultant is represented by the diagonal PC of the rectangle PACB, and is inclined at angle β to PA or OPA.

$$\text{Resultant Intensity} = \sqrt{\left(\frac{2M \cos \theta}{d^3}\right)^2 + \left(\frac{M \sin \theta}{d^3}\right)^2}$$

$$= \frac{M}{d^3}\sqrt{4 \cos^2\theta + \sin^2\theta}$$

$$= \frac{M}{d^3}\sqrt{3 \cos^2\theta + 1}$$

and $$\tan \beta = \frac{AC}{PA} = \frac{M \sin \theta . d^3}{d^3 . 2M \cos \theta}$$

$$= \frac{\tan \theta}{2}$$

or $$\beta = \tan^{-1}\left(\frac{\tan \theta}{2}\right)$$

Q. 141. How did Gauss prove that the force between two magnetic poles varies inversely as the square of the distance between them? (*Calcutta, 1930*)

Ans. **Gauss's Proof of Inverse Square Law.** If the magnetic force between two magnetic poles varies inversely as the nth power of the distance between them, the magnetic intensity at a point at a distance d from a pole of strength m is equal to $\dfrac{m}{d^n}$. Let a point P be on the axis of a *small* magnet

NS of pole strength m, length $2l$, and magnetic moment M (Fig. 117). If P be at a *large* distance d from the *middle* point O of the magnet, then NP$=(d-l)$, and SP$=(d+l)$. As l is very small as compared with d, $\frac{l}{d}$ is a very small quantity, and its higher powers are *negligible*, so that

$$(d-l)^n = d^n\left(1-\frac{l}{d}\right)^n = d^n\left(1-\frac{nl}{d}\right)$$

and $$(d+l)^n = d^n\left(1+\frac{nl}{d}\right)$$

Fig 117.

Intensity at P due to north pole $= \dfrac{m}{(d-l)^n}$

$$= \frac{m}{d^n\left(1-\frac{nl}{d}\right)} \text{ along NP}$$

Intensity at P due to south pole $= \dfrac{m}{(d+l)^n}$

$$= \frac{m}{d^n\left(1+\frac{nl}{d}\right)} \text{ along PS.}$$

As these two intensities are along the same line but in *opposite* directions.

Resultant intensity F_1 at P $= \dfrac{m}{d^n\left(1-\frac{nl}{d}\right)} - \dfrac{m}{d^n\left(1+\frac{nl}{d}\right)}$

$$= \frac{m\left(1+\frac{nl}{d}-1+\frac{nl}{d}\right)}{d^n\left(1+\frac{n^2l^2}{d^2}\right)}$$

$$= \frac{m \times 2nl}{d^{n+1}\left(1-\frac{n^2l^2}{d^2}\right)}$$

$$= \frac{n\text{M}}{d^{n+1}} \text{ along NP} \quad . \quad . \quad . \quad . \quad (1)$$

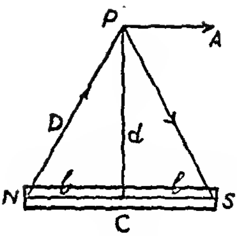


Fig. 118.

as $\frac{n^2l^2}{d^2}$ is negligible as compared with 1.

When the point P is on the equator of the magnet, its distance from the two poles is the same and equal to $(d^2+l^2)^{\frac{1}{2}}=D$ (Fig. 118).

Intensity at P due to north pole $=\frac{m}{D^n}$ along NP.

,, ,, ,, south pole $=\frac{m}{D^n}$ along PS.

These two intensities at P are equal and, may be represented, in magnitude and direction, by NP and PS respectively taken in order. Then, according to the law of triangle of forces (intensities), their resultant intensity F_2 at P is represented by the third side NS of the triangle NPS taken in the opposite order.

$$\therefore \frac{F_2}{NS}=\frac{\frac{m}{D^n}}{NP}$$

$$F_2=\frac{m\times 2l}{D^{n+1}}$$

$$=\frac{M}{(d^2+l^2)^{\frac{n+1}{2}}}$$

$$=\frac{M}{d^{n+1}\left(1+\frac{l^2}{d^2}\right)^{\frac{n+1}{2}}}$$

$$=\frac{M}{d^{n+1}\left(1+\frac{(n+1)l^2}{2d^2}\right)}$$

$$=\frac{M}{d^{n+1}} \text{ along PA parallel to NS} \quad \ldots \quad (2)$$

as $\frac{(n+1)l^2}{2d^2}$ and other terms are negligible as compared with 1. Thus with a small magnet, the magnetic intensity at a point at large distance on its axis is n times its value at the *same* distance on its equator.

If with a deflection magnetometer the deflections produced by this magnet in the "end on" and "broadside on" positions are equal to θ_1 and θ_2 respectively, and H is the horizontal component of the magnetic field of the earth,

$$H \tan \theta_1 = F_1 = \frac{nM}{d^{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$H \tan \theta_2 = F_2 = \frac{M}{d^{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Dividing (3) by the corresponding sides of (4), we get

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{F_1}{F_2} = n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But experiment shows that $\tan \theta_1/\tan \theta_2$ is equal to 2. Therefore n is equal to 2, that is, the force between two magnetic poles varies inversely as the *square* of the distance between them.

Q. 142. What factors determine the time of oscillation of a magnet when free to swing in a horizontal plane? Obtain mathematically an expression for the period of an oscillating magnet.

Two bar magnets are placed one above the other and suspended so as to oscillate in a horizontal plane. When the like poles of the two magnets are together, the period of swing is 12 secs., and when the unlike poles are together, the period is 16 secs. Compare the magnetic moments of the two magnets. (*Bombay, 1927*)

Ans. When a magnet is free to rotate in a horizontal plane and about a vertical axis and oscillates about its normal position in the magnetic meridian, its time of oscillation depends on its magnetic moment M and moment of inertia K about the axis of oscillation and the horizontal component H of the earth's field,

When the magnet is turned through a very small angle θ *radian* out of the magnetic meridian, its poles of strength m experience equal, parallel and opposite forces, each being equal to m H (Fig. 112). They tend to restore the magnet to its original position in the magnetic meridian, and the restoring couple is equal to MH sin θ.

$$\text{Angular acceleration} = \frac{\text{MH}\sin\theta}{\text{K}}$$
$$= \frac{\text{MH}\,\theta}{\text{K}},$$

as θ being *very small*, its sine is practically equal to its radian measure. Therefore, the motion of the magnet is *simple harmonic*, and its time period t is given by

$$t = \frac{2\pi}{\sqrt{\frac{\text{MH}}{\text{K}}}} = 2\pi\sqrt{\frac{\text{K}}{\text{MH}}}$$

Problem. If M_1 and M_2 are the magnetic moments of the magnets and K_1 and K_2 are their respective moments of inertia, the resultant magnetic moment in the first and second cases is equal to M_1+M_2 and M_1-M_2 respectively, while in both cases the total moment of inertia is equal to K_1+K_2.

$$12 = 2\pi\sqrt{\frac{K_1+K_2}{(M_1+M_2)H}}$$

$$16 = 2\pi\sqrt{\frac{K_1+K_2}{(M_1-M_2)H}}$$

$$\therefore \quad \frac{16}{12} = \sqrt{\frac{M_1+M_2}{M_1-M_2}}$$

or $$\frac{256}{144} = \frac{M_1+M_2}{M_1-M_2}$$

or $$256\,M_1 - 256M_2 = 144M_1 + 144M_2$$

or $$112M_1 = 400M_2$$

or $$\frac{M_1}{M_2} = \frac{400}{112}$$
$$= \frac{25}{7}$$

Q. 143. Define what is meant by the moment of a magnet and give a method of determining it.

Prove that the magnetic length of a bar magnet is $2\left[\dfrac{d_1^{3}\tan\theta_1-d_2^{3}\tan\theta_2}{2(d_1\tan\theta_1-d_2\tan\theta_2)}\right]^{\frac{1}{2}}$, where θ_1 and θ_2 are the deflections of the magnetometer needle at distances d_1 and d_2 from the centre of the bar. (*Bombay, 1929*)

Ans. The effect of a magnet at an external point is determined by its magnetic moment, which depends on its pole strength and length. It is a measure of the restoring couple which it experiences when it is placed inclined to a magnetic field and is equal to the product of its pole strength and the distance between its poles.

Determination of the Moment of a Magnet. A magnet of moment M is placed horizontally in a stirrup, suspended by a vertical thread, in a box with glass sides. When the magnet lies in the magnetic meridian, there should be no torsion in the thread. Another magnet is slowly brought near the glass box, with its one pole pointing towards a similar pole of the suspended magnet, so that, the latter is *slightly rotated* about the suspending thread. When the outside magnet is carefully removed, the suspended magnet begins to *rotate* back, under the restoring couple (MH sin θ) due to the field of the earth, with increasing speed; but, owing to its inertia, it does not stop in its normal position. It continues to move on the other side, with decreasing speed, until it comes to rest at the other extreme. It then moves back, and, if its amplitude be *small*, it vibrates with a definite time period t given by the equation

$$t=2\pi\sqrt{\frac{K}{MH}},$$

where K is the moment of inertia of the magnet about the axis of suspension and H is the horizontal component of the magnetic field of the earth. From the time of a *large* number of vibrations, the time of one vibration is calculated. The above equation may be put in the form

$$t^2=\frac{4\pi^2K}{MH}$$

or

$$MH=\frac{4\pi^2K}{t^2} \quad \ldots\ldots\ldots \quad (1)$$

The value of K is fouud from the mass of the magnet and its length and breadth.

A deflection magnetometer is arranged with its arms horizontal and perpendicular to the magnetic needle, that is, perpendicular to the magnetic meridian. The same magnet, as used in the vibration magnetometer, is placed on the east arm of the deflection magnetometer, with its north pole pointing east and its middle at a distance d from the middle of the magnetometer needle, and the deflection of both the ends of the pointer is found. Then the magnet is turned round, so that its north pole points west, and again both ends of the pointer are read. These observations are repeated with the magnet on the west arm of the magnetometer, with its middle at the same distance d from the magnetometer needle.

If θ be the mean of these eight readings and $2l$ the magnetic length of the magnet, the intensity F of its magnetic field at the magnetic needle is equal to $\frac{2\mathrm{M}d}{(d^2-l^2)^2}$, perpendicular to the magnetic meridian. As this intensity is *perpendicular* to the horizontal component of the earth's field,

$$\therefore \tan\theta = \frac{\text{Deflecting field}}{\text{Controlling field}} = \frac{\mathrm{F}}{\mathrm{H}}$$

or

$$\mathrm{H}\tan\theta = \mathrm{F} = \frac{2\mathrm{M}d}{(d^2-l^2)^2}$$

or

$$\frac{\mathrm{H}}{\mathrm{M}} = \frac{2d}{(d^2-l^2)^2\tan\theta} \quad \cdots\cdots \quad (2)$$

Dividing (1) by (2), we get

$$\mathrm{MH}\times\frac{\mathrm{M}}{\mathrm{H}} = \frac{4\pi^2\mathrm{K}}{t^2}\times\frac{(d^2-l^2)^2\tan\theta}{2d}$$

$$\therefore \mathrm{M} = \frac{2\pi(d^2-l^2)}{t}\sqrt{\frac{\tan\theta}{2d}}$$

If the poles of the magnet are not at its ends, its magnetic length is not equal to its geometrical length. To eliminate or determine the magnetic length, the deflection magnetometer experiment is repeated with the magnet at a different distance from the needle.

To Eliminate l

$$\frac{M}{H}=\frac{\tan\theta_1(d_1^2-l^2)^2}{2d_1}$$

$$=\frac{\tan\theta_1 . d_1^4\left(1-\frac{l^2}{d_1^2}\right)^2}{2d_1}=\frac{d_1^3\tan\theta_1}{2\left(1-\frac{l^2}{d_1^2}\right)^{-2}}$$

$$=\frac{d_1^3\tan\theta_1}{2\left(1+2\frac{l^2}{d_1^2}+\text{negligible terms}\right)}$$

$$=\frac{d_1^5\tan\theta_1}{2(d_1^2+2l^2)}$$

$$=\frac{d_2^5\tan\theta_2}{2(d_2^2+2l^2)}\text{ for second distance}$$

$$=\frac{d_1^5\tan\theta_1-d_2^5\tan\theta_2}{2(d_1^2+2l^2-d_2^2-2l^2)}$$

$$=\frac{d_1^5\tan\theta_1-d_2^5\tan\theta_2}{2(d_1^2-d_2^2)}$$

Magnetic Length.

$$\frac{M}{H}=\frac{\tan\theta_1(d_1^2-l^2)^2}{2d_1}=\frac{\tan\theta_1\, d_1^4}{2d_1}\left(1-\frac{l^2}{d_1^2}\right)^2$$

$$=\frac{d_1^3\tan\theta_1}{2}\left(1-2\frac{l^2}{d_1^2}+\text{negligible terms}\right)$$

Similarly, for second distance, $\frac{M}{H}=\frac{d_2^3\tan\theta_2}{2}\left(1-2\frac{l^2}{d_2^2}\right)$

$$\therefore d_1^3\tan\theta_1-2l^2d_1\tan\theta_1=d_2^3\tan\theta_2-2l^2d_2\tan\theta_2$$

$$l^2=\frac{d_1^3\tan\theta_1-d_2^3\tan\theta_2}{2(d_1\tan\theta_1-d_2\tan\theta_2)}$$

$$l=\left\{\frac{d_1^3\tan\theta_1-d_2^3\tan\theta_2}{2(d_1\tan\theta_1-d_2\tan\theta_2)}\right\}^{\frac{1}{2}}$$

$$\therefore \text{ Magnetic length }\quad 2l=2\left\{\frac{d_1^3\tan\theta_1-d_2^3\tan\theta_2}{2(d_1\tan\theta_1-d_2\tan\theta_2)}\right\}^{\frac{1}{2}}$$

Q. 144. Define the terms Declination and Dip. Describe a method of accurately determining the dip at a place. (*Punjab, 1931*)

Ans. **Declination.** When a magnet is free to rotate at a place about a *vertical* axis, the line joining its poles is called its magnetic axis and the vertical plane passing through this axis is called the magnetic meridian of the place. The angle which the magnetic meridian makes with the geographical meridian is called the angle of declination there.

Dip. When a magnet is free to rotate in the magnetic meridian and about a *horizontal* axis, the angle which its magnetic axis makes with the horizontal is called the angle of dip.

Determination of Dip. A dip circle consists of a magnetic needle free to rotate, in front of a vertical circular scale, about a horizontal axle passing through its centre of gravity and the centre of the vertical circular scale. The vertical circular scale is divided into four quadrants and is contained in a box, with glass sides, which can be rotated about a vertical axle passing through the centre of a horizontal graduated circular scale. This horizontal scale is attached to the base of the instrument,. and the whole instrument is supported on screws used for levelling it.

The instrument is levelled, and the vertical box is turned about the vertical axle until the magnetic needle stands *vertically*. In this position the needle is under the rotating action of the vertical component only of the earth's field, the horizontal component is ineffective in rotating the needle, and, therefore, the horizontal axle of the needle must be *parallel* to the horizontal component. As the axle is now in the magnetic meridian, the plane of rotation of the needle is *perpendicular* to the magnetic meridian. Then the vertical box is turned through 90° to set the needle in the magnetic meridian. In this position the angle of inclination of the needle to the horizontal gives the angle of dip.

To eliminate certain errors both ends of the needle are read; the vertical box is turned through 180°, and again readings are taken. The needle is turned round back for front, and its both ends are read; the vertical box is again

turned through 180°, and the ends of the needle are read. Finally the needle is magnetised in the opposite direction, and the above eight readings are again taken. The mean of these sixteen readings gives the true value of dip at that place.

Q. 145. Describe in detail how you would determine accurately the absolute value of the horizontal component of the earth's magnetic field at any place. Obtain the necessary formula. (*Bombay, 1930*)

Ans. **Determination of the Horizontal Component H.** The method for finding the horizontal component H of the earth's magnetic field is the same as used for measuring the magnetic moment of a magnet in Q. 143.

Multiplying (1) by (2), we get

$$MH\times\frac{H}{M}=\frac{4\pi^2K}{t^2}\times\frac{2d}{(d^2-l^2)^2\tan\theta}$$

$$\therefore\ H=\frac{2\pi}{t(d^2-l^2)}\sqrt{\frac{2Kd}{\tan\theta}}$$

Q. 146. How will you determine experimentally the value of the vertical component of the earth's magnetic field ?

The correct value of the dip at a certain place is 69°. What is the apparent dip if the circle be turned 45° out of the magnetic meridian ? (Trignometric tables will be provided.) (*Punjab, 1933*)

Ans. See Q. 145 for finding the horizontal component H of the earth's magnetic field and Q. 144 for the angle of dip θ. From these two determinations the vertical component V of the earth's field is found by the relation

$$V=H\tan\theta.$$

Problem. Angle of dip=69°

$$\therefore\ \frac{V}{H}=\tan 69°=2\text{·}605$$

When the vertical circular scale carrying the magnetic needle is turned about a vertical axis, through θ out of the magnetic meridian, the angle between the horizontal component H of the earth's field and the axle about which the needle rotates decreases from 90° to $(90°-\theta)$, and the component of H perpendicular to this axle and effective in

rotating the needle becomes equal to H $\cos(90° - \theta) = H \sin\theta$. The vertical component of the earth's field remains *fully* effective, and, therefore, if ϕ is the apparent angle of dip in this new position,

$$\tan\phi = \frac{V}{H\sin\theta} = \frac{\tan 69°}{\sin 45°}$$

$$= \frac{2{\cdot}605}{0{\cdot}7071} = 3{\cdot}684$$

and $\phi = 74{\cdot}8°$.

Q. 147. Derive an expression for the intensity of the field due to a small magnet at a large distance from it. Hence deduce that in case of the earth

$$2 \tan\lambda = \tan\delta$$

where δ is the angle of dip at a place whose magnetic latitude is λ. (*Punjab, 1938*)

Ans. It is shown in Q. 140 that the magnetic intensity at a point P in the field, and at a distance d from the centre, of a small magnet of magnetic moment M is equal to $\frac{M}{d^3}\sqrt{1+3\cos^2\theta}$, where θ is the angle which the line joining the point with the middle point O of the magnet makes with the line joining the middle point of the magnet with its *north* pole. This intensity is inclined at an angle β to the line OP produced such that

$$\tan\beta = \tfrac{1}{2}\tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The magnetic field of the earth may be supposed to be due to a small magnet NS placed at its centre O with its *north* pole pointing towards the *south* magnetic pole of the earth. If the magnetic equator passes through two points E and F at the ends of a diameter of the earth, OE is perpendicular to SN.

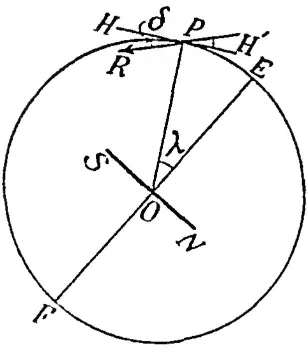


Fig. 119.

If λ is the magnetic latitude of a point P,

$$\lambda = \angle POE$$

and $\theta = \angle PON = (90° + \lambda) \quad . \quad . \quad (2)$

The horizontal line HPH′ at P is *perpendicular* to the radius OP, and dip δ is the angle which the

direction PR of the resultant magnetic intensity at P makes with PH. Therefore angle β which PR makes with line OP produced is given by

$$\beta=(90^\circ+\delta) \quad \ldots\ldots\ldots \quad (3)$$

Putting the values of θ and β in (1), we get

$$\tan(90^\circ+\delta)=\tfrac{1}{2}\tan(90^\circ+\lambda)$$

or

$$-\cot\delta=-\tfrac{1}{2}\cot\lambda$$

or

$$\frac{1}{\tan\delta}=\frac{1}{2\tan\lambda}$$

$$\therefore\ 2\tan\lambda=\tan\delta$$

PART VI

ELECTROSTATICS

Q. 148. Explain the meaning of electrostatic potential, and find an expression for it at a point in the field of a charge Q, the dielectric constant of the medium being K.

Ans. **Potential.** Two similarly charged bodies A and B, repel each other. The greater the distance of B from A, the smaller is the force of repulsion, so that at an infinite distance the repulsive force is zero. When B is brought towards A, work is done against this force, and this increases the electrical potential energy of B; the smaller the distance of B from A the greater is the potential energy of B. At different points in the electric field of A the electrical potential energy of B is different. At any point its value is directly proportional to the charge of A and the charge of B, and is inversely proportional to the distance of the point from A and the dielectric constant K of the medium.

The last three factors determine an electric property of the point. It is called its electric potential due to the charge of A, and is equal to this charge divided by K and the distance of the point from A. Thus the electric potential energy of the charge of B is equal to the product of its charge and the electric potential of the point where it is placed in the electric field. Therefore electric potential at a point in the field of a charged body is equal to the amount of work done in bringing a *unit* positive charge from infinity to that point. The smaller the distance of the point from the charged body, the greater is the potential. If the body is positively charged, the unit positive charge, left to itself, will recede from the body, or move from a point of higher potential to a point of lower potential. Thus *the potential at a point determines the direction of flow of electricity at the point.*

Value of Potential. The electric intensity F in the field of a charge changes from point to point, but for two points at a *very small* distance dx apart and *in* its direction its value is the *same*, and the amount of work done in carrying a unit

positive charge from the farther point to the nearer point is equal to $F(-dx)$, as the distance from the charge is *decreased* and, therefore, dx is negative. This is also equal to the potential difference dV between them.

$$\therefore \ dV = -F dx.$$

But the electric intensity F at a distance x from a charge Q and in a medium of dielectric constant K is equal to $\frac{Q}{Kx^2}$, and the potential V at a point is equal to the amount of work done in bringing a unit positive charge from infinity to that point.

$$\therefore \ V = \int_{\infty}^{x} -F dx$$

$$= \int_{\infty}^{x} -\frac{Q}{Kx^2} dx$$

$$= \left[\frac{Q}{Kx}\right]_{\infty}^{x}$$

$$= \frac{Q}{Kx} - \frac{Q}{K\infty}$$

$$= \frac{Q}{Kx},$$

as any quantity divided by ∞ is equal to zero.

Q. 149. Explain :—Electrical capacity of a conductor, an electrical condenser, and specific inductive capacity.

Two insulated spheres of radii 30 and 45 cm. have potentials of 48 and 32 units respectively. Calculate the potential and loss of energy when they are connected by a wire. Derive the equation you use.

(*Bombay, 1926*)

Ans. **Capacity.** When two isolated spherical conductors of different radii are given *equal* charges, their potentials are raised by *different* amounts : the sphere of smaller radius has higher potential than the sphere of greater radius. In order to raise both of them to the same potential the sphere of greater radius has to be given more charge than the other

sphere. Thus the charge required to raise the potential of a given conductor by a given amount depends not only on the value of the potential but also on the size of the conductor. It also depends on the shape of the conductor and the surrounding medium. Further, for a given conductor the charge required is *proportional* to the change of potential.

The capacity of a conductor is defined as equal to the ratio of its charge to its potential produced by *this* charge, and is measured by the amount of charge required to raise its potential by one unit. If a charge of Q units raises the potential of the conductor by V units, its capacity is equal to $\frac{Q}{V}$ units.

Condenser. The capacity of a conductor depends not only on its dimensions and the nature of the surrounding medium but also on the presence of neighbouring conductors. It is *increased* on account of such conductors, and the effect is the greatest when a neighbouring conductor is *earthed.* If an earthed conductor B is brought near a charged conductor A, *opposite charge is induced on B. The potential of A is lowered* due to the *opposite induced* potential, and thus its capacity is increased.

This arrangement of the two conductors is called a **condenser,** and its capacity is equal to the amount of charge required to produce a unit potential difference between *them* (coatings).

Specific Inductive Capacity. While an insulating medium does not allow flow of electricity in it, electric induction takes place through it, and different media are strained by different amounts by the same charge. The facility with which the lines of force are transmitted through it depends on its nature, and this property is called its inductive capacity. The **specific induction capacity** of a substance is equal to the capacity of a condenser with that substance as its dielectric divided by the capacity of the *same* condenser with air (vacuum) between its coatings.

Loss of Energy. The energy of a conductor of capacity C and raised to potential V is equal to $\frac{1}{2}CV^2$. When two conductors of capacities C_1 and C_2, and corresponding potentials V_1 and V_2 are connected by a thin wire, their charges are redistributed so that they acquire a common potential V. The thin wire being of negligible capacity takes up no charge, and

the total charge on the conductors remains *unchanged*, but some energy is dissipated and the sum of their final energies is *less* than the sum of their energies before they were connected.

$$\text{Initial total energy} = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2$$

$$\text{Total charge} = C_1V_1 + C_2V_2$$
$$= C_1V + C_2V$$

$$\therefore \text{ Final common potential } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \quad . \; . \; . \; . \quad (1)$$

$$\text{Final total energy} = \tfrac{1}{2}C_1V^2 + \tfrac{1}{2}C_2V^2$$
$$= \tfrac{1}{2}(C_1 + C_2)V^2$$
$$= \tfrac{1}{2}(C_1 + C_2)\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2$$
$$= \tfrac{1}{2}\,\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$\therefore \text{ Loss of energy} = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2 - \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - 2C_1V_1C_2V_2 - C_2^2V_2^2}{2(C_1 + C_2)}$$

$$= \frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{2(C_1 + C_2)}$$

$$= \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

Whether $(V_1 - V_2)$ is positive or negative, its square is positive, that is, loss of energy always occurs except when V_1 is *equal* to V_2.

Problem. The capacity of an isolated spherical conductor is equal to its radius.

Capacity of first sphere $A = 30$ e.s. units.
" " second " $B = 35$ " "
Potential of first sphere $= 48$ " "
" " second " $= 32$ " "

Putting the values of C_1, C_2, V_1, and V_2 in (1) and (2), we get

$$\text{Final potential} = \frac{30 \times 48 + 45 \times 32}{30 + 45}$$
$$= 38{\cdot}4 \text{ e.s. units.}$$

and $$\text{Loss of energy} = \frac{30 \times 45(48-32)^2}{2(30+45)}$$
$$= 2304 \text{ ergs.}$$

Q. 150. Define normal electrical induction and tubes of force. State and prove Gauss's theorem. Deduce from the theorem that the intensity of field near a charged surface of density σ is $4\lambda\sigma$. (*Punjab, 1935*)

Ans. **Tubes of Force.** A line of force in an electric field indicates the path of a *free* positive charge, and at each point the tangent to it shows the direction of the electric intensity there. When these lines are grouped into tubes in such a way that $\frac{4\pi}{K}$ tubes come out of a unit charge in a medium of dielectric constant K, they are called **Maxwell unit tubes of force.**

Normal Electrical Induction. When the lines of induction are grouped into tubes in such a way that 4π tubes come out of a unit charge, their number *perpendicular* to, and per *unit* area of, a surface in the electric field is called **normal electrical induction.**

Gauss's Theorem. *The total normal induction over a closed surface in an electric field is equal to 4π times the total charge inside it.*

Proof. Let a closed surface surround a charge Q at O, and consider a *very small* element of it of area δa at P and at a distance x from O (Fig. 120). The electric intensity F at P is directed *outward* along OP produced, and has practically the same value $\frac{Q}{Kx^2}$ over area δa. It makes angle θ with the normal PN at P, and its component along PN is equal to F cos θ.

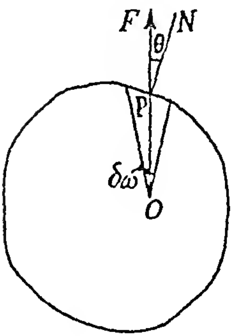


Fig. 120.

Normal intensity at P = F cos θ

" induction at P = KF cos θ,

where K is the dielectric constant of the medium.

$\therefore$ Total normal induction on $\delta a = \text{KF} \cos\theta.\delta a$

$$= \frac{\text{KQ}.\delta a}{\text{K}x^2} \cos\theta$$

$$= \frac{\text{Q}\delta a \cos\theta}{x^2},$$

as F is equal to $\frac{\text{Q}}{\text{K}x^2}$. The component of area δa *normal* to OP at P is equal to $\delta a.\cos\theta$, as the angle between two areas is equal to the angle between their normals, and $\frac{\delta a.\cos\theta}{x^2}$ is the *solid angle* $\delta\omega$ subtended by it at O, so that the above result gives.

Total normal induction on $\delta a = \text{Q}.\delta\omega$.

Proceeding in this way the total normal induction over any part of the surface may be found, and its value for the whole of the closed surface is equal to $\Sigma\text{Q}.\delta\omega$.

$\therefore$ Total normal induction over the closed surface $= \Sigma\text{Q}\ \delta\omega$

$$= 4\pi\text{Q}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

as Q is constant and the solid angle subtended by any closed surface on a point inside it is equal to 4π. If there are other charges inside the closed surface, the same treatment applies to all cases, and the total outward normal induction is equal to 4π times the *total* charge inside it.

Intensity near a Charged Surface. Let a charged conductor have σ units of charge per unit area. As it is a *conductor*, its surface is equipotential, and at any point on it the electric intensity F is outward and along the *normal* there. Consider a point A *very close* to the charged surface and a point B very near its inside, and draw a tube with faces at A and B parallel to the surface, its sides everywhere *normal* to the surface, and enclosing a very *small area* δa at it. As A and B are very close to the surface, the areas of its faces at A and B are equal to δa. There is no induction on the sides of this tube as intensity is parallel to

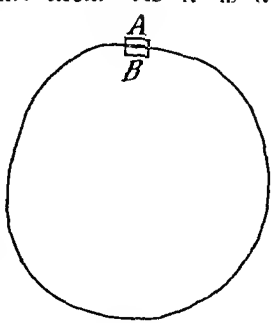


Fig. 121.

them, nor is there any induction on face B as it is within the closed surface and surrounds no charge, and thus there is normal induction on its face A only.

$$\text{Normal induction at } A = KF$$

$$\text{Total} \;\;,,\;\;\;\; ,, \;\;\;\; ,, \;,, = KF\delta a$$

$$\text{Total charge inside the tube} = \sigma.\delta a$$

$$\therefore \; KF.\delta a = 4\pi\sigma.\delta a$$

or
$$F = \frac{4\pi\sigma}{K}.$$

The dielectric constant of air is equal to 1, and therefore, F is equal to $4\pi\sigma$.

If the surface is not closed and is open, there is normal induction on face B also, but in that case the inner side of the conductor is also charged and has surface density σ. The electric intensity at B is also equal to F, and

$$KF.2\delta a = 4\pi\sigma.2\delta a$$

or
$$F = \frac{4\pi\sigma}{K}.$$

Q. 151. State Gauss's theorem. Prove that every element of a charged conductor experiences an outward force equal to $2\pi\sigma^2$ per unit area, where σ is the charge per unit area at the point considered.

A sphere of radius 100 cm. is charged to a potential of 1500 e.s. units. Calculate the force per unit area of the sphere. (*Punjab, 1938*)

Ans. See Q. 150 for the statement of Gauss's theorem and showing that the electric intensity very close to a charged conductor is equal to $\frac{4\pi}{K}$ time its surface density σ.

The electric intensity F_1 at A (Fig. 121) due to the charge on the *very small* area δa of the enclosed surface is in the *same* direction as the intensity F_2 there due to the charge on the *rest* of the surface, and, therefore, the resultant intensity F is equal to $F_1 + F_2$.

At B the intensity due to the charge on area δa is *equal* and *opposite* to that at A, while that due to the charge on the rest of the surface is the *same* as at A, as A and B are

very close to each other. But there is no resultant intensity at B.

$$\therefore \quad -F_1+F_2=0$$

or
$$F_2=F_1=\frac{F}{2}=\frac{2\pi\sigma}{K}.$$

Thus area δa is placed in a field of intensity $\frac{2\pi\sigma}{K}$, and as the charge on it is equal to $\sigma.\delta a$, the total force that it experiences *outward* is equal to $\frac{2\pi\sigma}{K}\times\sigma.\delta a=\frac{2\pi\sigma^2.\delta a}{K}$

$$\therefore \quad \text{Force per unit area}=\frac{2\pi\sigma^2.\delta a}{K.\delta a}=\frac{2\pi\sigma^2}{K}$$

For air, $K=1$, and the force per unit area is equal to $2\pi\sigma^2$.

Problem. The capacity of an isolated spherical conductor is numerically equal to its radius.

Capacity of the sphere $=100$ e.s. units

Potential „ „ „ $=1500$ e.s. units

$\therefore$ Charge „ „ „ $=100\times1500$ e.s. units

Surface area „ „ „ $=4\pi\times100^2$ sq. cm.,

$$\text{Charge per unit area}=\frac{100\times1500}{4\pi\times100\times100}$$

$$=\frac{15}{4\pi}\text{ e.s. units sq. cm.}$$

$$\therefore \text{Outward force „ „ „}=2\pi\left(\frac{15}{4\pi}\right)^2$$

$$=\frac{2\times15\times15}{16\times3{\cdot}142}=8{\cdot}95\text{ dynes}$$

Q. 152. (*a*) **Find an expression for the force per sq. cm. of surface on a conductor due to its charge.**

(*b*) **Find the mechanical stress per sq. cm. on the glass plates of a condenser, charged to a potential of 30,000 volts. S.I.C. of glass=4 and thickness=4 mm.**

(*Punjab, 1932*)

Ans. (*a*) A conductor charged to surface density σ and placed in a medium of specific inductive capacity K experiences an outward force $\frac{2\pi\sigma^2}{K}$ per unit area. See Q. 151 for finding this expression.

(*b*) Potential difference $= 30{,}000$ volts

$$= \frac{30000}{300} = 100 \text{ e.s. units}$$

S. I. C. of glass $= 4$

Electric intensity in glass $= \frac{4\pi\sigma}{4} = \pi\sigma$

Thickness of glass $= 0{\cdot}4$ cm.

$\therefore$ Potential difference $= \pi\sigma \times 0{\cdot}4 = 100$

or $$\sigma = \frac{100}{0{\cdot}4\pi}$$

$\therefore$ Force per unit area $= \frac{2\pi}{4}\left(\frac{100}{0{\cdot}4\pi}\right)^2$

$$= \frac{100 \times 100}{0{\cdot}32 \times 3{\cdot}142}$$

$= 1591$ dynes per sq. cm.

Q 153. What is electrostatic pressure? Find its value and show that electricity would escape from a charged conductor if the surface density be sufficiently high. Does this escape of electricity depend upon curvature? Explain fully. (*Bombay, 1934*)

Ans. The electrostatic pressure of a charged conductor is equal to the outward mechanical force that its unit area experiences due to the charge on the *rest* of the surface. See Q. 151 for showing that its value is $2\pi\sigma^2$ for a conductor whose surface density of charge is σ and is placed in air.

As this pressure is proportional to the square of surface density σ, the conductor experiences an enormous force if σ is sufficiently high. The conductor cannot be torn, but the charge escapes from it. The greater the surface density the greater is the tendency for leakage.

Surface Density and Curvature. Let two spheres, A and B, of radii R_1 and R_2 respectively be connected to an electric machine and thereby raised to a common potential V. As the capacity of a sphere is equal to its radius, and the charge on a conductor is equal to the product of its capacity and potential, charges on A and B are equal to R_1V and R_2V respectively.

$$\text{Surface area of } A = 4\pi R_1^2$$

$$\therefore \quad \text{Density of charge on } A = \frac{\text{charge}}{\text{area}} = \frac{R_1 V}{4\pi R_1^2}$$

$$= \frac{V}{4\pi R_1} \quad \cdots\cdots \quad (1)$$

$$\text{Surface area of } B = 4\pi R_2^2$$

$$\therefore \quad \text{Density of charge on } B = \frac{R_2 V}{4\pi R_2^2} = \frac{V}{4\pi R_2} \quad \cdots\cdots \quad (2)$$

Dividing (1) by (2) we get

$$\frac{\text{Density of charge on A}}{\text{Density of charge on B}} = \frac{V}{4\pi R_1} \cdot \frac{4\pi R_2}{V} = \frac{R_2}{R_1}$$

$$= \frac{\text{Radius of B}}{\text{Radius of A}}.$$

This shows that the surface density of charge on a spherical conductor, raised to a given potential, is inversely proportional to its radius of curvature. The radius of curvature of the pointed parts of a conductor is very small, and, therefore, surface density of charge at these parts is much greater than at other parts. The surface density of these pointed parts soon reaches the maximum value, and then they begin to leak.

Q. 154. What is normal induction? Find with the help of Gauss's theorem the electric field near (*i*) a charged infinite plane, (*ii*) an infinitely long charged cylinder. (*Punjab, 1936*)

Ans. **Normal Induction.** An electric charge tends to induce charge on a conductor placed in its field. The induced charge may be regarded as the flux (flow) of a vector quantity. In vacuum the direction of this vector at any point is the same as that of the electric intensity there. It is called **induction** and its magnitude is proportional to the field strength.

A unit charge gives rise to 4π tubes of induction, so that $4\pi Q$ such tubes emanate from a charge Q. If a spherical surface of radius R and centre at the charge is considered, these tubes are uniformly distributed over its surface and are everywhere *perpendicular* to it. The number of tubes per *unit* area is equal to $\frac{4\pi Q}{4\pi R^2} = \frac{Q}{R^2}$ and measures the **normal induction** over it.

According to Gauss's theorem, the total normal induction over a closed surface is equal to 4π times the total charge *inside* it.

(*i*) **Uniformly charged Infinite Plane.** Let AB be a part

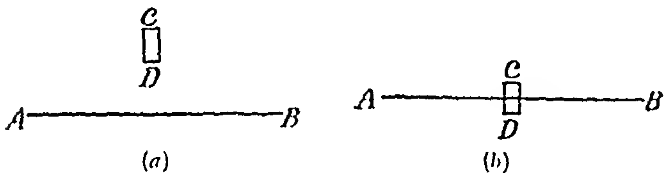


Fig. 122.

of the infinite plane charged *uniformly* and having surface density σ on *each* side, and consider a cylinder CD with its plane faces C and D of area S *parallel* to the charged surface, so that its cylindrical surface is perpendicular to AB [Fig. 122 (*a*)]. As the charged surface is infinite and conducting, the lines of force and, therefore, electric intensity are *everywhere perpendicular* to it.

If F and F′ are the values of electric intensity at C and D respectively, the corresponding values of *normal* induction are KF and KF′, where K is the dielectric constant of the medium. The total outward normal induction over the face C is equal to SKF, while over the face D it is *into* the cylinder and equal to SKF′, and there is *no* normal induction over the sides of the cylinder. As the cylinder contains *no* charge inside it, the total normal induction over it is zero.

$$SKF - SKF' = 0$$

or

$$F = F' \quad \ldots \ldots \quad (1)$$

This shows that the electric intensity is the *same everywhere.*

Next consider the cylinder lying with its ends on the opposite sides of the charged surface [Fig. 122 (*b*)]. In this case, while there is again no normal induction over its sides, the normal induction over *both* its ends is *outward*, and it contains a charge $2\times S\sigma$ inside it. Therefore, according to Gauss's theorem.

$$SKF+SKF=4\pi\times 2S\sigma$$

$$F=\frac{4\pi\sigma}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If σ denotes the surface density of the charge on the *two* faces of the plane *taken together*,

$$F=\frac{2\pi\sigma}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(*ii*) **Uniformly charged Infinite Cylinder.** Let AB be a part of the infinite cylinder of radius R having its axis along CD, and charged uniformly with σ units per *unit area* (Fig. 123). If electric intensity F be required at a point P at a distance x from CD, consider a coaxial cylinder of length l passing through P and having its plane faces perpendicular to CD.

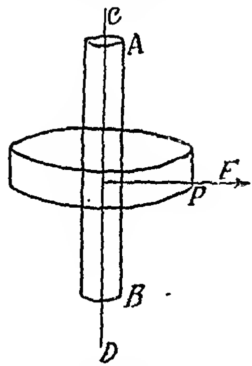


Fig. 123.

As the cylinder is conducting and uniformly charged, the lines of force leave its surface along the normals, and the electric intensity at any point is perpendicular to its length and is the *same* at all points at the same distance from its axis. There is *no* normal induction over the plane faces of the outer cylinder, and the total outward normal induction over its cylindrical surface is equal to $2\pi xlKF$. As the total charge inside it is equal to $2\pi Rl\sigma$,

$$\therefore \quad 2\pi xlKF=4\pi\times 2\pi Rl\sigma$$

or

$$F=\frac{4\pi R\sigma}{Kx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the point P is very close to the charged surface, x is almost equal to R, and equation (3) gives

$$F=\frac{4\pi\sigma}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Q. 155. Describe the electrostatic field between two parallel charged plates a small distance apart.

Define *specific inductive capacity* and find an expression for the capacity of a parallel plate condenser with a dielectric partly glass and partly air.

(Punjab, 1939)

Ans. **Field between Parallel Plates.** As the distance between the plates is *small*, they may be considered to be infinite. If they are oppositely charged, the charges are mainly confined to their *inner* faces. The lines of force between them are *parallel* to one another and *perpendicular* to the plates, and the surface density σ is uniform, except near the edges, where the lines of force bulge outward. The intensities due to the two charges are in the *same* direction between the two plates and help each other, but they oppose each other at all *external* points. Each plate gives rise to an intensity $2\pi\sigma$.

Specific Inductive Capacity. The specific inductive capacity of a substance is equal to the capacity of a condenser with it as the dielectric divided by the capacity of the same condenser with air (vacuum) between its plates.

Capacity of Parallel Plate Condenser. Consider two parallel metallic plates, B and D, at a small distance d apart (Fig. 124). Each is of area S (one face only). The surface density of the charge on the upper plate B is σ, and as the

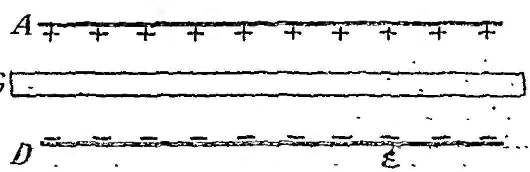


Fig. 124.

lower plate is earthed, its surface density is equal to $-\sigma$. A *larger* glass plate G of thickness t and specific inductive capacity K is placed between them, so that the thickness of the remaining air column is equal to $(d-t)$.

The electric intensity at a point between the plates is from A towards D due to the positive charge on B and in the *same*

direction due to the *negative* charge on D, so that the resultant intensity is equal to the *sum* of these two component intensities. As σ is the surface density of the charge on the *two* faces of the coatings *taken together*, according to Q. 154 equation (3).

Intensity in air due to each plate $=2\pi\sigma$

" " " both the plates $=4\pi\sigma$

" glass due to each plate $=\dfrac{2\pi\sigma}{K}$

" " " both the plates $=\dfrac{4\pi\sigma}{K}$

By definition intensity at a point is equal to the force experienced by a unit positive charge there, and the potential difference between two points is equal to the amount of work done in carrying a unit positive charge from one point to the other. The potential difference between the two coatings is equal to the sum of the fall of potential in air and the glass slab, and each of these is equal to the product of the thickness of the dielectric and the electric intensity in it.

Fall of Potential in air $=4\pi\sigma\ (d-t)$

Fall of Potential in glass $=\dfrac{4\pi\sigma}{K}t$

$\therefore$ Potential difference V between the coatings

$$=4\pi\sigma(d-t)+\frac{4\pi\sigma t}{K}$$

$$=4\pi\sigma\left(d-t+\frac{t}{K}\right)$$

Charge Q on each coating $=S\sigma$

$$\therefore \text{ Capacity of the condenser}=\frac{Q}{V}=\frac{S\sigma}{4\pi\sigma\left(d-t+\dfrac{t}{K}\right)}$$

$$=\frac{S}{4\pi\left(d-t+\dfrac{t}{K}\right)}$$

Due to the spreading of the lines of force near the edges of

the coatings, the actual capacity of the condenser is slightly greater than this value.

Q. 156. Find the electric intensity at a point between two large parallel plates, one of which is earthed, the surface density of the charge on the insulated plate being σ.

Two large metal plates are fixed horizontally at a distance of $\frac{1}{2}$ cm. from each other. What potential in volts should be applied between the plates, if a droplet of oil of mass $1 \cdot 5 \times 10^{-11}$gm., and carrying a charge of $4 \cdot 9 \times 10^{-10}$ E.S.U., is to be held at rest between the plates? [$g = 980$; one E.S.U. of potential = 300 volts.]

Mention any application of this arrangement.

(*Punjab, 1936*)

Ans. **Intensity between Parallel Plates.** As the surface density of the charge on the insulated plate, both faces *taken together*, is σ, that of the earthed plate is $-\sigma$. The two plates being *very large* as compared with the distance between them, they behave like plates of infinite area (Q. 154). The intensity at a point *between* them is *perpendicular* to them and is the *same* everywhere. The intensity due to the positively charged plate is directed *away* from it, while that due to the negatively charged plate is towards itself, that is, in the *same* direction as the first, and each is numerically equal to $\frac{2\pi\sigma}{K}$, where K is the dielectric constant of the medium between them. Therefore the resultant intensity at *any* point between the plates is equal to $\frac{4\pi\sigma}{K}$ and is directed from the positively charged plate towards the other plate With air between the plates, the value of intensity $= 4\pi\sigma$.

Problem. As the drop is balanced, the downward force of gravity on it is counterbalanced by an equal upward electric force which is equal to the product of electric intensity and the charge on it, and the potential difference between the

plates is equal to the product of electric intensity and the distance between them.

$$\text{Weight of the drop} = 1{\cdot}5 \times 10^{-11} \times 980 \text{ dynes.}$$

$$\text{Charge on the drop} = 4{\cdot}9 \times 10^{-10} \, e.\,s. \text{ units}$$

$$\therefore \text{ Electric intensity} = \frac{1{\cdot}5 \times 10^{-11} \times 980}{4{\cdot}9 \times 10^{-10}}$$

$$= 30 \; e.\,s. \text{ units.}$$

Distance between the plates $= 0{\cdot}5$ cm.

$$\therefore \text{ Potential diff. ,, ,, ,,} = 30 \times 0{\cdot}5$$

$$= 15 \; e.\,s. \text{ units.}$$

$$= 15 \times 300$$

$$= 4500 \text{ volts.}$$

Application. In Millikan's method for measuring the charge on electrons, oil droplets are produced and charged between two metal plates, and the potential difference between the plates is adjusted to make the droplets move upward or downward with any velocity or remain stationary.

Q. 157. Show that the capacity of the parallel plate condenser is $C = \dfrac{KA}{4\pi d}$.

Two Leyden jars are exactly similar in size and shape, but one has glass as dielectric and the other ebonite. The glass jar is charged but when charge is shared between the two jars, the potential falls to 0·6 of its original value. If S.I.C. of ebonite is 2, find the S I.C. of glass. (*Bombay, 1926*)

Ans. See Q. 155. The whole of the space between the coatings is filled with the substance of dielectric constant K and thickness d, and each plate is of area A.

$$\text{Electric Intensity} = \frac{4\pi\sigma}{K}$$

$$\text{Potential difference} = \frac{4\pi\sigma}{K} \times d$$

$$\text{Charge} = A\sigma$$

$$\therefore \text{ Capacity} = \frac{A\sigma \times K}{4\pi\sigma d}$$

$$= \frac{KA}{4\pi d}.$$

Problem. As the two Leyden jars are exactly similar, their capacities are proportional to the specific inductive capacities of their dielectrics. Let K be the specific induction capacity of glass, and C be the capacity of the ebonite Leyden jar, then the capacity of the glass Leyden jar is equal to $\frac{C \times K}{2}$. If V be the original potential difference produced between the coatings of the glass jar, it becomes 0·6V when its charge is shared with the ebonite jar.

$$\therefore \quad \frac{CK}{2} \times V = \left(C + \frac{CK}{2}\right) \times 0{\cdot}6V$$

$$= 0{\cdot}6VC\left(1 + \frac{K}{2}\right)$$

or

$$\frac{K}{2} = 0{\cdot}6 + 0{\cdot}3K$$

or

$$0{\cdot}2K = 0{\cdot}6$$

$$\therefore \ K = 3.$$

Q. 158. A condenser is formed out of two parallel plates each of area A and separated by a dielectric slab of thickness *d* and of dielectric constant K. Find the velues of the forces between the two plates when (*a*) they are oppositely charged with a surface density σ, and (*b*) a potential difference V is maintained between them. Show how you can use one of the arrangements for measuring the dielectric constant of a solid.

(*Calcutta, 1930*)

Ans. See Q. 155 for showing that the electric intensity at *any* point between the two plates is equal to $\frac{4\pi\sigma}{K}$. Half of this is due to the charge on one plate and the other half is due to the opposite charge on the other plate. Force experienced by the charge on one plate is due to the charge on the *other* plate.

(*a*) Force on a unit charge of one plate $= \frac{4\pi\sigma}{K}$

Charge on one plate $= A\sigma$

$\therefore$ Force exerted on each plate due to the charge on the other

$$= \frac{2\pi\sigma}{K} \times A\sigma$$

$$= \frac{2\pi\sigma^2 A}{K}.$$

(*b*) The potential difference V between the two plates is equal to the product of the electric intensity $\frac{2\pi\sigma}{K}$ between the plates and the distance d between them, as the potential difference between two points is equal to the amount of work done in carrying a unit positive charge from one point to the other and the electric intensity at a point is equal to the force experienced by a unit positive charge there.

$$V=\frac{4\pi\sigma}{K}\times d$$

or
$$\sigma=\frac{KV}{4\pi d}$$

$\therefore$ Force exerted on each plate $=\frac{2\pi\sigma^2 A}{K}$

$$=\frac{2\pi A}{K}\left(\frac{KV}{4\pi d}\right)^2$$

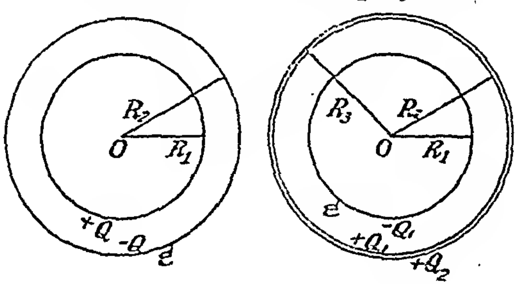
$$=\frac{KV^2A}{8\pi d^2}.$$

Measurement of Dielectric Constant. See Q. 162 for measuring the dielectric constant of a solid with an attracted disc electrometer.

Q. 159. Find the capacity of two concentric spheres when (*i*) outer one is earthed and inner one is insulated, (*ii*) inner one is earthed and outer one is insulated.

(Punjab, 1936)

Ans. **Capacity of a Spherical Condenser.** (*i*) *Outer sphere earthed.* The two *concentric* conducting spheres of radii R_1 and R_2 are placed in a medium of dielectric constant K [Fig. 125 (*a*)]. If a charge Q is given to the inner insulated sphere, an equal and opposite charge $-Q$ is induced on the outer sphere. As the



(*a*) (*b*)

Fig. 125.

outer sphere is earthed, its potential is *zero*, and is due to its own negative potential and its induced positive potential due to the charge on the inner sphere. Similarly, the potential of the inner sphere is due to its own charge and the induced opposite charge on the outer sphere. The potential difference between the two spheres is equal to that of the inner sphere, as the potential of the outer sphere is zero.

$$\text{Potential of inner sphere} = \frac{Q}{KR_1} + \frac{-Q}{KR_2}$$

$$= \frac{Q(R_2 - R_1)}{KR_1R_2}$$

$$\therefore \text{ Capacity of the spherical condenser} = \frac{Q}{\dfrac{Q(R_2 - R_1)}{KR_1R_2}}$$

$$= \frac{KR_1R_2}{R_2 - R_1} \quad . \quad . \quad . \quad (1)$$

(*ii*) *Inner sphere earthed.* In this case charge Q is given to the outer sphere, and it is divided into two parts, Q_1 on its inner surface and Q_2 on its outer surface of radius R_3 [Fig. 125(*b*)]. The inner sphere is earthed and charge $-Q_1$ (not $-Q$) is induced on it. Its potential is equal to zero on account of its own potential $\frac{-Q_1}{KR_1}$ and induced potentials $\frac{Q_1}{KR_2}$ and $\frac{Q_2}{KR_3}$. The potential of the outer sphere is due to its two charges Q_1 and Q_2, and the induced charge $-Q_1$ on the inner sphere, and is equal to the potential difference between the two spheres.

$$\text{Potential of inner sphere} = \frac{Q_2}{KR_3} + \frac{Q_1}{KR_2} - \frac{Q_1}{KR_1} = 0$$

or

$$\frac{Q_2}{R_3} = \frac{Q_1}{R_1} - \frac{Q_1}{R_2}$$

$$= \frac{Q_1(R_2 - R_1)}{R_1R_2}$$

or

$$Q_1 = \frac{Q_2R_1R_2}{R_3(R_2 - R_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Potential V of outer sphere $=\frac{Q_2}{KR_3}+\frac{Q_1}{KR_2}-\frac{Q_1}{KR_2}$

$$=\frac{Q_2}{KR_3}$$

$\therefore$ Capacity of the spherical condenser $=\frac{Q}{V}$

$$=\frac{(Q_1+Q_2)KR_3}{Q_2}$$

$$=\left\{\frac{Q_2R_1R_2}{R_3(R_2-R_1)}+Q_2\right\}\frac{KR_3}{Q_2} \text{ from (2)}$$

$$=\frac{KR_1R_2}{(R_2-R_1)}+KR_3 \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

Second Method. This arrangement gives two condensers connected in parallel. The inner surface of the outer sphere forms with the inner earthed sphere a spherical condenser of capacity $\frac{KR_1R_2}{(R_2-R_1)}$ as in (*i*), and the outer surface of the outer sphere of radius R_3 forms a condenser of capacity KR_3 with the *distant* earthed conductors.

Q. 160. Show that the energy stored per unit volume in an electrostatic field is $\frac{KE^2}{8\pi}$, where K is the dielectric constant of the medium and E is the electric intensity. (*Punjab, 1935*)

Ans. See Q. 151 for showing that a unit area of a charged conductor experiences an outward mechanical force $\frac{2\pi\sigma^2}{K}$, where σ is its surface density of charge and K is the dielectric constant of the medium. But the electric intensity E in its immediate neighbourhood is equal to $\frac{4\pi\sigma}{K}$.

$$\therefore \quad \text{Force per unit area} = \frac{2\pi\sigma^2}{K}$$

$$= \frac{2\pi}{K}\left(\frac{KE}{4\pi}\right)^2$$

$$= \frac{KE^2}{8\pi}$$

If an area δA of the charged surface is considered to be moved through a distance δx *perpendicular* to itself and *opposite* to the direction of the mechanical force, work done in overcoming this force is equal to $\frac{KE^2}{8\pi}\delta A.\delta x$, and this becomes the energy of the new electric field produced by the motion of the charged surface. But the volume added to the electric field by this motion is equal to $\delta A \times \delta x$,

$$\therefore \quad \text{Energy per unit volume} = \frac{KE^2 \delta A \times \delta x}{8\pi \times \delta A \times \delta x}$$

$$= \frac{KE^2}{8\pi}$$

Q. 161. How was the inverse square law in electrostatics established theoretically and experimentally by Cavendish? (*Punjab, 1936*)

Ans. **Cavendish Proof.** Fig. 126 represents a hollow metal sphere charged *uniformly* with the surface density of charge equal to σ. Through a point A, *not* at its centre C, draw lines to form two opposite cones of a *very small* solid angle ω and cut *very small* areas S_1 and S_2 from the sphere at their opposite ends and at distances from A equal to R_1 and R_2 respectively. Then

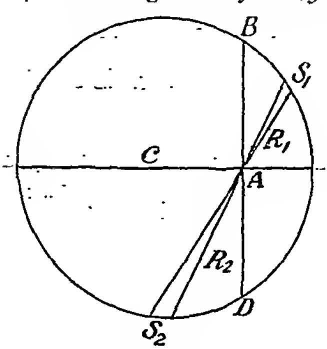


Fig. 126.

$$S_1 = R_1^2\omega$$

and $$S_2 = R_2^2\omega$$

$$\text{Charge on } S_1 = R_1^2\omega\sigma$$

$$\text{,, \quad ,, } S_2 = R_2^2\omega\sigma$$

If the electric intensity varies inversely as the nth power of the distance,

$$\text{Intensity at A due to charge on } S_1 = \frac{R^2{}_1\omega\sigma}{R_1{}^n}$$

$$= \frac{\omega\sigma}{R_1{}^{n-2}} \text{ along } S_1A$$

and " " " " " " " $$S_2 = \frac{R_2{}^2\omega\sigma}{R_2{}^n}$$

$$= \frac{\omega\sigma}{R_2{}^{n-2}} \text{ along } S_2A.$$

In this way the whole of the spherical surface can be considered to be divided by pairs of cones, one on the right of the plane BD and the other on its left. The intensity at A due to the charge on an area to the right of BD is from right towards A, while that due to the charge on an area on the left of BD is from left towards A. The direction of the resultant intensity at A depends on the value of n, and three cases arise.

(1) If n is greater than 2, $\frac{1}{R_1{}^{n-2}}$ is greater than $\frac{1}{R_2{}^{n-2}}$, as R_1 is smaller than R_2, and the intensity at A due to the charge on the part of the surface to the right of BD is greater than that due to the charge on the surface to the left of BD, that is, the resultant intensity at A is from *right to left towards* the centre C.

(2) If n is less than 2, $\frac{1}{R_1{}^{n-2}}$ $(=R_1{}^{2-n})$ is smaller than $\frac{1}{R_2{}^{n-2}}$. The intensity at A due to the charge on the right part of the spherical surface is smaller than that due to the charge on its left part, and the resultant intensity at A is from *left to right away* from the centre C.

(3) If n is equal to 2, $R_1{}^{n-2} = R_2{}^{n-2} = 1$, and the resultant intensity at A is *zero*. There is *no* intensity anywhere within the hollow sphere and the potential is the *same* throughout, because the electric intensity at a point is equal to the rate of change of potential there.

Experiment. Cavendish placed a small metallic sphere within the hollow sphere charged positively and at a point

other than its centre, and connected the two together for a moment. After disconnecting the two, he took out the small sphere, and found that it was *without* any charge.

In the first case where the resultant intensity is towards the centre, the small sphere should have acquired a positive charge, while in the second case of the resultant intensity being directed away from the centre of the hollow sphere, the small sphere should have been charged negatively. In the third case, there is no change of potential within the hollow sphere and no charge is given to the small sphere. Thus this experiment proved the third case, that is, electric intensity varies inversely as the square of the distance.

Q. 162. Describe and explain the use of an attracted disc electrometer. How do you make use of this to measure the dielectric constant of a solid ?

(*Punjab, 1935*)

Ans. **Attracted Disc Electrometer.** A circular metal plate A of face area S is suspended in a horizontal position by delicate springs from a plate L, L attached to a vertical rod N, and its height can be adjusted with a screw. (Fig. 127). It is surrounded by a ring R, called guard ring, forming a part of the bottom of a box, with a very narrow annular space between them. The lower plate B is supported parallel to A by a vertical rod. By using a micrometer screw M, its position can be adjusted and thereby its change of position is measured. The whole apparatus is placed in a glass jar whose inside is coated with tin foil and earthed to eliminate any disturbance due to the surrounding objects.

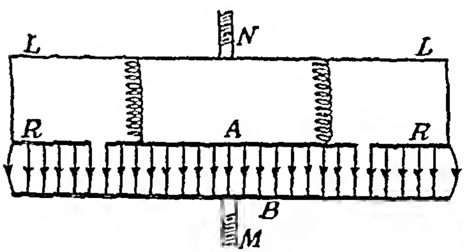


Fig. 127.

At first plates A and B and the guard ring R are earthed. A known mass m is placed on A to depress it below R, and the screw carried by N is adjusted to bring A in level with R. Then this mass is removed when plate A is pulled up by the springs. Both the plates and the guard rings are insulated,

and A and R are raised to and maintained at a *constant* potential V_A, while plate B is connected to one of the two points between which the potential difference is to be measured. The guard ring maintains *uniform* field at the edges of the plate A.

If the potential of this point is V_1, the potential difference between A and B is equal to $V_A - V_1$, and due to the mutual attraction between them, plate A is pulled down to some extent. By moving the micrometer screw M the position of plate B is adjusted to bring A in level with R, and the position of M is noted. In this position the force of attraction exerted by B on A is equal to mg, where g is the acceleration due to gravity. Let the distance between A and B be equal to d_1.

If σ be the surface density of charge on A, the electric intensity at a point between the two plates is equal to $4\pi\sigma$, and the potential difference $(V_A - V_1)$ between them is equal to $4\pi\sigma \times d_1$, so that

$$\sigma = \frac{V_A - V_1}{4\pi d_1}$$

But force of attraction on $A = 2\pi\sigma^2 S$

$$= 2\pi S \left(\frac{V_A - V_1}{4\pi d_1}\right)^2$$

$$= \frac{S}{8\pi d_1^{\,2}} (V_A - V_1)^2$$

$$= mg$$

$$\therefore\ V_A - V_1 = d_1 \sqrt{\frac{8\pi mg}{S}} \qquad \ldots\ldots \quad (1)$$

Then plate B is connected to the other point at potential V_2 and the micrometer screw is adjusted to bring A once more in level with R. If d_2 be the distance between A and B in this position,

$$V_A - V_2 = d_2 \sqrt{\frac{8\pi mg}{S}} \qquad \ldots\ldots \quad (2)$$

Subtracting (2) from (1), we get

$$V_2 - V_1 = (d_1 - d_2) \sqrt{\frac{8\pi mg}{S}}, \qquad \ldots \quad (3)$$

where $(d_1 - d_2)$ is the distance through which the micrometer screw is moved in the second case.

If $(d_1 - d_2)$ is measured in centimetre, m in gram, g in cm. per second per second, and S in square cm., $(V_2 - V_1)$ is obtained in e.s. units. This instrument is used for the measurement of rather *high* potential differences, and is called absolute electrometer as it measures potential in absolute or mechanical units.

Determination of Dielectric Constant. The attracted disc A is connected to the positive pole of a battery, whose other pole is earthed, and thereby it is raised to a potential V. Plate B is earthed and its position is adjusted so that A is in level with the guard ring when the distance between A and B is equal to d. Then

$$V = d\sqrt{\frac{8\pi mg}{S}}$$

Plate A forms a condenser with B and its capacity is equal to $\frac{S}{4\pi d}$. A slab of thickness t of the solid, whose dielectric constant K is to be measured, is placed on B. The capacity of the condenser increases and as the potential difference V between its plates is kept constant, they take up more charge, and A is pulled down as it experiences greater force of attraction. Then plate B is lowered through a distance x to bring A back to its former position.

As A now experiences the same force and its potential is the same as in the first adjustment, the surface density σ of the charge on it is also the same, and the capacity of the condenser is reduced to its original value $\frac{S}{4\pi d}$.

Electric intensity in air $= 4\pi\sigma$

" " in solid slab $= \frac{4\pi\sigma}{K}$

Thickness of air space $= d - t + x$

Fall of potential in " " $= 4\pi\sigma(d - t + x)$

" " " " solid slab $= \frac{4\pi\sigma}{K} \times t$

$\therefore$ Potential difference between A and B $= 4\pi\sigma(d - t + x)\frac{4\pi\sigma t}{K}$

$$= 4\pi\sigma\left(d - t + x + \frac{t}{K}\right)$$

and Capacity $$=\frac{S\sigma}{4\pi\sigma\left(d-t+x+\frac{t}{K}\right)}$$

$$=\frac{S}{4\pi\left(d-t+x+\frac{t}{K}\right)}$$

$$\therefore \frac{S}{4\pi d}=\frac{S}{4\pi\left(d-t+x+\frac{t}{K}\right)}$$

or $$d=d-t+x+\frac{t}{K}$$

or $$Kt-Kx=t$$

$$\therefore K=\frac{t}{t-x}$$

Q. 163. Describe the construction of the Dolezalek Quadrant Electrometer. How is it used to measure (*a*) small, and (*b*) large potential differences?

(*Punjab, 1934*)

Ans. **Dolezalek Quadrant Electrometer.** A hollow cylindrical brass box, of about 2·5 cm. radius and 1 cm. height, is cut into four quadrants A, B, C, D, and these are mounted on separate amber supports (Fig. 128). Inside the box is suspended a needle consisting of two silvered papers in contact at the edges and pressed apart at the middle, by a *very thin* wire of phosphor bronze or quartz made conducting. The suspension wire carries a mirror for measuring small deflections.

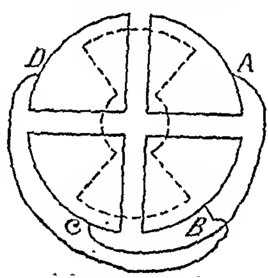


Fig. 128.

Opposite quadrants A and C, and B and D, are connected together. The edges of the needle are made circular so that when it is deflected, the increase of its area from under one pair of quadrants is *equal* to the decrease under the other. The points between which the potential difference is to be measured are connected to the two pairs of quadrants. When the needle and the two pairs of quadrants are at zero potential, the needle hangs symmetrically between them. Its radial

edges should be well within the quadrants so that the irregularities of the electric field of the quadrants remain constant and do not change with its deflection.

If the needle, pair of quadrants A and C, and B and D are given the potential V, V_1, and V_2 respectively, the needle is deflected so that its area *increases* under that pair of quadrants whose potential differs from its potential by a *greater* amount than that of the other pair of quadrants. Usually V is greater than V_1 or V_2, and therefore the needle is deflected towards the quadrants of *lower* potential. The deflection is opposed by the torsion produced in the suspension, and equilibrium is reached when the restoring couple becomes equal to the deflecting couple. If θ is the deflection produced,

$$\theta \propto (V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

or

$$= K\left(V - \frac{V_1 + V_2}{2}\right)(V_1 - V_2) \quad . \quad . \quad . \quad . \quad (1)$$

where K is the constant of proportionality.

(*a*) *Small potential difference.* When the potential difference to be measured is small, the needle is charged to a *high* potential by connecting it with one pole of a battery of constant *e.m.f.*, and the other pole of the battery is earthed. As V is very large as compared with V_1 or V_2, $\left(V - \frac{V_1 + V_2}{2}\right)$ is almost equal to V, and the above equation becomes

$$\theta = KV(V_1 - V_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus θ is proportional to the potential difference to be measured.

(*b*) *Large potential difference.* If the potential difference to be measured is large, the needle is not given a separate potential, but is connected to one pair of quadrants and thus its potential is made equal to, say, V_1. Then

$$\theta = K\left(V_1 - \frac{V_1 + V_2}{2}\right)(V_1 - V_2)$$

$$= \frac{K}{2}(V_1 - V_2)(V_1 - V_2)$$

$$= K'(V_1 - V_2)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Where $K' = \frac{K}{2}$. In this case the deflection is proportional to

the *square* of the potential difference to be measured, and the arrangement can as well be used for the measurement of an *alternating* potential difference.

Q. 164. How would you determine the specific inductive capacity of a solid substance? Give a sketch of the experimental arrangement you would employ. *(Punjab, 1931)*

Ans. **Hopkinson's Method.** A battery B consisting of an *even* number of exactly similar cells is earthed at its *middle*

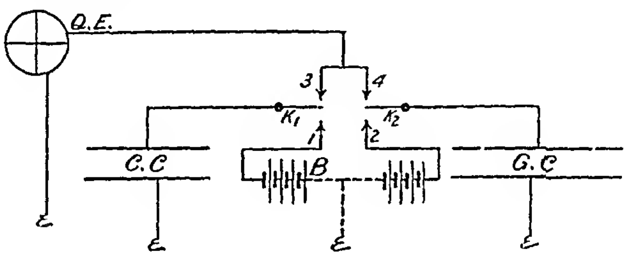


Fig. 129.

point, so that the potentials of its poles, connected to studs 1 and 2, are *equal* and *opposite* (Fig. 129).

The lower plate of a guard-ring condenser G.C., the outer cylinder of the sliding cylindrical condenser C.C., and a pair of opposite quadrants of a quadrant electrometer Q.E. are also earthed to eliminate any disturbance due to the surrounding objects. Spring keys K_1 and K_2 are connected with the inner cylinder of C.C. and the upper plate of G. C. respectively, and they are provided with ebonite knobs to press them.

On pressing the knobs, the inner cylinder of C C. and the upper plate of G. C. are charged to potentials $-V$ and $+V$ respectively. On releasing the keys they come in contact with the studs 3 and 4 connected to the other pair of opposite quadrants of the electrometer. In this process the opposite charges of the two condensers are mixed and their resultant charge is shared with the electrometer.

By adjusting the position of the inner cylinder of C.C., the capacity of this condenser is changed so that on mixing the charges the electrometer shows *no* deflection. This shows that the two charges are exactly equal and opposite, and as the potential differences of the two condensers are also equal

and opposite, therefore their capacities are *equal.* If A be the mean area of the central plate of G.C. and the gap and d the distance between the two plates, its

$$\text{Capacity} = \frac{A}{4\pi d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The capacity of the cylindrical condenser is kept *constant*, and a large slab of thickness t of the solid, whose specific inductive capacity K is to be measured, is placed on the lower plate of G. C This increases the capacity of this condenser, and by lowering its lower plate through a distance x, its capacity is lowered to its original value, so that on sharing the charges the electrometer gives *no* deflection.

Let σ be the surface density of the charge on its plates.

Electric intensity in the air space $= 4\pi\sigma$

Thickness of ,, ,, $= d - t + x$

$\therefore$ Fall of potential in ,, ,, $= 4\pi\sigma(d - t + x)$

Electric intensity in the slab $= \dfrac{4\pi\sigma}{K}$

$\therefore$ Fall of potential ,, ,, ,, $= \dfrac{4\pi\sigma}{K} t$

Hence

$$V = \frac{4\pi\sigma t}{K} + 4\pi\sigma(d - t + x)$$

$$= 4\pi\sigma\left(d - t + x + \frac{t}{K}\right)$$

and

$$\text{Capacity} = \frac{\sigma A}{4\pi\sigma\left(d - t + x + \frac{t}{K}\right)}$$

$$= \frac{A}{4\pi\left(d - t + x + \frac{t}{K}\right)} \qquad (2)$$

This is also given by (1)

$$\therefore \quad \frac{A}{4\pi d} = \frac{A}{4\pi\left(d - t + x + \frac{t}{K}\right)}$$

or

$$d = d - t + x + \frac{t}{K}$$

or

$$Kt - Kx = t$$

$$\therefore \quad K = \frac{t}{t - x}$$

PART VII

CURRENT ELECTRICITY

Q. 165. Find an expression for the insulation resistance of a cable.

If the insulation resistance of a cable between two stations is 20,000 ohms, and that between one station and an intermediate station is 30,000 ohms, find the insulation resistance between the intermediate station and the other station.

Ans. **Insulation Resistance.** Let l be the length of the cable, S the specific resistance and r_1 and r_2 the internal and external radii of the insulation (Fig. 130). The leakage of current takes place from the conductor inside to the outside perpendicular to the insulation and not along it. In the conductor current flows from its one end to the other *along* its length and *perpendicular* to its cross-section area, but in the case of the insulation, current leaks through its thickness and perpendicular to its inner cylindrical surface. The face area of the insulation *perpendicular* to which the current travels increases outward, but for a very thin sheath of internal radius x and thickness dx it is practically the same on the two sides and equal to $l \times 2\pi x$.

$$\text{Resistance of sheath} = \frac{S\,dx}{2\pi l x}$$

$$\therefore \text{ Resistance of insulation} = \int_{r_1}^{r_2} \frac{S\,dx}{2\pi l x}$$

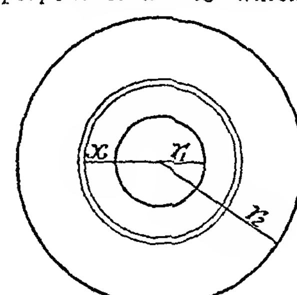


Fig. 130.

$$= \frac{S}{2\pi l}\Big[\log_e x\Big]_{r_1}^{r_2}$$

$$= \frac{S}{2\pi l}\log_e \frac{r_2}{r_1}$$

Thus the insulation resistance is proportional to the logarithm of the ratio of its external and internal radii and is *inversely* proportional to its length.

Problem. The two insulation resistances between the intermediate station and the other two stations are in *parallel*. Therefore, if x ohms is the insulation resistance between the intermediate station and the other station,

$$\frac{1}{x}+\frac{1}{30000}=\frac{1}{20000}$$

or
$$\frac{1}{x}=\frac{1}{20000}-\frac{1}{30000}$$

$$=\frac{3-2}{60000}$$

$$=\frac{1}{60000}$$

and
$$x=60{,}000 \text{ ohms.}$$

Q. 166. State and explain Kirchoff's laws on the distribution of currents in a network of conductors.

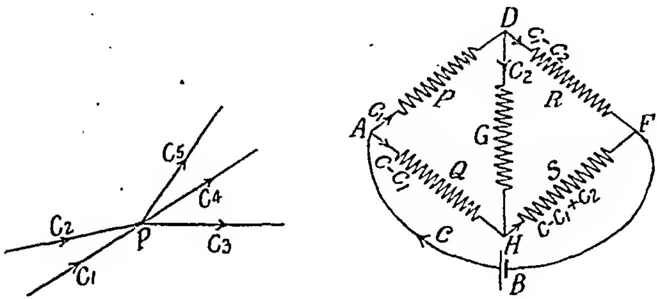
A Grove cell of *e.m f.* 2 volts and resistance 1 ohm is joined in parallel to a Daniel cell of *e.m.f.* 1 volt and resistance 1 ohm by two wires of resistances 2 ohms each. The electrical middle points of the connecting wires are joined by a wire of resistance 9 ohms. Calculate the current through the Grove cell. (*Calcutta, 1922*)

Ans. **Kirchoff's Laws.** 1. *At any point in a circuit in which current is flowing the algebraic sum of the currents meeting there is zero.*

Current at a point is equal to the rate of flow of electricity there. As there is *no accumulation* of electricity, the *rate* of inflow of electricity is equal to its rate of outflow at the

same point. The incoming currents are called positive while the outgoing currents are called negative. At a point P the sum of the incoming currents (C_1+C_2) is equal to the sum of the outgoing currents $(C_3+C_4+C_5)$, Fig. 131 (*a*).

or $$C_1+C_2-C_3-C_4-C_5=0$$



(*a*) (*b*)

Fig. 131.

(2) *In any closed path in a network of conductors the algebraic sum of the products of the current in, and resistance of, each part of the circuit is equal to the sum of the electromotive forces in it.*

In Fig. 131 (*b*) resistances P, Q, R, S, and G are connected as shown, and B is a battery of electromotive force E and internal resistance B. At A, current C is divided into C_1 along P and $(C-C_1)$ along Q, at D; C_2 flows through G leaving (C_1-C_2) in R, and current $(C-C_1+C_2)$ passes through S. For mesh ADH

$$PC_1+GC_2-Q\ (C-C_1)=0$$

or $$PC_1+GC_2=Q\ (C-C_1)$$

But PC_1, GC_2, and $(C-C_1)$ give the fall of potential from A to D, D to H, and A to H respectively, and the above equation shows that the fall of potential from A to H is the same through Q as through P and G. This is according to the law of conservation of energy.

For mesh HDF,

$$R(C_1-C_2)-S(C-C_1+C_2)-GC_2=0$$

and for BAF,

$$BC+Q(C-C_1)+S(C-C_1+C_2)=E.$$

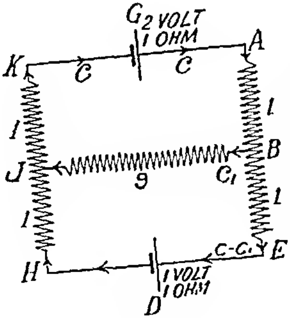


Fig. 132.

Problem. A Grove cell G of e.m.f. 2 volts and resistance 1 ohm. is connected in parallel with a Daniel cell of e.m.f. 1 volt and resistance 1 ohm. and other resistances are connected as shown (Fig. 132). Current C ampere flows through G, AB, and JK, C_1 ampere through BJ, and $(C-C_1)$ ampere through BE, D, and HJ.

For mesh KABJ,

$$1C+1C+9C_1+1C=2$$

or $$3C+9C_1=2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For mesh BFHJ

$$1(C-C_1)+1(C-C_1)+1(C-C_1)-9C_1=-1$$

or $$3C-12C_1=-1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (2) from (1),

$$21C_1=3$$

or $$C_1=\tfrac{1}{7} \text{ amp.}$$

Putting this value of C_1 in (1),

$$3C+\tfrac{9}{7}=2$$

or $$C=\frac{2-\frac{9}{7}}{3}$$

$$=\tfrac{5}{21}=0{\cdot}2381 \text{ amp.}$$

Q. 167. The electrodes of a quadrant electrometer are joined to the terminals of a battery of 5 cells in series. In what ratio will the deflection of the needle be altered if the electrodes are also joined to the terminals of a battery of 3 cells in series similarly arranged, all the cells being alike and the conducting wires thick.

(*Calcutta, 1920*)

Ans. Let the electro-motive force and internal resistance of each cell be equal to E and R respectively, and θ_1 be the electrometer deflection in the first case and θ_2 in the second case.

E.M.F. of the battery of 5 cells $=5E$

" " " 3 cells $=3E$

Internal resistance of the battery of 5 cells $=5R$

" " " " 3 cells $=3R$

In the first case there is no current in the battery and the potential difference between the two pairs of quadrants is equal to its e.m f.

$$\therefore \quad \theta_1 \propto 5E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

On connecting the second battery of 3 cells in parallel with the first of greater e.m.f., current C flows through the two, *out* of the positive of the bigger battery and *into* the positive of the smaller one. If E′ be the effective e.m.f. of the two batteries taken together,

$$5E-3E=5R\times C+3R\times C$$

or $$E=4R\times C$$

or $$C=\frac{E}{4R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and $$E_1=5E-5R\times C$$

$$=5E-5R\times\frac{E}{4R} \qquad \text{[from (2)}$$

$$=\frac{15E}{4}$$

$$\therefore \quad \theta_2 \propto \frac{15E}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and $$\frac{\theta_2}{\theta_1}=\frac{15E}{4\times 5E}=\tfrac{4}{3}.$$

Q. 168. A skeleton cube is formed of wires such that two of its opposite faces are ABCD and EFGH. A current I enters A and comes out at the opposite corner G. Calculate the equivalent resistance of the whole cube if each side is of length 5 cm. and the resistance of 100 cm of the wire is one ohm. (*Punjab, 1930*)

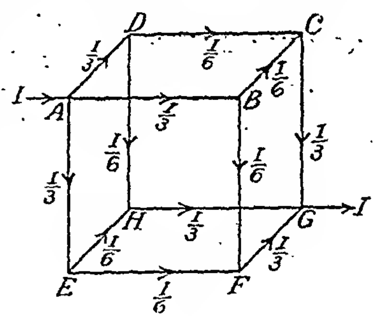


Fig. 133.

Ans. The skeleton cube is formed of 12 wires each of $\frac{5}{100}=0·05$ ohm resistance (Fig. 133). ABCD and EFGH are its upper and lower faces respectively, and current I enters at A and leaves at G. All the paths from A to G are *symmetrical*, so that the current

at A is divided into three *equal* parts. At B, D and E a further sub-division into two *equal* parts takes place; at C, F, and H these equal parts combine to give $\frac{I}{3}$, and three such equal currents pass out of G.

The fall of potential from A to G along any path is equal to the algebraic sum of the products of resistance of, and current in, each of its parts and is also equal to IR, where R is the equivalent resistance of the whole cube. Taking the path ADCG,

$$IR = \frac{0{\cdot}05I}{3} + \frac{0{\cdot}05I}{6} + \frac{0{\cdot}05I}{3}$$

$$= \frac{0{\cdot}125I}{3}$$

$$\therefore \quad R = 0{\cdot}0417 \text{ ohm.}$$

Q. 169. State the laws which are the basis of calculation of the resistance of a network of electrical conductors.

Calculate the effective e.m.f. of three cells of unequal e.m.fs. joined in parallel. (*Punjab, 1925*)

Ans. **Kirchoff's Laws.** See Q. 166.

Effective E.M.F. Let three cells of e.m.fs. E_1, E_2, and E_3 and internal resistances b_1, b_2, and b_3 respectively be connected in parallel and their poles joined to an external resistance R, so that the corresponding currents in them are C_1, C_2, and C_3 (Fig. 134). Then current C in the external circuit is equal to the algebraic sum of the internal current.

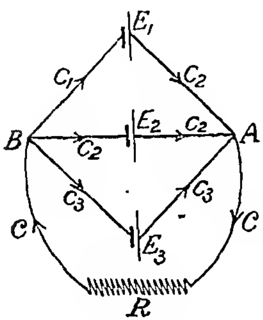


Fig. 134.

$$C = C_1 + C_2 + C_3 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Applying Kirchoff's second law to the three closed circuits formed by the cells with the external resistance, we get

$$CR + C_1 b_1 = E_1$$

or $$\frac{CR}{b_1} + C_1 = \frac{E_1}{b_1} \quad . \quad . \quad . \quad (2)$$

Similarly, $$\frac{CR}{b_2} + C_2 = \frac{E_2}{b_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and $$\frac{CR}{b_3}+C_3=\frac{E_3}{b_3} \quad \ldots\ldots\ldots\ldots \quad (4)$$

Adding the corresponding sides of (2), (3), and (4), and using (1), we get

$$C\left(\frac{R}{b_1}+\frac{R}{b_2}+\frac{R}{b_3}\right)+(C_1+C_2+C_3=C)=\frac{E_1}{b_1}+\frac{E_2}{b_2}+\frac{E_3}{b_3}$$

or $$C\left\{\frac{R(b_2b_3+b_1b_3+b_1b_2)+b_1b_2b_3}{b_1b_2b_3}\right\}=\frac{E_1b_2b_3+E_2b_1b_3+E_3b_1b_2}{b_1b_2b_3}$$

$$\therefore \text{ Total current } C=\frac{E_1b_2b_3+E_2b_1b_3+E_3b_1b_2}{R(b_2b_3+b_1b_3+b_1b_2)+b_1b_2b_3} \quad \ldots \quad (5)$$

Total internal resistance of the cells

$$=\frac{1}{\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}}$$

$$=\frac{b_1b_2b_3}{b_2b_3+b_1b_3+b_1b_2}$$

$$\text{Total resistance } =R+\frac{b_1b_2b_3}{b_2b_3+b_1b_3+b_1b_2}$$

$$=\frac{R(b_2b_3+b_1b_3+b_1b_2)+b_1b_2b_3}{(b_2b_3+b_1b_3+b_1b_2)} \quad \ldots \quad (6)$$

$\therefore$ Effective *e.m.f.* = *Total resistance* × *Total current*

$$=\frac{E_1b_2b_3+E_2b_1b_3+E_3b_1b_2}{b_2b_3+b_1b_3+b_1b_2} \quad \ldots\ldots \quad (7)$$

Q. 170. Describe the different ways of grouping of cells and show how you would decide the best arrangement for the strength of the current required.

Find the minimum number of cells, each of e.m.f. 0·8 volt and resistance 4 ohms, which will supply 2 watts to an external circuit of resistance 8 ohms.

(*Bombay, 1930*)

Ans. When an external resistance R is connected with the poles of a cell of electromotive force E and internal resistance b,

$$\text{Current}=\frac{E}{R+b} \quad \ldots\ldots\ldots \quad (1)$$

If a number of similar cells are available, they may be combined in three ways.

(1) **Series Grouping.** Here n cells are connected in series so that the positive pole of the first cell is connected with the negative pole of the second; and so on, and the negative of the first and the positive of the last cell are the poles of the battery [Fig. 135 (*a*)].

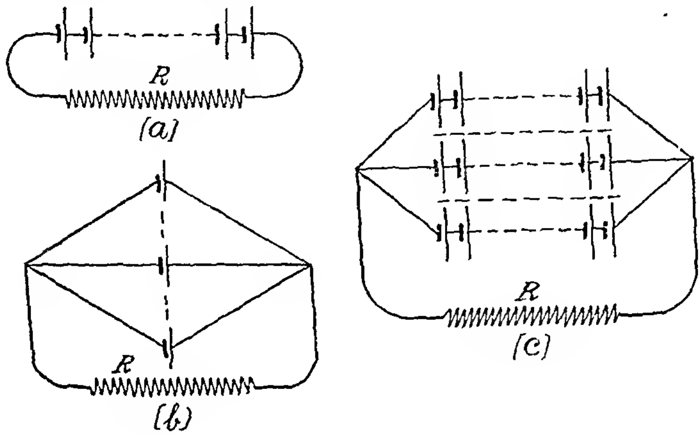


Fig. 135.

$$\text{Electromotive force of the battery} = nE$$

$$\text{Total internal resistance} = nb$$

$$\text{Total resistance} = R + nb$$

$$\therefore \text{ Current} = \frac{nE}{R+nb} \qquad . \quad . \quad (2)$$

$$= \frac{E}{\dfrac{R}{n}+b}$$

This currrent is *greater* than that obtained with a single cell, but the difference depends on the *relative* values of R and b. The same current passes through each cell, and the rate at which the material of the battery is consumed is more than n times that in the case of a single cell. This grouping is to be used when the external resistance R is *very large* as compared with the internal resistance b of each cell, for then this arrangement gives a very strong current as compared with (1).

(2) **Parallel Grouping.** In this arrangement all the negative poles of n cells are connected together, and so are their positive poles, so that the *e.m.f.* of the battery is the *same* as that of *one* cell [Fig. 135 (*b*)]. The internal resistance of the battery is decreased very much, and only $\frac{1}{n}$th of the total current passes through each cell.

$$\text{Total internal resistance} = \frac{b}{n}$$

$$\text{Total resistance} = R + \frac{b}{n}$$

$$\therefore \text{ Current} = \frac{E}{R + \frac{b}{n}} \quad . \quad . \quad (3)$$

This current is *greater* than that obtained with a single cell, but its excess depends on the relative values of R and b, and this arrangement is to be used when R is *very small* as compared with b.

(3) **Mixed Grouping.** Here $n x m$ cells are divided into m rows. Each row contains n cells in series, and m such rows are connected in parallel [Fig. 135 (*c*)].

$$\text{Electromotive force of the battery} = nE$$

$$\text{Internal resistance of each row} = nb$$

$$\text{,, \quad ,, \quad ,, the battery} = \frac{nb}{m}$$

$$\text{Total resistance} = R + \frac{nb}{m}$$

$$\therefore \text{ Current} = \frac{nE}{R + \frac{nb}{m}} \quad . \quad . \quad (4)$$

$$= \frac{E}{\frac{R}{n} + \frac{b}{m}}$$

This arrangement is used when the relative values of R and b are not according to the extreme conditions of the previous arrangements. The product of $\frac{R}{n}$ and $\frac{b}{m}$ is constant, and

their sum is minimum, and current is maximum, when they are equal, that is, the internal resistance R is *equal* to the internal resistance of the battery.

This is, however, not the most efficient arrangement, as the internal resistance being equal to the external resistance 50 per cent. of the energy supplied by the battery is wasted in overcoming its internal resistance. To reduce this wastage to a minimum, the internal resistance of the battery should be minimum, and for this the cells should be connected in *parallel*.

Problem. Here $E=0{\cdot}8$ volt, $b=4$ ohms, and $R=8$ ohms. For a given current, the number of cells is minimum

if $$R=\frac{nb}{m}$$

or $$8=\frac{n}{m}\times 4$$

or $$\frac{n}{m}=2$$

$$\text{Maximum current}=\frac{n\times 0{\cdot}8}{8+\frac{n}{m}\times 4}$$

$$=\frac{0{\cdot}8n}{16}=\frac{n}{20}\text{ ampere.}$$

But Watts $=(\text{amps.})^2\times$ ohms.

$$\therefore\ 2=\left(\frac{n}{20}\right)^2\times 8$$

or $$\left(\frac{n}{20}\right)^2=\tfrac{1}{4}$$

$$\therefore\quad n=10$$

and $$m=\tfrac{10}{2}=5$$

Hence Total no. of cells $=10\times 5=50$.

Q. 171. Find the intensity of the magnetic field due to a circular coil carrying a current, at a point on its axis. How can it be experimentally determined ?

How can a local uniform field be obtained with two such coils ?

(*Punjab, 1936*)

Ans. **Magnetic Field on the Axis of a Circular Coil.**

Let a current C flow through a circular coil of radius a and n turns placed perpendicular to the plane of paper and P be a point on its axis and at a distance x from its centre O (Fig. 136).

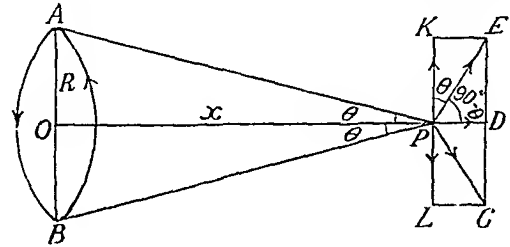


Fig. 136.

The direction of current in the coil is anti-clockwise when looked at from the right. Consider a *very small* length δl *perpendicular* to the plane of paper and join its middle point A with P, so that AP is *perpendicular* to this element, then the intensity δF of the magnetic field at P due to the current in it is equal to $\frac{\delta l . C}{AP^2}$ and is perpendicular to the plane containing the element and the point P.

This intensity is in the plane of paper, and is represented by PE. If PA makes angle θ with PO, angle EPD is equal to $(90^\circ - \theta)$, and the intensity δF has a rectangular component $\delta F \cos(90^\circ - \theta) = \delta F \sin\theta$ along the axis and represented by PD, and the other component $\delta F \cos\theta$ is perpendicular to the axis and is represented by PK.

The magnetic intensity at P due to the current in an *equal* element at B at the other end of the diameter AOB is also in the plane of paper and is represented by PG = PE. Its rectangular components along and perpendicular to the axis are equal to $\delta F \sin\theta$ and $\delta F \cos\theta$ and are represented by PD and PL respectively. The components along the axis are equal and in the *same* direction, while those perpendicular to it are *equal* and *opposite*, and cancel out each other.

Dividing one turn of the coil in this way, the resultant intensity perpendicular to the axis is equal to zero, while the component intensities along the axis are all in the *same* direction, and their sum gives the resultant intensity.

Intensity at P due to one turn $= \Sigma \dfrac{\delta l\ C}{AR^2} \sin\theta$

$$= \frac{2\pi RC}{(R^2+x^2)} \times \frac{R}{\sqrt{R^2+x^2}}$$

$$= \frac{2\pi R^2 C}{(R^2+x^2)^{\frac{3}{2}}} \text{ along PD.}$$

and " " " n turns $= \dfrac{n\ 2\pi R^2 C}{(R^2+x^2)^{\frac{3}{2}}}$ along PD.

Experimental Determination. A *small* magnetic needle is suspended or pivoted at P and the circular coil is set *in* the magnetic meridian so that the magnetic intensity F at P due to it is horizontal and perpendicular to the horizontal component H of the earth's magnetic field. The magnetic needle is under the action of two perpendicular magnetic fields, and it is deflected through *an angle* ϕ *from its normal position.* Then

$$F = H \tan\phi$$

Uniform Field due to Two Coils. The magnetic intensity F changes with x, and its rate of change $\dfrac{dF}{dx}$ also changes.

$$F = \frac{2\pi nCR^2}{(R^2+x^2)^{3/2}}$$

$$\frac{dF}{dx} = 2\pi nCR^2\{-\tfrac{3}{2}(R^2+x^2)^{-\frac{5}{2}} \times 2x\}$$

$$\frac{d^2F}{dx^2} = 6\pi nCR^2\{-(R^2+x^2)^{-\frac{5}{2}} + \tfrac{5}{2}(R^2+x^2)^{-\frac{7}{2}} \times 2x^2\}$$

The *rate* of change of F with x is constant when $\dfrac{d^2F}{dx^2}$ is equal to zero.

$$\therefore\ (R^2+x^2)^{-\frac{5}{2}} = 5x^2(R^2+x^2)^{-\frac{7}{2}}$$

or
$$1 = \frac{5x^2}{R^2+x^2}$$
$$5x^2 = R^2 + x^2$$
$$4x^2 = R^2$$
$$\therefore \quad x = \pm \frac{R}{2}$$

If two equal coils are placed parallel to, and at a distance R, from each other, the rate of change of the magnetic intensity, due to either coil, at a point on their common axis and *midway* between them is constant. Then over a small region here the increase of intensity due to one coil is *equal* to the decrease due to the other, and their sum is *constant* and equal to

$$\frac{2 \times 2\pi n C R^2}{\left\{R^2 + \left(\frac{R}{2}\right)^2\right\}^{3/2}} = \frac{32\pi n C}{5\sqrt{5} \times R}$$

Q. 172. Discuss the equivalence of a magnetic shell and a circuit carrying current. Calculate the field at a point on the axis of a plane circular shell or coil of wire in which a current flows. (*Punjab, 1933*)

Ans. The intensity of the magnetic field at a point on the axis, and at a distance x from the centre, of a coil of *one* turn of radius R and carrying current C is equal to $\frac{2\pi R^2 C}{(x^2+R^2)^{\frac{3}{2}}}$ along the axis, and this reduces to $\frac{2\pi R^2 C}{x^3}$ when R is very small as compared with x. See Q. 171 for finding this relation. If A is the face area of the coil, it is equal to πR^2 and the magnetic intensity is equal to $\frac{2AC}{x^3}$.

A plane magnetic shell has a very small thickness (length) and the magnetic intensity at a point on its axis at a distance x from it is equal to $\frac{2M}{x^3}$, where M is the magnetic moment of the shell See. Q. 138 for finding this. If A is its face area and ϕ its strength, $M = A\phi$, and magnetic intensity $= \frac{2A\phi}{x^3}$.

This resembles the above expression for the magnetic intensity due to a coil, and the two fields are equal if the strength ϕ of the shell is equal to the strength C of current in the coil and their face areas are equal. Thus a coil of wire carrying current acts at *external* points in the same manner as a very thin magnet of the *same* size and shape and of appropriate magnetic moment. From this the electromagnetic unit of current is defined as that current which when flowing in a coil is equivalent to a magnetic shell of unit strength if the two are of the same size and shape and are placed in *air*.

The magnetic potential at a point in the field of a magnetic shell is equal to the product of the strength ϕ of the shell and the solid angle ω subtended by it at the point. Similarly, for an equivalent coil carrying current C, the potential at a point is equal to $C\omega$. But this equivalence is only for *external* points. When a unit north pole is taken round a magnetic shell from its one face to the other, the amount of work done is practically equal to $4\pi\phi$. If a hole is bored in the shell, the direction of the magnetic force in it is *reversed*, and when the pole is brought to its initial position, the total amount of work done is *zero*. On the other hand work done in the case of a coil carrying current is equal to $4\pi C$ for its each turn, and this is obtained from the *source* of the current.

Work done in the case of the shell *depends* on the nature of the surrounding medium, while that in the case of a coil carrying current is *independent* of it, for even when the surrounding medium is magnetised, the magnetic circuit is complete and no work is done. Further, the potential at a point due to a shell has only *one* value, but in the case of a coil it is *multi-valued*, depending on the number of times that a unit north pole has been taken round the coil.

Q. 173. Explain what is meant by a simple magnetic shell, and find the potential of such a shell at any point.

Calculate the magnetic force at a point inside a long solenoid of N turns carrying a current of A amperes.

(*Punjab, 1931*)

Ans. See. Q. 138 for magnetic shell and Q. 172 for finding the potential of a point in its field.

It is proved in Q. 139 that the amount of work done in carrying a unit north pole from one face of a magnetic shell of strength ϕ to the other is equal to $4\pi\phi$. In Q. 172 it is shown from the equivalence between a magnetic shell and a coil carrying current that the amount of work done in carrying a unit north pole *once* round a coil of *one* turn and carrying current C is equal to $4\pi C$.

A *long* solenoid is very closely wound, and the magnetic field at a point *well within* it is *uniform* and *parallel* to its axis there. Outside it the magnetic field is very *weak* and *negligible* as compared with its strength inside it.

Let the length of the solenoid be l so that the number of turns per unit length is equal to $\frac{N}{l}$. Fig. 137. shows a part of the solenoid carrying current of C *e.m.* units. If a unit north pole is taken from A to B *parallel* to its axis, the amount of work done is equal to $F \times AB$, where F is the intensity of the magnetic field in the solenoid from right to left. No work is done in carrying the north pole from B to D and from E to A at right angles to F, and as the field outside the solenoid is negligible, no appreciable work is done in taking the pole from D to E.

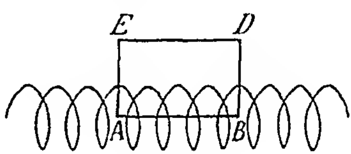


Fig. 137.

The number of turns of the solenoid from A to B is equal to $\frac{N}{l} \times AB$, and as the amount of work done in carrying a unit north pole *once* round *one* turn is equal to $4\pi C$, the total amount of work done in once going round $\frac{N}{l} \times AB$ turns is equal to

$4\pi C \times \frac{N}{l} \times AB$.

$$\therefore \quad F \times AB = \frac{4\pi CN}{l} \times AB$$

or
$$F = \frac{4\pi NC}{l}$$
$$= \frac{4\pi NA}{10l},$$

as A amperes $= \frac{A}{10}$ e. m. units.

Q. 174. Determine the magnetic field at the centre of a long solenoid. Show that the field at the end of such a solenoid is one-half that at the centre.

(Bombay, 1928)

Ans. Fig. 138 represents a part of the long and closely wound solenoid whose turns of radius R lie between the upper and lower lines, AB and DE, and the axis passes along the middle line. The magnetic field inside the solenoid is *uniform* and *parallel* to its axis and if the direction of current is anti-clockwise when looked at from the right, the intensity of the magnetic field is from left to right.

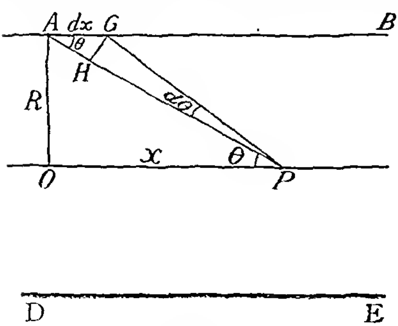


Fig. 138.

Consider the magnetic field at P due to the current C in the turns of the solenoid in a *very small* length dx from A to G. P is at *a* distance x from the centre O of this thin circular coil containing $n.dx$ turns, where n is the number of turns of the solenoid per *unit* length, and as shown in Q. 171, the intensity dF of the magnetic field at P is equal to $\frac{2\pi n dx . R^2 C}{(R^2+x^2)^{\frac{3}{2}}}$ from left to right.

Draw GH perpendicular to AP. Then

$$GH = dx \sin\theta$$

also
$$GH = GP . d\theta = (R^2+x^2)^{\frac{1}{2}} d\theta$$

or
$$\sin\theta \, dx = (R^2+x^2)^{\frac{1}{2}} d\theta$$

and
$$dx = \frac{(R^2+x^2)^{\frac{1}{2}} d\theta}{\sin\theta}$$

$$\therefore \; dF = \frac{2\pi n R^2 C . (R^2+x^2)^{\frac{1}{2}} d\theta}{(R^2+x^2)^{\frac{3}{2}} \sin\theta}$$

$$= 2\pi nc \sin\theta \, d\theta \quad \left[\because \frac{R^2}{R^2+x^2} = \sin^2\theta\right]$$

Integrating this expression for the limits θ_1 and θ_2, the values of the angle APO when point A is at the left and right ends of the solenoid respectively, we get the total magnetic intensity F at P given by

$$F = \int_{\theta_1}^{\theta_2} 2\pi nc \sin\theta \, d\theta$$

$$= 2\pi nc\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= 2\pi nc \,(\cos\theta_1 - \cos\theta_2) \quad . \quad . \quad . \quad (1)$$

When the solenoid is *very long* and P is *far* removed from its two ends, $\theta_1 = 0°$, and $\theta_2 = 180°$.

$$\therefore \quad F = 2\pi nc\,(\cos 0° - \cos 180°)$$

$$= 4\pi nc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If P is at the right end, $\theta_1 = 0°$, and $\theta_2 = 90°$, and

$$F = 2\pi nc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Similarly, when P is at the other end, $\theta_1 = 90°$, and $\theta_2 = 0$, and F is numerically equal to $2\pi nc$. Thus the magnetic intensity at the end of a *very* long solenoid is one-half that at a point well within it.

Q. 175. (*a*) **Describe the construction of a moving coil galvanometer, and derive an expression for the relation between the current and deflection.**

(*Punjab, 1938*)

(*b*) **Explain how with the help of an additional resistance it can be used as an ammeter or as a voltmeter.** (*Punjab, 1935*)

Ans. **Moving Coil Galvanometer.** A rectangular coil of length l and breadth b made of fine wire is suspended by a phosphorbronze wire in a narrow gap between the poles of a very strong permanent horse-shoe magnet, with its faces parallel to the magnetic field (Fig. 139). The ends of the poles are made *circular*, and a soft iron cylinder is fixed

between them to make the magnetic field strong and *radial*. The coil rotates round this cylinder, and in all its positions the lines of force are *parallel* to its breadth. At the bottom of the coil is connected a fine spiral, and this and the suspension wire serve as the leads for passing current through the coil.

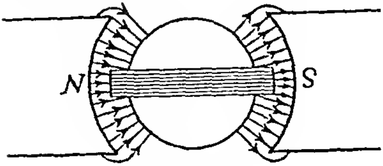


Fig. 139.

If a current C passes through the coil, its sides experience two equal, parallel and opposite forces, each equal to $n\mathrm{FC}l$, where n is the number of turns in the coil and F is the intensity of the magnetic field. These forces are perpendicular to both the length of the coil and the magnetic field, that is, perpendicular to the face of the coil, and form a couple whose arm is equal to its breadth b.

$$\text{Moment of deflecting couple} = n\mathrm{FC}l \times b$$
$$= n\mathrm{FCA}, \quad \ldots \ldots \quad (1)$$

where A is the face area of the coil.

When the coil is deflected, a twist is produced in the suspension and this opposes its deflection. The restoring couple increases with the angle of deflection until at deflection θ it becomes equal to the twisting couple when equilibrium is reached between the two couples. If K is the restoring couple per unit angular displacement (radian), its value for θ radian is equal to $\mathrm{K}\theta$.

$$\therefore \quad n\mathrm{FCA} = \mathrm{K}\theta$$

or

$$\mathrm{C} = \frac{\mathrm{K}\theta}{n\mathrm{FA}}$$
$$= \mathrm{K}'\theta \quad \ldots \ldots \ldots \quad (2)$$

where K' is equal to $\dfrac{\mathrm{K}}{n\mathrm{FA}}$ and is a constant for a given galvanometer.

If no soft iron ore is used to make the magnetic field radial, it is parallel to NS, and when the coil is deflected through θ, the arm of the deflecting couple is $b\cos\theta$ and its

moment in that position is equal to nFCA cos θ. In this case

$$C = \frac{K\theta}{nFA\cos\theta}.$$

(*b*) A moving coil galvanometer can be used as an ammeter or voltmeter by using a suitable resistance. The moving coil is not suspended but is pivoted between jewel bearings and is kept in position by a spring. A pointer is connected through a lever with the spring, and thus the deflection of the coil is shown magnified by it.

Ammeter. It is connected in *series* with the circuit whose current it is to measure in amperes. It has a *very low* resistance so that its introduction in the circuit may not increase its resistance appreciably and decrease the strength of the current. A shunt of suitable *very low* resistance is connected in *parallel* with the coil of the galvanometer, and the instrument is graduated.

Most of the current flows through the shunt, and a very small fraction of it which passes through the galvanometer produces its deflection on a graduated scale. If a current of C ampere produces full scale deflection of a galvanometer of resistance G ohms, the current through a shunt of resistance S ohm is equal to $\frac{C \times G}{S}$ amp., and, therefore, total current is equal to $C + \frac{C \times G}{S} = \frac{C(G+S)}{S}$ amp. The smaller the shunt resistance, the greater is the total current for a given galvanometer current, and thus by using shunts of different resistances, the same instrument can be used for different ranges.

Voltmeter. It is used for measuring potential difference, in volts, between two points of an electric circuit, and for this purpose, it is connected in *parallel* with the part of the circuit between these points. This potential difference is equal to the product of the resistance of *that* part of the circuit and the current passing through it, and if the voltmeter is to measure this P.D. correctly, it should take inappreciable current, so that current in that part of the circuit remains practically the same as before. For this reason resistance of a voltmeter is made *very high*, and a moving coil galvanometer

can be used as a voltmeter by connecting a very high resistance in *series* with it, and calibrating its deflection in volts.

When a resistance of R ohms is connected in series with the galvanometer and its full scale deflection is obtained, current through it must be equal to C amp, and the fall of potential over this resistance and the galvanometer is equal to C(R+G) volts. This is the potential difference between the two points of the circuit to which the ends of the instrument are connected. The greater the value of R, the greater is the fall of potential through it and the galvanometer, for a given galvanometer current, and thus the same instrument may be used for different ranges of voltage.

Q. 176. Describe the construction of a suspended coil galvanometer and explain its action. How would the deflection produced by a given potential difference be altered by rewinding the coil with twice as many turns of wire of half the original cross-section?

(*Punjab, 1936*)

Ans. **Suspended Coil Galvanometer.** See Q. 175 for its construction and action and the terms used below.

Problem. Let S ohm-cms. be the specific resistance of the wire and E volt be the potential difference applied to the galvanometer.

First Case. Total length of the wire $= n2(l+b)$ cm.

Cross-section area „ „ „ $= a$ sq. cm.

Resistance „ „ „ $= \dfrac{2Sn(l+b)}{a}$ ohms.

$$\therefore \text{ Current} = \frac{Ea}{2Sn(l+b)} \text{ amp.}$$

$$= \frac{Ea}{20Sn(l+b)} \textit{ e.m.} \text{ units}$$

Force on each turn of the coil $= \dfrac{EaFl}{20Sn(l+b)}$ dynes

„ „ n turns „ „ „ $= \dfrac{EaFl}{20S(l+b)}$ dynes

Moment of the deflecting couple $= \dfrac{EaFlb}{20S(l+b)}$ c.g.s. units

$$\therefore \frac{EaFlb}{20S(l+b)} = K'\theta$$

or
$$\theta = \frac{EaFlb}{20S(l+b)K'} \quad . \quad . \quad . \quad . \quad (1)$$

[The deflection is *independent* of n. If n is doubled, coil resistance is doubled, and the current in it is reduced to half. The force on each turn becomes half, but the number of turns being doubled, the total force on the coil and the deflecting couple remain unchanged.]

Second case. The cross-section of the wire is reduced to half, but other terms in the value of θ in (1) remain the same. Therefore deflection in the second case is *half* of the deflection in the first case.

Q. 177. A galvanometer, a resistance R, and a battery of resistance B are joined in a circuit. When a shunt S is put across the terminals of the galvanometer, a certain deflection is produced. If the resistance R is changed to R′, and the shunt S is changed to S′, same deflection is produced. State the relation between the change of R to R′ and that of S to S′, and find the resistance of the galvanometer in terms of the given quantities. (*Bombay, 1930*)

Ans. Let E be the *e.m.f.* of the battery and G the resistance of the galvanometer. The galvanometer and the shunt being connected in parallel their combined resistance in the first and second case is equal to $\dfrac{GS}{G+S}$ and $\dfrac{GS'}{G+S'}$ respectively, and the galvanometer current is equal to the total current multiplied by the shunt resistance and divided by the sum of the galvanometer and shunt resistances.

(*i*) *First case.*

$$\text{Total resistances} = R + B + \frac{GS}{G+S}$$

$$\text{Total current} = \frac{E}{R + B + \dfrac{GS}{G+S}}$$

$$\therefore \text{ Galvanometer current} = \frac{E}{\left(R+B+\frac{GS}{G+S}\right)} \times \frac{S}{(G+S)} \quad \ldots(1)$$

(*ii*) *Second case*

$$\text{Total resistance} = R' + B + \frac{GS'}{G+S'}$$

$$\text{Total current} = \frac{E}{R'+B+\frac{GS'}{G+S'}}$$

$$\text{and Galvanometer current} = \frac{E}{\left(R'+B+\frac{GS'}{G+S'}\right)} \times \frac{S'}{(G+S')} \ldots(2)$$

Equating (1) and (2), we get

$$\left(R+B+\frac{GS}{G+S}\right)\frac{(G+S)}{S} = \left(R'+B+\frac{GS'}{G+S'}\right)\frac{(G+S')}{S'}$$

Multiplying both sides by SS′ and simplifying,

$$RGS'+RSS'+BGS'+BSS'+GS'$$
$$= R'GS+R'S'S+BGS+BS'S+GS$$

or
$$SS'(R'-R) = G(RS'+BS'+S'-R'S-BS-S)$$
$$= G\{RS'-R'S+B(S'-S)\}$$

$$\therefore \; G = \frac{SS'(R'-R)}{\{RS'-R'S+B(S'-S)\}}$$

Q. 178. Explain the theory of the ballistic galvanometer and show how it can be employed in the determination of a quantity of electric charge.

(*Bombay, 1929*)

Ans. **Ballistic Galvanometer.** It is used for the measurement of electric charge Q which is passed through it for a *very short* time. The suspended part (magnetic needle or coil) is of large moment of inertia K to make its period of vibration T *large*, so that it takes up motion very slowly and even when the current passing through it has ceased, it has *not* moved appreciably from its normal position. This makes the angle of deflection independent of the time δt for which the charge passes through it. Damping is reduced to a minimum, and even for that correction is applied.

(*a*) **Suspended Magnet Type.** The coil is set in the magnetic meridian, and a small magnetic needle of pole strength m, length $2l$, and moment M is suspended at its centre. If the current at any instant is C, the intensity of the magnetic field produced by it at the centre of the coil and perpendicular to the magnetic meridian is equal to GC, where G is the galvanometer constant or field produced by a unit current, and two equal, parallel and opposite forces mGC act on the poles of the needle and perpendicular to its length.

$$\text{Moment of the couple acting on the needle} = 2l \times m\text{GC} = \text{MGC}$$

$$\text{Angular impulse} = \int \text{MGC}\,dt = \text{MG}\int \text{C}\,dt = \text{MGQ}$$

Moment of momentum $= \text{K}\omega$.

where ω is the angular velocity at the start.

$$\therefore \quad \text{K}\omega = \text{MGQ}$$

or $$\text{K}^2\omega^2 = \text{M}^2\text{G}^2\text{Q}^2. \qquad (1)$$

If H is the horizontal component of the earth's magnetic field, work done in deflecting the needle through angle β is equal to $\text{MH}(1-\cos\beta)$, and this is equal to its kinetic energy $\frac{1}{2}\text{K}\omega^2$ at the start.

$$\tfrac{1}{2}\text{K}\omega^2 = \text{MH}(1-\cos\beta) = 2\text{MH}\sin^2\left(\frac{\beta}{2}\right)$$

or $$\text{K}\omega^2 = 4\text{MH}\sin^2\left(\frac{\beta}{2}\right). \qquad (2)$$

Dividing (1) by (2) we get

$$\text{K} = \frac{\text{MG}^2\text{Q}^2}{4\text{H}\sin^2\left(\frac{\beta}{2}\right)} \qquad (3)$$

also $$\text{T} = 2\pi\sqrt{\frac{\text{K}}{\text{MH}}}$$

or $$\text{K} = \frac{\text{T}^2\text{MH}}{4\pi^2} \qquad (4)$$

Equating the values of K in (3) and (4), we get the value of Q.

$$\frac{MG^2Q^2}{4H\sin^2\left(\frac{\beta}{2}\right)}=\frac{T^2MH}{4\pi^2}$$

or

$$Q^2=\frac{T^2H^2\sin^2\left(\frac{\beta}{2}\right)}{\pi^2G^2}$$

$$\therefore \quad Q=\frac{TH}{\pi G}\sin\left(\frac{\beta}{2}\right) \cdot \quad \cdot \quad \cdot \quad \cdot \quad (5)$$

(*b*) **Suspended Coil Type.** A coil of N turns, length l, and breadth b is suspended between the poles of a strong magnet in a magnetic field of strength F by a fine wire for which the twisting couple per unit angular displacement (radian) is equal to n. If the current at any instant is equal to C, the two arms of the coil experience equal, parallel, and opposite forces each equal to NFCl and perpendicular to its face.

$$\text{Moment of the couple}=NFCl\times b=FCA,$$

where A is N times the face area lb of the coil.

$$\text{Total angular impulse}=\int FACdt$$

$$=FA\int Cdt$$

$$=FAQ$$

$$\text{Angular momentum}=K\omega$$

$$\therefore \quad K\omega=FAQ$$

and

$$K^2\omega^2=F^2A^2Q^2 \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (6)$$

The restoring couple is proportional to the angle of twist of the suspension, and the amount of work done in twisting the suspension through an angle β is equal to $\int_0^\beta n\beta d\beta=\frac{1}{2}n\beta^2$,

and this is equal to the kinetic energy $\frac{1}{2}K\omega^2$ of the coil at the start.

$$\frac{1}{2}K\omega^2=\frac{1}{2}n\beta^2 \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

Dividing (6) by (7) we get

$$K=\frac{F^2A^2Q^2}{n\beta^2} \quad \dots\dots\dots\dots \quad (8)$$

also

$$T=2\pi\sqrt{\frac{K}{n}}$$

or

$$K=\frac{T^2n}{4\pi^2} \quad \dots\dots\dots\dots \quad (9)$$

Equating the values of K in (8) and (9), we get

$$\frac{F^2A^2Q^2}{n\beta^2}=\frac{T^2n}{4\pi^2}$$

or

$$Q^2=\frac{T^2n^2\beta^2}{4\pi^2F^2A^2}$$

$$\therefore \quad Q=\frac{Tn\beta}{2\pi FA} \quad \dots\dots\dots\dots \quad (10)$$

The value of $\frac{H}{G}$ in the first case and $\frac{n}{FA}$ in the second case is found by passing a *steady* current I through it and noting the deflection α produced.

Suspended magnet. $\quad I=\frac{H}{G}\tan\alpha$

or

$$\frac{H}{G}=\frac{I}{\tan\alpha}=\frac{I}{\alpha} \quad \dots\dots\dots\dots \quad (11)$$

if α is very small and expressed in *radian*.

Suspended coil. $NFIl\times b=n\alpha$.

or

$$FIA=n\alpha$$

or

$$\frac{n}{FA}=\frac{I}{\alpha} \quad \dots\dots\dots\dots \quad (12)$$

Q. 179. A condenser is charged to a P.D. of 1 volt and then discharged through a ballistic galvanometer. The deflection is 4·16 cm. on a scale placed one metre from the mirror. The time of 20 complete oscillations (without appreciable damping) is found to be 175 seconds and the steady deflection corresponding to a current of one microampere is 11·6 cm. What is the capacity of the condenser? (*Bombay, 1933*)

Ans. See Q. 178.

$$\alpha = \frac{11{\cdot}6}{100} \text{ radian}$$

$$I = 10^{-6} \text{ ampere}$$

$$\therefore \quad \frac{I}{\alpha} = \frac{10^{-6} \times 100}{11{\cdot}6} = \frac{10^{-4}}{11{\cdot}6}$$

$$T = \frac{175}{20} = 8{\cdot}75 \text{ seconds}$$

$$\beta = \frac{4{\cdot}16}{100} \text{ radian}$$

$$\therefore \quad \text{Charge } Q = \frac{8{\cdot}75 \times 10^{-4} \times 4{\cdot}16}{2 \times 3{\cdot}142 \times 11{\cdot}6 \times 100}$$

$$= 0{\cdot}4992 \times 10^{-6} \text{ coulomb}$$

Potential difference $= 1$ volt

$$\therefore \text{ Capacity of the condenser} = \frac{0{\cdot}4992 \times 10^{-6}}{1}$$

$$= 0{\cdot}4992 \times 10^{-6} \text{ Farad.}$$

Q. 180. Show that the amount of heat generated per second in a conductor by the passage of current through it is inversely proportional to the conductivity of the conductor.

A railway carriage is lit up by thirteen 9 candle power lamps each taking 1·22 ampere at 15 volts. What is the resistance of each lamp, how much heat in calories is generated per second in each lamp, and what is the total power in watts used in lighting the compartment? (*Calcutta, 1933*)

Ans. The potential difference V between two points is equal to the amount of work done in carrying a unit positive charge from the point at the lower potential to the other. If Q units of charge are passed at a *uniform rate* in a time t, the strength of current C is equal to $\frac{Q}{t}$. Electrical energy is spent in doing work in overcoming the resistance R of the conductor between the two points, and this energy is converted into heat.

$$\text{Work done} = V \times Q = VCt = C^2Rt \qquad (\because\ V = CR)$$

$$\text{Heat produced} = \frac{C^2Rt}{J},$$

where J is the mechanical equivalent of heat,

and $$\text{Rate of production of heat} = \frac{C^2Rt}{J \times t} = \frac{C^2R}{J}$$

Thus the rate of generation of heat, for a *given current*, is proportional to the resistance of the conductor or inversely proportional to its conductivity, which is proportional to $\frac{1}{R}$.

Problem. Resistance of each lamp $= \frac{15}{1{\cdot}22} = 12{\cdot}30$ ohms.

$$\text{Heat generated per second in each lamp} = \frac{VC}{J} = \frac{(1{\cdot}22)^2 \times 15}{4{\cdot}16 \times 1{\cdot}22}$$
$$= 4{\cdot}399 \text{ calories.}$$

because joules per sec. (= watts) = volts × amps, and one *calorie* = 4·16 *Joules*.

Power of each lamp = V × C = 15 × 1·22 watts
as watts = volts × amps

Power of 13 lamps = 15 × 1·22 × 13
= 237·9 watts.

Q. 181. Explain Faraday's laws of electrolysis.

A current passes through a coil of wire immersed in a vessel of water containing 3 kilos. of water and then through a copper voltameter. The resistance of the wire is 5 ohms, and it is found that the temperature of water rises 10°C per minute. How much copper is deposited per minute? (Electro-chemical equivalent of copper in c.g.s. units = 0·003281). (*Bombay, 1934*)

Ans. **Faraday's Laws of Electrolysis. First Law.** When current is passed through a solution of an electrolyte, the amount of an ion liberated at either electrode is proportional to the strength of the current and the time for which it is passed.

This means that the amount of an ion liberated is proportional to the product of current C and time t, that is,

proportional to the *total* quantity of electricity $Q(=Ct)$ passed through the electrolyte. A weak current passed for a long time sets free the same amount of an ion as a strong current sent for a short time, provided the product of current and time is the same in both the cases. The same amount of electricity liberates the same mass of an ion But this does not mean that the same mass of different ions is set free at the two electrodes. This law indicates that the ions are responsible for carrying electricity through the electrolyte, and that the mass of each ion liberated is proportional to the amount of electricity passed.

Second Law. When the *same current* is passed through different electrolytes for the *same time*, the masses of the different ions liberated are proportional to their respective chemical equivalent weights.

Suppose solutions of copper sulphate and silver nitrate are placed in separate voltameters, connected in series, and current is passed through them. Copper and silver will be deposited at the cathodes of their respective voltameters, while the *same* mass of oxygen will be set free at the two anodes. It will be found that the amounts of copper, silver, and oxygen set free are in the ratio of 31·5, 108, and 8 respectively: but these are their respective chemical equivalent weights.

$$\therefore \frac{\text{Amount of ion A liberated}}{\text{Amount of ion B liberated}}=\frac{\text{Chemical equivalent of A}}{\text{Chemical equivalent of B}}.$$

Therefore the same quantity of electricity is required to liberate the gm. equivalent of *any ion*. The mass of *an ion* liberated by a unit quantity of electricity is proportional to its chemical equivalent weight, and is called its *electro-chemical equivalent*.

$$\frac{\text{Electro-chemical equivalent of A}}{\text{Electro-chemical equivalent of B}}=\frac{\text{Chemical equivalent of A}}{\text{Chemical equivalent of B}}$$

Problem. Mass of water $=3000$ gms.

Rise of temperature per minute $=10°C$

$\therefore$ Heat produced per minute $=3000\times 10$ calories

$=30000\times 4{\cdot}16$ Joules.

Resistance of the wire $=5$ ohms.

$$\therefore \text{Current}=\sqrt{\frac{30000\times 4{\cdot}16}{5\times 60}}\text{ amp.}$$

as, Energy (Joules) = (current in amps.)^2 × resistance (ohms) × time (seconds).

Electricity passed in one minute $= 60\sqrt{4{\cdot}16}$ coulombs.

Electrochemical equivalent of copper $= 0{\cdot}003281$ gm./e.m. unit $= 0{\cdot}0003281$ gm./coulomb

∴ Amount of copper deposited in one minute

$$= 0{\cdot}0003281 \times 60\sqrt{4{\cdot}16} = 0{\cdot}4016 \text{ gm.}$$

Q. 182. A normal Daniell's cell has an e.m.f. of 1·07 volts and resistance of 2 ohms. Its terminals are connected by two wires in parallel of resistances 3 and 4 ohms. Assuming that the electro-chemical equivalent of copper is 0·000328 gram per coulomb, calculate the weight deposited in the cell, and also the heat developed (*a*) in the cell, (*b*) in each of the wires, during an hour of the working of the cell. (*Punjab, 1931*)

Ans. (i) Combined external resistance

$$= \frac{4 \times 3}{4 + 3} = \frac{12}{7} \text{ ohms.}$$

$$\text{Total resistance} = \frac{12}{7} + 2 = \frac{26}{7} \text{ ohms.}$$

$$\text{Total current} = \frac{1{\cdot}07 \times 7}{26} \text{ amp.}$$

$$\text{Charge passed in 1 hour} = \frac{1{\cdot}07 \times 7}{26} \times 60 \times 60 \text{ coulombs.}$$

$$\text{Copper deposited in 1 hour} = \frac{1{\cdot}07 \times 7 \times 60 \times 60 \times 0{\cdot}000328}{26}$$

$$= 0{\cdot}3402 \text{ gm.}$$

Heat developed in one hour in the cell

$$= \frac{\left(\frac{1{\cdot}07 \times 7}{26}\right)^2 \times 2 \times 60 \times 60}{4{\cdot}16}$$

$$= 143{\cdot}6 \text{ calories}$$

as work (Joules) = (current in amps.)^2 × resistance (ohms) × time (seconds), and one calorie = 4·16 Joules.

The current in the external circuit is divided into two parts in the *inverse* ratio of the resistances.

Current in 3 ohms resistance wire $= \frac{1{\cdot}07 \times 7}{26} \times \frac{4}{7} = \frac{2{\cdot}14}{13}$ amp.

" 4 " " " $= \frac{1{\cdot}07 \times 7}{26} \times \frac{3}{7} = \frac{3{\cdot}21}{26}$ amp.

$\therefore$ Heat developed in 1 hour in 3 ohms resistance wire

$$= \frac{2{\cdot}14^2 \times 3 \times 60 \times 60}{13^2 \times 4{\cdot}16}$$

$= 70{\cdot}37$ calories

and Heat developed in 1 hour in 4 ohms. resistance wire

$$= \frac{3{\cdot}21^2 \times 4 \times 60 \times 60}{26^2 \times 4{\cdot}16}$$

$= 52{\cdot}78$ calories

Q. 183. Explain how, from the phenomena of electrolysis, we get an idea of the atomic nature of electricity.

A current of 2 amperes is passed through a copper sulphate solution. The area of the cathode surface is 1·5 sq. metre Calculate the average increase in the thickness of the copper deposit per minute. (Electro-chemical equivalent of copper = 0·0003294. Density of copper = 8·9) (*Calcutta, 1936*)

Ans. **Atomic Nature of Electricity.** As in electrolysis the amount of a substance deposited is proportional to the amount of electricity passed, it follows that a *definite* charge of electricity is associated with a definite mass of the substance. Further, the amounts of different ions deposited by the passage of the *same* quantity of electricity are proportional to their chemical equivalent weights, or the *same* amount of charge is required to deposit a gram-equivalent of *any* ion. Thus the charge carried by all the monovalent ions is the same, because the number of atoms in a gram-atom of any element is the same. Again, the charge of all the bivalent ions is the same, but is twice the charge on a monovalent ion. Similarly, the charge of all the trivalent ions is the same, but is three times that of the monovalent ions.

The *smallest* charge met with in electrolysis is that carried by a monovalent ion, and all other ionic charges are *integral*

multiples of it. Thus electricity has atomic nature, that is, it has an *ultimate indivisible* unit, and any charge consists of an integral number of such atoms of electricity.

Problem. Current $=2$ amps.

Electricity passed in 1 minute $=2\times 60$ coulombs

Electrochemical equivalent of copper $=0{\cdot}0003294$ gm./coulomb.

$\therefore$ Amount of copper deposited $=0{\cdot}0003294\times 120$ gms.

Density of copper $=8{\cdot}9$ gms./c.c.

$$\text{Volume of copper deposited}=\frac{0{\cdot}00329\times 120}{8{\cdot}9}\text{ c.c.}$$

Surface area of cathode $=1{\cdot}5\times 100\times 100$ sq. cms.

$\therefore$ Average thickness produced per minute

$$=\frac{0{\cdot}00329\times 120}{8{\cdot}9\times 1{\cdot}5\times 100\times 100}$$

$$=2{\cdot}957\times 10^{-7}\text{ cm.}$$

Q. 184. The heat of combustion of hydrogen and oxygen to form water is 34,200 calories for each gramme of hydrogen burnt. A C.G.S. unit current decomposes in one second 0·000945 gm. of water. The mechanical equivalent of heat being $4{\cdot}2\times 10^7$ ergs, find in volts the smallest e.m.f. which can decompose water. (*Punjab, 1932*)

Ans. When hydrogen and oxygen combine together to form water, heat is set free, and, therefore, when they are separated the same amount of heat is absorbed and is equal to 34,200 calories for 1 gm. of hydrogen. As 1 gram-molecule of water (18·016 gms.) consists of 16 gms. of oxygen and 2·016 gms. of hydrogen,

Heat required for decomposing 1 gm. of water

$$=\frac{34200\times 2{\cdot}016}{18{\cdot}016}\text{ calories}$$

$$=\frac{34200\times 2{\cdot}016\times 4{\cdot}2\times 10^7}{18{\cdot}016}\text{ ergs}\quad\ldots\quad(1)$$

Amount of water decomposed by 1 e.m. unit of electricity $=0{\cdot}000945$ gm.

$\therefore$ Amount of electricity for decomposing 1 gm. of water

$$=\frac{1}{0{\cdot}000945} \text{ e.m. units.}$$

If V e.m. units is the potential difference between the two electrodes of the voltameter, amount of work done in passing $\frac{1}{0{\cdot}000945}$ e.m. units of electricity, or

Energy spent for decomposing 1 gm. of water

$$=\frac{V}{0{\cdot}000945} \text{ ergs} \quad \ldots\ldots\ldots \quad (2)$$

Equating (1) and (2), we get

$$\frac{V}{0{\cdot}000945}=\frac{34200\times 2{\cdot}016\times 4{\cdot}2\times 10^7}{18{\cdot}016}$$

$\therefore$ Smallest e.m.f. required

$$=\frac{34200\times 2{\cdot}016\times 4{\cdot}2\times 10^7\times 0{\cdot}000945}{18{\cdot}016}$$

$$=1{\cdot}518\times 10^8 \text{ e.m. units}$$

$$=1{\cdot}518 \text{ volts.}$$

Q. 185. Explain the terms : molecular conductivity, transport ratio, and the degree of dissociation. Describe an experimental arrangement for determining the conductivity of an electrolyte. (*Punjab, 1935*)

Ans. **Molecular Conductivity.** In a solution of an electrolyte its molecules are dissociated into ions, the degree of dissociation depending on the dilution of the solution. A molecule is broken up into a positive and a negative ion, and when a potential difference is produced between the two electrodes, positively charged ions move towards the cathode, while the negatively charged ions move in the opposite direction towards the anode.

The strength of the current in the electrolyte depends on the potential difference applied, the concentration of the electrolyte, and the velocities of its ions. The conductivity K of the electrolyte depends on the last two factors, and is equal to the reciprocal of its specific resistance. If the concentration of an electrolyte is equal to *m* gram-equivalents

per litre of the solution, its **molecular conductivity** (or *equivalent conductivity*) is equal to K/m. The conductivity of a *very dilute* solution is *proportional* to its concentration, but its molecular conductivity is *constant.*

Degree of Dissociation. All the molecules of an electrolyte are not dissociated. As the concentration of its solution is decreased, more of its molecules are dissociated into ions. The number of charged particles is increased, and this leads to an increase in the value of its molecular conductivity. Its **degree of dissociation** is the fraction of its molecules ionized, that is, the ratio of the number of molecules dissociated to the total number of molecules, and is equal to its molecular conductivity divided by its molecular conductivity for *infinite* dilution.

Transport Ratio. The velocities u and v of the kations and anions respectively of an electrolyte are usually different, and each depends on the nature of the ion and the potential gradient in the solution. The number of anions and cations deposited in a given time is the same, but as their velocities are different, the concentration of the solution *near* the two electrodes changes by *different* amounts, if it has no chemical action on the electrodes, though it remains the same in the middle portion of the solution. The decrease of concentration near an electrode is proportional to the velocity of the ions *going away* from it.

$$\frac{\text{Anode space loss}}{\text{Kathode space loss}}=\frac{\text{Velocity of kations}}{\text{Velocity of anions}}=\frac{u}{v} \quad . \quad . \quad (1)$$

$$\therefore \quad \frac{\text{Anode space loss}}{\text{Total loss}}=\frac{u}{u+v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{and} \quad \frac{\text{Kathode space loss}}{\text{Total loss}}=\frac{v}{u+v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The fractions given by (2) and (3) are called the **transport ratio** (or *migration constant*) of the anions and kations respectively.

Conductivity of an Electrolyte. Two arms, DE and FE, of a wheatstone bridge contain equal and large resistances P, arm AF contains a glass tube containing the electrolyte, while the fourth arm AD contains a smaller tube containing

the electrolyte and a variable resistance (Fig. 140). A high resistance galvanometer G is connected between D and F, and a battery B of high voltage is used. The two tubes containing the electrolyte are of exactly the *same uniform* and narrow bore, and the electrodes placed very near their ends are of platinum coated with platinum black.

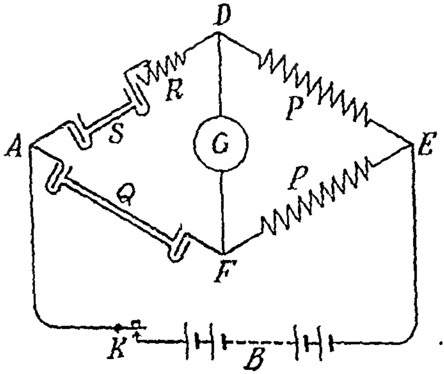


Fig. 140.

By adjusting the variable resistance the third and fourth arms are also made of equal resistances. Then

$$R+S=Q$$

or

$$Q-S=R$$

If the longer and shorter tubes are of lengths l_1 and l_2 respectively and cross-section area a, and ρ is the specific resistance of the electrolyte,

$$\text{Resistance } Q=\frac{\rho l_1}{a}$$

$$\text{,, } \quad S=\frac{\rho l_2}{a}$$

$$Q-S=\frac{\rho}{a}(l_1-l_2)$$

$$=R$$

or

$$\rho=\frac{Ra}{(l_1-l_2)}$$

and

$$\text{Conductivity } K=\frac{1}{\rho}$$

$$=\frac{(l_1-l_2)}{Ra}$$

Q. 186. What do you understand by transport numbers?

Show how a knowledge of the transport numbers together with the knowledge of the molecular conductivity of an electrolyte enables us to determine the migration velocities of ions. (*Bombay, 1931*)

Ans. **Transport Numbers.** See Q. 185. If u and v cm./sec. are the velocities of the kations and anions respectively,

$$\text{Transport number of kations} = \frac{u}{u+v} \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{anions} = \frac{v}{u+v} \quad . \quad . \quad . \quad . \quad (2)$$

The strength of current in an electrolytic solution depends on (1) the number of ions per c.c, (2) charge on each ion, and (3) velocities of ions. When the solution is very dilute, *all* the molecules of the solute may be assumed to be dissociated. Let n gram-equivalents per c.c. be the concentration of the solution, Q e.m. units be the charge carried by a gram-equivalent of the ions, which is the same for all ions, and consider an area A sq. cm. of the solution parallel to the two electrodes.

When potential difference is applied to the electrodes, the two kinds of ions move in opposite directions. All the kations contained in a volume uA c.c. on the side of the area at higher potential pass through it in one second, while all the anions contained in a volume vA c.c. on the other side cross it in the opposite direction in the same time.

$$\text{Charge on kations} = QnuA$$

$$\text{,,} \quad \text{,,} \quad \text{anions} = -QnvA$$

As the flow of positive charge in any direction gives the same direction of current as the flow of the negative charge in the opposite direction,

$$\text{Current through area A} = QnuA + QnvA$$

$$= QnA(u+v) \text{ e.m. units} \quad (3)$$

If δE e.m. units be the fall of potential over a length δx cm. in the electrolyte of conductivity K e.m. units, the resistance of the electrolyte column of length δx cm. and cross-

section area A sq. cm. is equal to $\frac{\delta x}{KA}$ e. m. units, and

$$\text{Current} = \frac{\delta E}{\frac{\delta x}{KA}} = KA\frac{\delta E}{\delta x} \text{ e.m. units,} \quad . \quad . \quad . \quad . \quad (4)$$

where $\frac{\delta E}{\delta x}$ or $\frac{dE}{dx}$ is the potential gradient.

Equating (3) and (4), we get

$$QnA \ (u+v) = KA\frac{dE}{dx}$$

or

$$(u+v) = \frac{K}{Qn} \cdot \frac{dE}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The concentration of the electrolyte is $1000n$ gram-equivalents per litre, so that the molecular conductivity is equal to $\frac{K}{1000n}$ and the value of Q is 9470 e.m. units. Thus knowing the values of molecular conductivity, charge Q, and potential gradient, the value of $(u+v)$ is determined, and combining this with (1) or (2), the migration velocities of the corresponding ions are found.

Q. 187. Give an account of the chemical changes which occur in a storage cell during charge and discharge.

An accumulator has a capacity of 28 ampere hours. What is theoretically the least weight of PbO_2 on its positive plates, given that the PbO_2 is reduced to PbO and that (*a*) the electro-chemical equivalent of hydrogen is 0·00001038 gm./coulomb, and (*b*) the atomic weight of lead is 207, and of oxygen 16 ?

(*Punjab, 1934*)

Ans. **Lead accumulator.** It consists of a positive pole of lead peroxide and a negative pole of lead, both placed in dilute sulphuric acid of suitable density. When current is taken from it, positively charged hydrogen ions appear on the positive and negatively charged oxygen ions on the negative pole. Hydrogen combining with lead peroxide produces lead oxide and water ; lead oxide then combines with sulphuric acid and

forms lead sulphate and water. These changes are given by the following equations:—

$$PbO_2 + 2H \longrightarrow PbO + H_2O$$
$$PbO + H_2SO_4 \longrightarrow PbSO_4 + H_2O$$

Oxygen combines with the lead of the negative pole to form lead oxide, and then lead oxide reacts with sulphuric acid to form lead sulphate and water according to the equations

$$Pb + O \longrightarrow PbO$$
$$PbO + H_2SO_4 \longrightarrow PbSO_4 + H_2O$$

In this process of discharge, water is formed at the expense of sulphuric acid, and, therefore, the density of the solution decreases. As the two poles become more and more similar, owing to lead sulphate formed on them, *e.m.f.* of the accumulator decreases; but long before the process of discharge is complete, current is stopped being taken from it, and it is then recharged.

The process of charging consists in sending current, of suitable strength, through the accumulator in the *opposite* direction; current enters the accumulator at its positive pole and leaves it at its negative pole, and the process is like the electrolysis of dilute sulphuric acid. Oxygen appears at the anode and in the presence of water combines with lead sulphate to form lead peroxide and sulphuric acid according to the equation.

$$PbSO_4 + H_2 + 2O \longrightarrow PbO_2 + H_2SO_4$$

Hydrogen is evolved at the cathode, and lead and sulphuric acid are formed, the reaction being given by

$$PbSO_4 + 2H \longrightarrow Pb + H_2SO_4$$

In charging sulphuric acid is formed at the expense of water, and, therefore, density of the solution increases Electrical energy is spent in charging the accumulator, that is, making its electrodes of different nature, and being placed in an electrolyte the electrodes acquire different potentials, like the poles of a simple cell. The accumulator is now ready for giving out electrical energy, anode becoming the positive pole and cathode the negative. When discharged the accumulator can be again charged.

Problem.

Chemical equivalent weight of hydrogen = 1·008

" " " " oxygen = 8

Electrochemical " " " hydrogen = 0·00001038 gm./coulomb

$\therefore$ " " " " oxygen $= \dfrac{0{\cdot}00001038 \times 8}{1{\cdot}008}$ gm./coulomb

Amount of electricity to be passed $= 28 \times 60 \times 60$ coulombs

During discharge PbO_2 of the positive pole is reduced to PbO, and the same amount of oxygen which leaves this pole combines with the lead of the negative pole.

$\therefore$ Amount of oxygen lost by PbO_2

$$= \frac{0{\cdot}00001038 \times 8 \times 28 \times 60 \times 60}{1{\cdot}008} \text{ gm.}$$

But for oxygen = 16, $PbO_2 = 207 + 32 = 239$

$\therefore$ Least amount of PbO_2

$$\frac{0{\cdot}00001038 \times 8 \times 28 \times 60 \times 60 \times 239}{1{\cdot}008 \times 16}$$

$= 124{\cdot}1$ grams.

Q. 188. Explain what is meant by thermo-electric power, Peltier effect, Thomson effect, and the neutral temperature.

If the thermo-electric power of iron is 1734 − 4·87T, and that of copper is 136 + 0·95T, where T is the temperature on the centigrade scale, show that the e.m.f. of a thermo-couple, the junctions of which are at 0°C and 100°C is 130700 c.g.s. units. Express the result in terms of micro-volts. *(Bombay, 1927)*

Ans. **Thermo-Electric Power.** When the two junctions of a thermo-couple are at different temperatures, *e.m.f.* is produced in it. If the temperature of the cold junction is kept constant, the magnitude E of this *e.m.f.* changes with the temperature T of the hot junction, and its rate of change $\dfrac{dE}{dT}$ with temperature is called the **thermo-electric power** of the

given thermo-couple for that temperature. It changes with the temperature and is *zero* when the hot junction is at the neutral temperature, and after that its sign is reversed.

Neutral Temperature For a given temperature of the cold junction, the thermo-electro-motive force increases with rise of the hot junction temperature. The rate of increase of *e.m.f.* decreases, and the *e.m.f.* becomes *maximum* at a temperature called the **neutral temperature.** After that the magnitude of *e.m.f.* decreases. The neutral temperature of a thermo-couple is *fixed*, whatever be the temperature of its cold junction, and is a characteristic of the two metals used. When the temperature (*temperature of inversion*) of the hot junction is as much above the neutral temperature as that of its cold junction is below it, the *e.m.f.* is reduced to zero. Thus the neutral temperature of a thermo-couple is the arithmetic mean of the temperatures of its junctions when its *e.m.f.* is zero though its junctions are at *different* temperatures.

Peltier Effect. When current is made to pass through a thermo-couple, its one junction is cooled (heat absorbed) and the other is heated (heat evolved). *Below* the neutral temperature that junction which for *this* direction of the thermo-electric current (Seebeck effect) should be heated (heat supplied) is cooled, while the other which should be kept cold (heat removed) is heated. Therefore the energy of the current produced in a thermo-couple is due to a part of the heat energy absorbed.

If the direction of current in the couple is *reversed*, *opposite* thermal effects are produced, that is, that junction which was cooled is now heated, while the other junction instead of being heated is cooled. This *reversible* thermal effect at the junction of two *different* conductors when current is passed through it is called **Peltier Effect.** The ends of the two metals at the junction are at *different* potentials, and heat is absorbed or evolved according as the current is passed from the lower potential metal to the higher potential metal or in the opposite direction.

Thomson Effect. The value of the potential difference at the junction of two given metals at any temperature T (absolute) is called the **Peltier coefficient** at that temperature,

and is equal to $T\frac{dE}{dT}$. If the hot junction of a thermo-couple is at the neutral temperature, $\frac{dE}{dT}$ is equal to zero. There is no potential difference between the two metals; no heat energy is absorbed at this junction, but still heat energy is given out at the cold junction and energy is supplied to the thermo-electric current produced. Therefore heat must be absorbed at some other point or points in the thermo-couple. It may be absorbed from one or both the conductors, or absorbed from one and given out to the other, the amount of heat absorbed being *greater* than the amount of heat evolved.

The junctions of the thermo-couple being at different temperatures, the temperature of each conductor rises from its cold end towards its hot end, and, due to this change of temperature, its different points are at *different potentials*. In some metals potential increases with temperature, while in others opposite is the case, and when current passes through a conductor, heat is absorbed or given out according as the current flows from its lower potential end to its higher potential end or in the opposite direction. This *reversible* absorption or production of heat in an *unequally* heated conductor due to flow of current in it is called **Thomson effect,** and if the potential difference δV between two points of a conductor at temperatures T and $T+\delta T$ is equal to $\sigma.\,\delta T$, $\sigma=\frac{\delta V}{\delta T}=\frac{dV}{dT}$ is called Thomson coefficient.

Problem. Here the thermoelectric powers $\frac{dE}{dT}$ of iron and copper are given in e. m. units with respect to a common metal, lead.

Thermo-electric power of iron at 0°C	=	1734
,, ,, ,, ,, ,, 100°C	=	$1734-4{\cdot}87\times100$
	=	1247
,, ,, ,, copper at 0°C	=	136
,, ,, ,, ,, ,, 100°C	=	$136+0{\cdot}95\times100$
	=	231

As at both the temperatures the value of $\frac{dE}{dT}$ for iron is *greater* than that of copper, they are on the *same* side (*below*) of the neutral temperature for a copper iron couple. According to the law of successive contacts, or intermediate metals, the thermo-electric power of iron with respect to copper is equal to $1734 - 136 = 1598$ and $1247 - 231 = 1016$ at 0°C and 100°C respectively, and its value at the mean temperature $= \frac{1598 + 1016}{2} = 1307$.

$$\therefore \text{Thermo-electro-motive force} = 1307 \times 100$$
$$= 130700 \text{ e. m. units,}$$
$$= \frac{130700}{100} = 1307 \text{ micro-volts.}$$

as for the two temperatures the same side of the neutral temperature, the *e.m.f.* is equal to the product of the difference of temperatures and the thermo-electric power at the mean temperature, and 1 micro-volt = 100 e. m. units.

Q. 189. Prove the relation $P = T\frac{dE}{dT}$, where P is the Peltier coefficient at a metallic junction, T the absolute temperature of the junction, and E is the total thermo E. M. F. in the circuit. (*Punjab, 1935*)

Ans. **Peltier Coefficient.** Let the junctions, D and F, of a thermo-couple of metals A and B of Thomson coefficients σ_A and σ_B respectively be kept at temperatures T and $T + dT$, and the Peltier coefficients be P and $P + dP$ at the corresponding junctions, A being *positive* with respect to B (Fig. 141). When a current δC passes for time *t* round the circuit from A to B at the cold junction and from B to A at the hot junction, the amount of electricity carried is equal to $\delta C.t$. At F it is pushed against a potential difference $P + dP$,

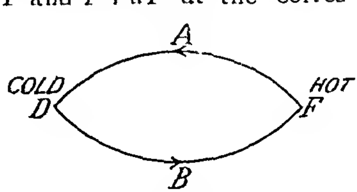


Fig. 141.

and the amount of heat energy *absorbed* is equal to $(P+dP)\,\delta C t$. Due to Thomson effect the potential of end F of A is higher than that of end D, by $\sigma_A .\, dT$, and heat energy *given out* is equal to $\sigma_A .\, dT .\, \delta C .\, t$. At D current falls through a potential P giving out heat energy $P\delta C . t$, while from D to F along the metal B, heat energy *absorbed* in going from lower potential end to the higher potential end is equal to $\sigma_B .\, dT\, \delta C .\, t$.

$\therefore$ Heat energy converted into electrical energy

$$=(P+dP-\sigma_A.dT-P+\sigma_B.dT)\delta C.t.$$
$$=\{dP+(\sigma_B-\sigma_A)\,dT\}\,\delta C.t.$$

If dE is the *e. m. f.* of the circuit.

$$\delta C.t.dE=\{dP+(\sigma_B-\sigma_A)dT\}\,\delta C.t.$$

or
$$dE=dP+(\sigma_B-\sigma_A)dT \quad . \quad . \quad . \quad (1)$$

Peltier and Thomson effects are proportional to current and are *reversible*. Joule heating effect is irreversible, but as it is proportional to the square of current, it is *negligible* as compared with the first two effects when the current is *very small*. As the process in the thermo-electric couple is reversible, according to thermo-dynamics the ratio of heat energy converted into work to the total amount of heat energy absorbed is equal to the ratio of the difference of the temperatures of the source and sink to the temperature of the source.

$$\frac{dT}{T+dT}=\frac{(dP+(\sigma_B-\sigma_A)\,dT)\delta C.t}{(P+dP)\,\delta C.t}$$
$$=\frac{dP+(\sigma_B-\sigma_A)\,dT}{P+dP}$$
$$=\frac{dE}{P+dP} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{from } (1)$$

or
$$\frac{dT}{T}=\frac{dE}{P}$$

$$\therefore\ P=T\frac{dE}{dT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Q. 190. Describe the construction and use of a thermo-couple for the measurement of temperatures.

A thermo-couple is made of iron and constantan wires. Find the e.m.f. developed per °C difference of temperature between the two junctions, given that the thermo-e.m.f's. of iron and constantan against platinum are respectively + 1600 and − 3440 micro-volts per 100°C difference of temperature.

What do the opposite signs in the above indicate?

(*Punjab, 1931*)

Ans. **Thermo-couple.** A thermo-couple consists of two *different* metals or alloys, whose choice depends on the range of temperature to be measured. Antimony-bismith couple is used for ordinary temperatures (0°—100°C), as it is very sensitive. Iron-copper couples are used upto 300°C, iron-nickel couples upto 600°C, and platinum—platinum-rhodium couples are used for higher temperatures, but *e.m.f.* developed is very small.

The hot junction is welded electrically or in an oxy-hydrogen flame. The wires near the hot junction are insulated by capillaries of fireclay, threaded through mica discs, and placed in a protecting tube of quartz or porcelain. The cold junction is placed in ice and is thus kept at a *constant* temperature.

For accurately measuring *e.m.f.* produced, it is balanced against the fall of potential in a part of the wire AB carrying current (Fig. 142). A cell C of *constant e.m.f.* is connected

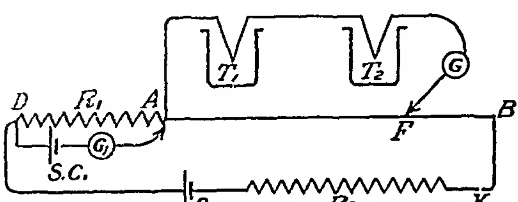


Fig. 142.

with resistances R_1 and R_2 and wire AB placed in series. The resistance of wire AB is very small as compared with that of the rest of the circuit as the thermo-electromotive force to be measured is very small. A standard cell S.C. is balanced

against the fall of potential in R_1 by adjusting resistance R_2. Wire AB is calibrated and its resistance for any length is known. The two junctions of the thermo-couple are kept at constant temperatures T_1 and T_2 and its free ends are connected to the wire AB through a galvanometer G. By adjusting the position of the jockey the balance point F is found. If the resistance of AF is equal to R,

$$\frac{\text{Thermo } e.m.f.}{e.m.f. \text{ of S. C.}} = \frac{R}{R_1}$$

Knowing the relation between temperature T and electromotive force E produced, such as, $E = aT + bT^2$, the value of T is calculated. To avoid ambiguity a thermo-couple is used for temperatures *below* its neutral temperature.

Problem. If the junctions of an iron-platinum couple are maintained at a difference of temperature of 1°C, the thermo-electromotive force produced is equal to 16 micro-volts, the thermo current flowing *from iron to platinum at the cold end*, while for a constantan-platinum couple *e.m.f.* produced for the same temperatures of the junction, is equal to 34 micro-volts and the current flows *from platinum to constantan at the cold end.* Therefore, according to the law of intermediate metals, the thermo-electromotive force produced in an iron-constantan couple is equal to $16 - (-34) = 50$ micro-volts for 1°C difference of temperature, and the current flows *from iron to constantan at the cold junction.*

Q. 191. State the laws of electromagnetic induction, and explain how Lenz's law follows from the law of conservation of energy.

Ans. **Laws of Electromagnetic Induction:**

(1) When the magnetic flux through a circuit changes, induced *e.m.f.* is produced in it, and lasts only as long as the change in the flux through the circuit continues.

(2) The induced *e.m.f.* is proportional to the rate of change of flux.

(3) The direction of the induced current is such that it (its magnetic field) tends to oppose the change or motion that produces it.

The third law is called **Lenz's Law.** When a magnet, with its north pole downward, is lowered towards a closed coil, the direction of the induced current is such that the upper face of the coil becomes a north pole, and tends to *oppose* the

motion of the magnet. Therefore, in lowering the magnet, work is done against the magnetic repulsion, and this work is converted into the electrical energy of the induced current in the coil. Thus Lenz's Law accounts for the energy of the induced current in the coil.

If the induced current were produced in the opposite direction, the upper face of the coil would have become a south pole: it would have attracted the north pole of the magnet, and accelerated its motion downward. The downward motion of the magnet would have increased the induced current further and added to its energy without the expenditure of work, which is contrary to the law of conservation of energy.

When the magnet is withdrawn, the direction of the induced current is reversed; the upper face of the coil becomes a south pole, and attracts the receding north pole of the magnet. Work is done in moving the magnet against this attraction, over and above that done in overcoming the force of gravity, and is converted into the electrical energy of the induced current. The same argument applies to the case when the change of flux is due to a change of current in a neighbouring circuit.

Q. 192. A square conducting frame with free ends is rotated in the earth's field about a vertical axis. Describe the variation in e.m.f. between its ends during one complete revolution.

Calculate the maximum e.m.f. when the frame is making 120 revolutions per minute, the side of the square being 25 cm., and the intensity of Earth's field 0·33 gauss. (*Punjab, 1929*)

Ans. **Rotating Frame.** As the frame is rotated about a *vertical* axis, induced *e.m.f.* in it is caused by the change of flux due to the *horizontal* component H only of the intensity of the earth's magnetic field and not the vertical component. In Fig. 143, in the position BD the plane of the frame of area A is perpendicular to the horizontal component of the intensity of the earth's magnetic field, and total flux through it is equal to AH. When it is rotated and is inclined at θ to its former position, its area perpendicular to H is A cos θ, or the component of H

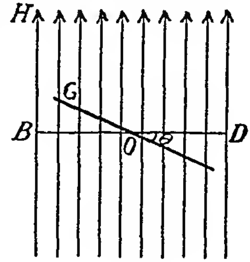


Fig. 143.

perpendicular to its face is $H \cos \theta$, so that the magnetic flux F in it is given by

$$F = AH \cos \theta$$

[Here the frame has only one turn. If it should have S turns, $F = SAH \cos \theta$.]

By differentiating F with respect to time t, the value of the *instantaneous* induced electromotive force E is obtained, and is given by

$$E = -\frac{dF}{dt} = -\frac{d(AH \cos \theta)}{dt}$$

$$= AH \sin \theta . \frac{d\theta}{dt}$$

$$= AH \sin \theta . \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ω is the angular velocity of the frame. It is equal to $2\pi n$, and θ is equal to $\omega t = 2\pi nt$ if n is the number of rotations made by the frame at a uniform speed in a unit time, so that

$$E = AH 2\pi n \sin (2\pi nt) \quad . \quad . \quad . \quad . \quad (2)$$

As ω is constant, equation (1) shows that the induced *e.m.f.* varies as the sine of θ. When θ is equal to $0°$, the induced *e.m.f.* is also zero, because then the sides of the frame move *parallel* to H and there is no change of flux. With increase in θ, the rate of change of flux increases and is maximum when θ is equal to $\pi/2$ radian. In this position the frame moves *perpendicular* to H, and the induced *e.m.f.* is equal to $AH\omega$ (maximum).

On further increasing θ, the inclination of the frame to H decreases and the induced *e.m f.* diminishes. It becomes zero when θ is equal to π, for then the frame moves parallel to H. For the next half rotation the induced *e.m.f.* is in the *opposite* direction. It increases from 0 to its maximum value $AH\omega$ for change of θ from π to $3\pi/2$, and then decreases to zero as the angle changes from $3\pi/2$ to 2π. These changes are then repeated over and over again. Thus an alternating *e.m.f.* of maximum value $AH\omega$ and period of variation of $2\pi/\omega$ is produced.

Problem. Face area of the frame $= 25 \times 25$ sq. cms.

Value of $H = 0{\cdot}33$ gauss.

Angular velocity $= \dfrac{2\pi \times 120}{60} = 4\pi$ radians/sec.

$\therefore$ Maximum induced *e.m.f.* $= 625 \times 0{\cdot}33 \times 4 \times 3{\cdot}142$

$= 2592$ e.m. units.

$= 2{\cdot}592 \times 10^{-5}$ volts.

Q. 193. What is an earth inductor? Explain how it can be employed for the determination of the total intensity of the earth's magnetic field.

(*Calcutta, 1935*)

Ans. **Earth Inductor.** It is a coil of n turns of insulated wire wound over a circular frame of face area A, and capable of rotation about an axis along its diameter. The position of the frame and the axis of rotation can be adjusted, and thus the coil can be rotated in any way required. The ends of the coil are connected to the two segments of a commutator, and a *low* resistance ballistic galvanometer is connected to two brushes pressing on the commutator segments.

If the coil is held perpendicular to a magnetic field of intensity F, the total flux N associated with it is equal to nAF. On rotating the coil rapidly, about an axis *perpendicular* to the magnetic field, through 180°, the flux through it is *reversed* and the change of flux is equal to $2n$AF.

$$\text{Induced } e.m.f. = \frac{-dN}{dt}$$

$$\text{,, current C} = \frac{-dN}{R \cdot dt},$$

where R is the resistance of the circuit consisting of the coil and galvanometer.

$$\text{Induced charge Q} = \int C \,.\, dt$$

$$= \int_{N_1}^{N_2} \frac{-dN}{Rdt} . dt$$

$$= \frac{\text{change of flux}}{R}$$

$$= \frac{2nAF}{R} \quad . \quad . \quad . \quad . \quad (1)$$

$$\therefore \quad \frac{2n\text{AF}}{\text{R}} = \text{K}\theta$$

$$\text{F} = \frac{\text{RK}\theta}{2n\text{A}}, \quad \ldots \quad (2)$$

where θ is the first corrected deflection of the galvanometer, and K is a constant of the galvanometer.

Determination of the Intensity of Earth's Field. The coil is placed *perpendicular* to the magnetic meridian, so that the horizontal component H of the earth's field is *perpendicular* to its plane, and is rotated rapidly through 180°, about a *vertical* axis *parallel* to the other (vertical) component. If θ_1 is the deflection,

$$\text{H} = \frac{\text{RK}\theta_1}{2n\text{A}} \quad \ldots \quad (3)$$

In this process *no* change of flux is due to the vertical component V of the earth's field.

Then the coil is placed in a *horizontal* position, with its axis of rotation *in* the magnetic meridian, so that on rotating it the change of flux in it is due to the vertical component *only* and *not* the horizontal component. If on rotating the coil rapidly through 180°, the galvanometer deflection is equal to θ_2,

$$\text{V} = \frac{\text{RK}\theta_2}{2n\text{A}} \quad \ldots \quad (4)$$

$$\therefore \text{ Intensity of earth's field} = \sqrt{\text{H}^2 + \text{V}^2}$$

$$= \frac{\text{RK}}{2n\text{A}} \sqrt{\theta_1^2 + \theta_2^2} \quad \ldots \quad (5)$$

Q. 194. What is meant by the co-efficient of self-induction of a circuit? Define the absolute and the practical units in which it is measured.

Calculate approximately the co-efficient of self-induction of a coil of 1000 turns of wire wound on an iron ring of 20 cm. mean diameter and 2 sq. cm. cross-section, the permeability of iron being 500.

(*Punjab, 1925*)

Ans. **Co-efficient of Self-Induction.** When current C passes through a circuit, the magnetic flux F associated with it

due to this current depends on the circuit and the strength of current.

$$F \propto C$$

or $$F = LC, \quad \ldots \ldots \quad (1)$$

where L is a constant for the given circuit. It is called its co-efficient of self-induction, and is equal to the flux due to a unit current. The practical unit of self-inductance is one **henry** and is equal to 10^9 e. m. units.

If the current in the circuit changes, its flux varies. Due to this change induced *e.m.f.* is set up in the circuit, and is proportional to the rate of change of flux.

$$\text{Induced } e.m.f. = -\frac{dF}{dt}$$

$$= -L\frac{dc}{dt} \quad \ldots \ldots \quad (2)$$

Thus the induced *e.m.f.* is proportional to the rate of change of current and depends on the circuit. The co-efficient of self-induction is one e. m. unit if the induced *e.m f.* is one e. m. unit when the rate of change of current is one e. m. unit per second. It is one **henry** if the rate of change of current at 1 ampere per second induces an *e.m.f.* of one volt.

As the induced electromotive force *c* is opposite to the applied *e.m f.* sending current in the circuit, work has to be done in overcoming this opposition, and thus energy is stored up in the magnetic field of the circuit. For establishing a current of C units, amount of work to be done is $\frac{1}{2}LC^2$. Thus self-induction of a circuit corresponds to the *inertia* of a material body, and it tends to diminish the rate of increase or decrease of current in it. It does not allow current to grow to its maximum value at once nor does it allow it to stop abruptly. When the applied *e.m.f.* is stopped, this latent energy of the magnetic field of the circuit is set free, and does not allow the current to decay suddenly. The co-efficient of self-induction is one e. m. unit if the amount of work done in establishing a current of 1 e. m. unit is $\frac{1}{2}$ erg, and it is equal to 1 henry if for establishing current of one ampere strength amount of work to be done is $\frac{1}{2}$ Joule.

Problem. The magnetic field in the coil may be assumed to be uniform and of strength $4\pi nC$, where n is the number of turns per *unit* length and C is the current.

Mean diameter $= 20$ cm.

Circumference $= \pi \times 20$ cm.

Total No. of turns $= 1000$

$\therefore$ Intensity of magnetic field due to C *e.m.* units of current

$$= \frac{4\pi \times 1000 \times C}{\pi \times 20} = 200C \text{ gauss.}$$

Permeability $= 500$

Cross-section area $= 2$ sq. cm.

Flux for each turn $= 500 \times 2 \times 200C$ *c.g.s.* units.

" " 1000 turns $= 1000 \times 200000C$ " "

But Total flux of the coil $= LC$ " "

$$\therefore \; LC = 2 \times 10^8 \times C$$

and $L = 2 \times 10^8$ *e.m.* units.

$= 0{\cdot}2$ henry.

Q. 195. What do you mean by the co-efficients of self and mutual induction?

Two coils, a primary of 400 turns and a secondary of 20 turms, are wound on an iron ring of mean diameter 20 cm. and cross-section 2 cm. radius. Find their mutual inductance if $\mu = 800$.

(*Punjab, 1939*)

Ans. **Co-efficient of Self-Induction.** See Q. 194.

Co-efficient of Mutual Induction. When a current C passes through a circuit (primary), its magnetic field is produced. The flux F in a neighbouring circuit (secondary) depends on the *two* circuits and their relative position, and is proportional to current C.

$$F \propto C$$

or $$F = MC, \quad \ldots \ldots \ldots \quad (1)$$

where M is called the co-efficient of mutual induction of the two circuits, and is equal to the flux in the secondary when a *unit* current flows in the primary.

When current varies in the primary, induced electromotive force e is produced in the secondary and is equal to the rate of change of flux in it.

$$e = -\frac{dF}{dt} = -\frac{d(MC)}{dt}$$

$$= -M\frac{dC}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the current in the primary changes at the rate of 1 unit per unit time, the co-efficient of mutual induction is equal to the *e.m.f.* induced in the secondary.

Problem. As the secondary coil is wound over the primary coil, the *whole* of the magnetic field of the primary is associated with the secondary.

Mean diameter of the ring = 20 cm.

Circumference „ „ „ $= \pi \times 20$ cm.

No. of turns of the primary = 400

Intensity of magnetic field due to C *e.m.* units of current

$$= \frac{4\pi \times 400 \times C}{\pi \times 20} = 80C \text{ gauss.}$$

Permeability = 800

Cross-section area of the *primary* coil $= \pi \times 2^2$ sq. cm.

$\therefore$ Flux for each turn of the secondary $= 800 \times 3{\cdot}142 \times 4 \times 80C$ *c.g.s.* units.

„ „ 20 turns „ „ „ $= 800 \times 3{\cdot}142 \times 4 \times 80C \times 20$

$= 16090000C$ *c.g.s.* units.

But Total flux of the secondary = MC „ „

$\therefore$ MC = 16090000C

or M = 16090000 *e.m.* units $= 1{\cdot}609 \times 10^{-2}$ henry.

Q. 196. Find the change of current at (*a*) make, and (*b*) break in a circuit containing resistance and inductance, and explain the meaning of its time constant.

Ans. (*a*) **Growth of current.** Let R and L be the resistance and inductance respectively of the circuit, E the applied *e.m.f.*, and C the current at any instant before the current reaches its final steady value. As the current is

changing, induced electromotive force $L\frac{dC}{dt}$ is produced and it is *opposite* to the applied *e.m.f.* Therefore the effective *e.m.f.* is equal to $E - L\frac{dC}{dt}$, and according to Ohm's Law this is equal to CR.

$$E - L\frac{dC}{dt} = CR \quad \ldots\ldots\ldots \quad (1)$$

or

$$E - CR = L\frac{dC}{dt}$$

or

$$\frac{dC}{E - CR} = \frac{dt}{L}$$

$$\therefore \quad \int_0^C \frac{dC}{E - CR} = \int_0^t \frac{dt}{L}$$

$$\left[-\frac{1}{R}\log_e (E - CR)\right]_0^C = \left[\frac{t}{L}\right]_0^t$$

$$\text{Log}_e \frac{E - CR}{E} = -\frac{Rt}{L}$$

$$\frac{E - CR}{E} = e^{-\frac{Rt}{L}}$$

$$CR = E\left(1 - e^{-\frac{Rt}{L}}\right)$$

and

$$C = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \quad \ldots \quad (2)$$

Here $\frac{E}{R}$ is the final steady value of the current. The result shows that the greater the inductance and the smaller the resistance of the circuit, the smaller is the rate of growth of current, and vice versa.

(*b*) **Decay of Current.** When the applied *e.m.f.* is suddenly removed, putting $E = 0$ in (1), we get

$$-L\frac{dC}{dt} = CR \quad \ldots\ldots\ldots \quad (3)$$

or
$$\frac{dC}{C} = -\frac{Rdt}{L}$$

$$\therefore \quad \int_{C_0}^{C} \frac{dC}{C} = \int_{0}^{t} -\frac{Rdt}{L},$$

where $C_o\left(=\frac{E}{R}\right)$ is the steady value of current at the time of breaking the circuit.

$$\Big[\log_e C\Big]_{C_o}^{C} = \Big[-\frac{Rt}{L}\Big]_{o}^{t}$$

$$\log \frac{C}{C_o} = \frac{-Rt}{L}$$

$$C = C_o e^{-\frac{Rt}{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Time Constant. In this case also the greater the inductance and the smaller the resistance, the smaller is the rate of change of the current, and vice versa. The ratio $\frac{L}{R}$ is a characteristic of the circuit and indicates how the current changes. If t is equal to $\frac{L}{R}$, equations (2) and (4) become

$$C = \frac{E}{R}\left(1 - \frac{1}{e}\right)$$

and
$$C = \frac{C_o}{e}$$

respectively. In the first case the initial value of current is zero, and the change of current is equal to $\frac{E}{R}\left(1-\frac{1}{e}\right)$. In the case of decay of current, the initial value of the current is C_o,

$$\therefore \quad \text{Change of current} = C_o - \frac{C_o}{e}$$

$$= C_o\left(1 - \frac{1}{e}\right)$$

Thus in both the cases the rate of change of current is the *same*, and *time constant* is the interval in which the current

decays to $\frac{1}{e}$ of its original steady value at break, or at make rises to $\left(1-\frac{1}{e}\right)$ of its final steady value, or both at make and break changes by $\left(1-\frac{1}{e}\right)$ of its steady value.

Q. 197. Describe the construction and explain the working of an induction coil fitted with a condenser.

Ans. **Induction Coil.** In an induction coil there is a primary coil, PP, (Fig. 144) of a small number of turns of thick insulated copper wire, wound over a soft iron core C, made of a number of insulated soft iron wires, and *surrounding* the primary coil is a secondary coil, SS, of a *very large* number of turns of very thin insulated copper wire. One end of the primary coil is joined to a pole of battery B, while the other end is connected to an elastic stand carrying a soft iron hammer, H, facing the iron core. The other pole of the battery is connected to a stand carrying a screw, K.

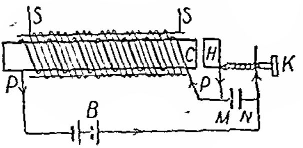


Fig. 144.

When screw K is moved forward and its tip is brought in contact with the hammer stand, current flows in the primary coil in the direction shown. This magnetises the iron core, and as the number of magnetic lines of force through the secondary coil increases, an induced *e.m.f.* is produced in it—the direction of the induced *e.m.f.* being *opposite* to that of the current in the primary coil. The hammer, being attracted towards the magnetised iron core, bends forward, and breaks contact with the screw. The primary circuit is broken, the iron core is demagnetised, number of magnetic lines of force through the secondary coil decreases, and an induced *e.m.f.*, in the *same* direction as the primary current, is set up in the secondary coil. Thus induced *e.m.f.* is set up in the secondary coil, at make in one direction and at break in the opposite direction, and as the number of turns of wire in the secondary coil is very large, the induced *e.m.f.* is very high.

The iron core being demagnetised, the hammer springs back and touches the screw, and then the above process is repeated.

Thus the function of the automatic break is to break and make the primary circuit *quickly*. The iron core, due to its very high permeability, increases enormously the number of lines of force threading the secondary coil and also helps in breaking the primary circuit.

Owing to the high resistance of the air gap, produced between H and K, the resistance of the primary circuit at break is much greater than that at make. The time constant $\left(\frac{L}{R}\right)$ of the primary at break is very large as compared with its value at make, so that the rate of fall of the primary current at break is much greater than that of the rise of current at make, and, therefore, induced *e.m.f.* at break is much greater than that at make.

But at make and break the number of lines of force through the primary coil also changes, and an induced current is produced in it due to its self-induction. At make this induced current is opposite to the primary current, and *retards* the rate of rise of the current; at break it is in the same direction as the primary circuit current, and *decreases* the rate of fall of the current. Therefore the induced *e.m.f.* both at make and break is decreased. The fall of current is very quick, and, therefore, the induced current at break—called extra current—is very strong (may be even stronger than the primary current), and produces a spark in the air gap between H and K. This spark not only prolongs the fall of the current—for the spark continues the current—but also burns the platinum tip of the screw.

To avoid this, a condenser MN is connected across the air gap between H and K. At break the induced current—in the same direction as the primary current—charges the condenser, and its plates N and M are charged positively and negatively respectively, but owing to its *large capacity* its potential difference does not become sufficiently high to pass a spark across the gap. Soon the condenser discharges, producing current in the direction NBPPM, *opposite* to that of the extra current, and quickly neutralizes the direct current, and, therefore, increases the induced *e.m.f.* at break. The condenser current, being stronger than the primary current, not only demagnetises the iron core

but even magnetises it in the opposite direction, so that at the next make the primary current will have first to demagnetise the core and then to magnetise it. Therefore, by using a condenser, the fall of current at break is made *very sudden* and its rise at make is *prolonged*, that is, induced *e.m.f.* at break is far greater than that at make, and the induced current in the secondary is almost unidirectional.

Q. 198. Describe the construction and explain the action of a D. C. dynamo, and discuss the special features of series, shunt, and compound wound machines.

Ans. **Direct Current Dynamo.** A rectangular conducting coil $ABDE$ (Fig. 145), called armature coil and mounted on an iron core, is revolved betweeen the poles N, S of the field magnet, and about a horizontal axis passing mid-way between AB and DE. In the position shown, arm AB is being moved upward, while arm DE is going downward. The coil cuts magnetic lines of force, running from left to right, and due to charge of flux, induced *e.m.f.* is produced in it. See Q. 192 for the change in the induced *e.m.f.* By applying Fleming's *right-hand* rule, the direction of the induced current is found, as shown in the figure.

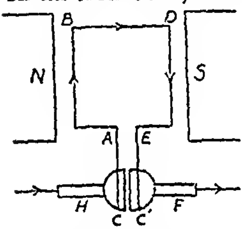


Fig. 145.

The direction of the induced current may also be found by applying Lenz's law. The upper face of the coil is being rotated towards the south pole of the field magnet, and the direction of the induced current should be such that it *opposes* this rotation. Hence the upper face becomes south pole, and the direction of the induced current in it is clockwise. Similarly, the lower face of the coil becomes north pole, as it is being made to set opposite the north pole of the field magnet, and the direction of the induced current, looked at from below, is *anti*-clockwise.

The ends of the coil are mounted with two halves, C and C', of a conducting cylinder, insulated from each other, and called *commutator*. The external circuit ends in two brushes, H and F, which are pressing against C and C' respectively. Current enters the external circuit at the brush F and leaves it by the other brush H.

After the coil becomes vertical, AB begins to descend and DE begins to ascend, and the direction of the induced current in it is reversed, that is, current flows from E to D and from B to A. *At the same time C comes in contact with F and C' with H*, so that the current again enters the external circuit through the brush F and leaves it by the other brush H, and thus the *direction of the current in the external circuit remains the same as before.*

With a single coil induced *e.m.f.* and current vary in strength, being maximum when the coil is passing through its horizontal position and zero when it is vertical. To make the induced *e.m.f.* strong and of uniform strength, a large number of coils, suitably placed with respect to one another and connected to a commutator with many segments, are used.

Field Magnet. The field magnet is usually an electromagnet, and a part or whole of the rectified current generated by the dynamo is used for exciting it. At the start there is enough residual magnetism in it to produce a small electromotive force. This increases the strength of the field, which in turn gives rise to stronger induced *e.m.f.*, and this mutual strengthening goes on until a steady state is reached.

Series Wound Dynamo. Here the windings of the field magnet are of *few* turns of *thick* wire, and are connected in *series* with the armature and the external circuit, joined with A and B, so that the *whole* of the current excites the field magnet Fig. 146 (a). When the load increases, the resistance of the external circuit decreases and the current

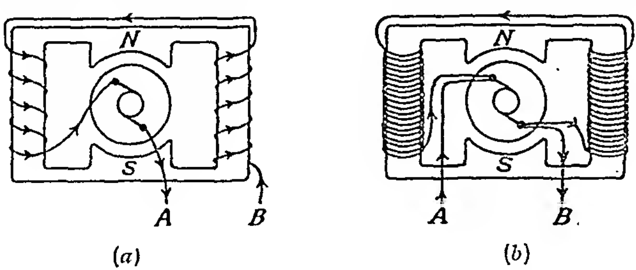


Fig. 146.

increases. The strength of the fieldmagnet increases, and, therefore, greater *e.m.f.* is induced. Similarly, when the load decreases, external resistance increases and current diminishes. This

decreases the strength of the field magnet, and the induced *e.m.f.* decreases. Thus the induced *e.m.f.* changes with the load and the machine does *not* supply a constant potential difference. Hence series winding is not suitable.

Shunt Wound Dynamo. In this case the field magnet coil consists of a *very large* number of turns of *fine* wire of *high* resistance and is connected with the brushes in parallel with the external circuit, so that the potential difference for the field coil and the external circuit is the *same* (Fig. 146 *b*). When the resistance of the external circuit decreases, it takes more current; the current in the field coil decreases, and the potential difference at the brushes diminishes. If the external circuit resistance increases, current in it decreases, that in the field coil increases, and the potential difference at the brushes rises.

This arrangement also does not supply current at constant potential, but the changes are opposite to those in the case of a series wound machine and are smaller in magnitude. It is used when there is no sudden change in load, and is particularly suitable for charging accumulators. At the start the cells send current in the field coil in the right direction to supply the magnetic field. As their charging proceeds, their *e.m.f.* increases, and more current is passed in the field coil, which gives rise to greater *e.m.f.* of the machine.

Compound Wound Dynamo. To make the potential difference supplied by the machine independent of the external load compound (both series and shunt) winding is used. The series coil has small resistance and is made of thick wire, whereas the shunt coil has high resistance and is made of fine wire. Their resistance and number are suitably adjusted, so that with change of load the change of *e.m.f.* due to one coil is *compensated* by the *opposite* change due to the other coil. Usually a field regulating resistance is placed in the shunt coil to keep the potential difference constant.

Q. 199. Describe in an elementary way the physical principles on which the working of an ordinary electric motor is based. (*Calcutta, 1931*)

Ans. **Electric Motor.** When a conductor carrying current is placed perpendicular to a magnetic field, it experiences a mechanical force, which is equal to the product of the length

of the conductor, strength of the current, and the intensity of the magnetic field, and is *perpendicular* both to the conductor and the field. This principle is used in the construction of an electric motor.

Armature coil ABDE (Fig. 145), with commutator segments C, C′ fixed at its ends, and capable of rotating about a horizontal axis passing mid-way between AB and DE, is placed between the poles N, S of the field magnet, which is usually an electro-magnet. Brushes, H and F, connected with the poles of a battery or to electric mains, press against C and C′ respectively. When current is passed in the coil in the direction shown, it begins to revolve, AB moving *downwards* and DE *upwards* (left-hand rule), for the upper face of the coil (current clockwise) becomes south pole and the lower face north pole, and the coil rotates so as to have its north and south poles opposite to the south and north poles of the field magnet respectively. When the coil becomes vertical, the mechanical force on AB is vertically downwards, while that on ED is upwards, and these forces do not help to rotate the coil any further. But owing to its inertia, the coil rotates on, and just then the commutator segments exchange contact with the brushes, F comes into contact with C and H with C′; direction of current in the coil is reversed, and so is the direction of motion of the arms AB and DE, that is, AB begins to ascend and DE to descend and, therefore, the coil keeps rotating in the same direction as before. For continuous rotation, current in the coil is reversed twice in each rotation.

In such a simple motor, the turning effect of the force is greatest when the coil is horizontal, and almost nothing when it is vertical; therefore the speed of this motor will not remain the same. In order to increase the speed and keep it uniform, a large number of coils, each of many turns, placed suitably with respect to one another and mounted on an iron core, are used along with a commutator of many segments.

As the armature revolves, magnetic flux passing through it changes and *e.m.f.* is induced in it in a direction *opposite* to the direction of the current in it. If V is the applied potential difference, E the induced *e.m.f.*, and R the resistance of the armature, current in it is equal to $\frac{V-E}{R}$. At the start the

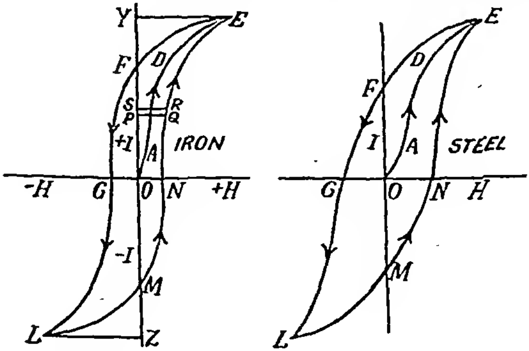
driving torque exerted by the motor is greater than the load torque. This accelerates the motor, the rate of change of flux and the induced *e.m.f.* increase, and current decreases until the driving torque, which is proportional to the armature current and the strength of the magnetic field, becomes equal to the load torque.

The electric motor is self-adjusting. When the load increases, its speed diminishes, the induced *e.m.f.* falls, and armature current increases to make the driving torque equal to the loading torque. Similarly when the load decreases, its speed and induced *e.m.f.* increase and armature current and driving torque decrease until equilibrium is attained.

Q. 200. What is meant by 'hysteresis' and by a cycle of magnetisation? Draw the hysteresis curves for (*a*) soft iron, and (*b*) steel.

Prove that the area of the H-B curve denotes 4π times the energy dissipated per c.c. of the magnetic substance during each cycle of magnetisation. (*Calcutta, 1933*)

Ans. **Hysteresis and Cycle of Magnetisation.** A piece of iron is magnetised by placing it in a solenoid in which current is passed. Its intensity of magnetisation I is measured, and by changing the strength of current a curve is drawn between the magnetising field H and I (Fig. 147). First H is increased until saturation condition is obtained and there is no further increase in I. This change is indicated by OADE. On decreasing H, the old curve is *not* retraced but a *new* curve EF is obtained, showing that the value of I



(*a*) (*b*)

during the decrease of H is greater than during its increase. When H is reduced to zero, there is a good deal of *residual magnetism* left, and this *retentivity* of the material is measured by OF.

On reversing the current and increasing the value of H in the opposite direction, I decreases along FG. At G the whole of the residual magnetism is removed, and the value of OG is a measure of the *coercivity* of the material. The increase of current is continued to its former maximum value in this direction to obtain the curve GL. The current is decreased to zero; it is reversed, and increased to its former value, and during this process the curve obtained is LMNE. This change of magnetisation from its maximum value in one direction to its maximum value in the opposite direction and back to its maximum value in the first direction is called one **cycle of magnetisation.** Then on repeating this process, curve EFGLMN is obtained over and over again.

This curve shows that the magnetic condition of iron tends to persist, or opposes any change. For the same value of H, I has two different values, one when H is increasing and the other when H is decreasing. If I has the same direction in the two cases, it is greater when H is decreasing than when it is increasing. This *lagging* of magnetisation behind the magnetising force is called **hysteresis.** The B-H curves are similar to the I-H curves, but at the saturation stage increase in B does *not* stop and is equal to the increase in H. Thus this part of the B-H curves, unlike the I-H curves, slopes upward and is not parallel to the H-axis.

Soft iron and Steel. Fig. 147 (*a*) is the H-I curve for soft iron while that for steel is shown in Fig. 147 (*b*) on the same scale. The two are similar in shape but the hysteresis loop of steel differs from that of soft iron in some respects. In the case of steel the saturation value of I and residual magnetism are lower, but the coercive force is greater, than for soft iron, and the area of the hysteresis loop of steel is greater than that of soft iron, which means that the loss of energy per cycle per unit volume of steel is greater than that of soft iron.

Dissipation of Energy. Suppose the molecular magnets, each of magnetic moment M, are inclined to the magnetising

field H at an angle θ radian. Their components $M\cos\theta$ along the magnetising field are in the same direction and their resultant is equal to $\Sigma M\cos\theta$, while their components $M\sin\theta$ perpendicular to the magnetising field *cancel* out each other. Therefore, if we consider a *unit* volume of the material,

$$I = \Sigma M\cos\theta, \quad \ldots\ldots\ldots \quad (1)$$

as I, the intensity of magnetisation, is equal to the magnetic moment per unit volume.

$$\therefore\ dI = -\Sigma M\sin\theta.\, d\theta \quad \ldots\ldots \quad (2)$$

This shows that as θ is *decreased* by $d\theta$, I *increases* by dI, and the rate of change of I with respect to θ is proportional to $\sin\theta$. But the restoring couple acting on a magnet of moment M and placed at θ to a magnetic field of strength H is equal to $MH\sin\theta$, and work dW done on it in increasing the angle by $d\theta$ is equal to $MH\sin\theta.d\theta$, while that done *on* it in *decreasing* it by $d\theta$ is equal to $-MH\sin\theta.d\theta$. Therefore in the case of a unit volume of the material in decreasing the angle of inclination of the molecular magnets by $d\theta$, and thereby increasing its intensity of magnetisation by dI,

$$dW = -\Sigma MH\sin\theta.\, d\theta$$

$$= H.dI \quad \ldots\ldots\ldots\ldots \quad (3)$$

In Fig. 147 (a), if OP$=$I, PQ$=$H, and PS$=dI$, $H.dI$ is given by the area PQRS. Draw EY and LZ perpendiculars on YOZ. Therefore in one cycle work done *on* a unit volume of the material is given by the area NEYFGLZM, while that done by the material (energy returned by it) is represented by the sum of the areas FEY and LZM, so that the net amount of work done on the material, or the energy dissipated and not returned, is given by the area of the H-I loop.

But $$B = H + 4\pi I$$

and $$dB = dH + 4\pi dI$$

or $$\int H dB = \int H \cdot dH + \int H . 4\pi dI$$

$$= 4\pi \int H \cdot dI$$

as $\int H \cdot dH$ (area of H-H loop) is equal to zero. Thus the area of the H-B loop is 4π times the area of the H-I loop,

and, therefore, the amount of energy dissipated per unit volume per cycle is equal to the area of the H-B loop divided by 4π.

Second Method. The energy of a unit volume of magnetic field of flux density B in a medium of permeability μ is equal to $\frac{B^2}{\mu 8\pi}$. If the flux density is increased by a *very small* amount δB, the new value of this energy is equal to $\frac{(B+\delta B)^2}{\mu 8\pi}$,

and

$$\text{Increase of energy} = \frac{(B+\delta B)^2}{\mu.8\pi} - \frac{B^2}{\mu.8\pi}$$

$$= \frac{B^2 + 2B.\delta B + \delta B^2 - B^2}{\mu.8\pi}$$

$$= \frac{B.\delta B}{\mu.4\pi}$$

$$= \frac{H.\delta B}{4\pi}, \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as δB^2 is negligible and $B = \mu H$. Here $H.\delta B$ is equal to the area lying between the H-B hysteresis curve, axis of B, and the abscissae at B and $B+\delta B$. Reasoning as before the value of $H.\delta B$ for one cycle is equal to the area of the H-B hysteresis loop, and the loss of energy per unit volume per cycle is equal to the area of this loop divided by 4π.

Q. 201. Define magnetic permeability and Susceptibility. How are the two related ?

Describe an experimental method of getting the hysteresis curve for iron and steel.

Point out briefly the importance of these curves in the construction of dynamos. (*Punjab, 1936*)

Ans. **Susceptibility and Permeability.** When an iron rod is placed in a magnetic field with its axis parallel to the field, it becomes magnetised, and the magnetic field in it is due to the magnetising force H as well as the magnetism induced in it. If the intensity of magnetisation produced is I, the total number of tubes of force per unit area taken perpendicular to them is called induction or flux density B, and is given by

$$B = H + 4\pi I, \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

as 4π tubes of force are given out from a unit pole.

Dividing (1) throughout by H,

$$\frac{B}{H}=1+4\pi\ \frac{I}{H}$$

or $$\mu=1+4\pi\kappa, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where μ and κ are equal to B/H and I/H respectively. Thus the **permeability** μ of a magnetic substance is equal to the *total* magnetic induction produced in it divided by the magnetising force, while its **susceptibility** is equal to the ratio of the intensity of magnetisation produced to the magnetising force producing it.

Magnetometer Method. A *very long, thin* rod, AB, of iron or steel of length l is placed vertically well within and along the axis of a solenoid, S, which is connected with another coil, C.C., a reversing key, R.K., galvanometer, G, battery, B, and a variable resistance, R (Fig. 148). The rod is placed to the east or west of a magnetometer needle N, which is, in level with the upper end of the needle and the axis of the coil C.C.

On passing current through the solenoid the rod is magnetised, and the magnetometer shows a deflection θ. If the intensity of magnetisation of the rod is I, and a its cross-section area, its pole strength is equal to Ia.

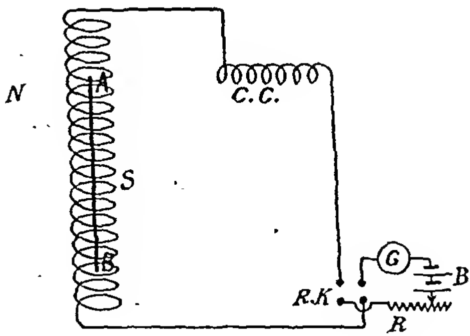


Fig. 148.

Intensity at N due to pole $A=\frac{Ia}{d^2}$ along AN, ... (3)

where AN is equal to d and the pole at A is supposed to be *a*

north pole. The lower pole at B (south pole) produces intensity $\frac{Ia}{(d^2+l^2)}$ at N along NB and its effective horizontal component along NA is equal to $\frac{Ia}{(d^2+l^2)} \times \frac{d}{(d^2+l^2)^{\frac{1}{2}}}$, while the vertical component perpendicular to NA is overcome by the suspension or pivot of the magnetometer needle.

$$\therefore \text{ Resultant intensity at N} = \frac{Ia}{d^2} - \frac{Iad}{(d^2+l^2)^{3/2}}$$

$$= Ia \left\{ \frac{(d^2+l^2)^{3/2} - d^3}{d^2(d^2+l^2)^{3/2}} \right\} \text{along AN} \qquad (4)$$

If h is the horizontal component of the earth's magnetic field,

$$Ia \left\{ \frac{(d^2+l^2)^{3/2} - d^3}{d^2(d^2+l^2)^{3/2}} \right\} = h \tan\theta$$

or
$$I = \frac{h \tan\theta \, d^2(d^2+l)^{3/2}}{a[(d^2+l^2)^{3/2} - d^3]} \quad \cdot \cdot \cdot \cdot \qquad (5)$$

The magnetising field H for the rod is equal to $4\pi n$C, where n is the number of turns of the solenoid per unit length and C is the strength of current. The current is increased step by step to its maximum value when the saturation condition of the magnetisation of the rod is reached. Then it is decreased to zero step by step: reversed and increased to its former maximum value, again decreased to zero, reversed, and increased to its maximum value. At each step the values of I and H are calculated and a curve is drawn, taking H as abscissa and I as ordinate. See Q. 200. Using equation (1), the value of B is calculated, and H—B curve is drawn.

The deflection of the needle is not only due to the rod AB but the solenoid also. To overcome this difficulty, a compensating coil, C.C., is used, Its axis is in level with A and N, and *before* the rod is introduced its position is so adjusted that when current is passed through it and the solenoid, the magnetometer needle shows no deflection.

The rod is also magnetised by the vertical component of the earth's field. To eliminate its effect, another coil is wound over the solenoid and current is passed through it so that the magnetisation of the rod due to it is equal and opposite to that

due to the vertical component of the earth's magnetic field. For this condition to be true, when *no* current is passed through the main solenoid and the rod is placed in it, current in the outer coil is adjusted until the magnetometer needle shows no deflection.

Importance of the Hysteresis Curves. See Q. 200 and Fig. 61. *When a piece of iron is carried through a cycle of* magnetisation, for every value H of the magnetising field, its intensity of magnetisation I has *two* values, one when H is increasing and the other when H is decreasing. The amount of energy used in magnetising it is not completely returned when it is demagnetised, and the loss of energy for its unit volume for each cycle is represented by the area of its I—H curve. Owing to its rapid rotation, the iron core of a dynamo has to go through these magnetic changes very quickly, and a good deal of energy is lost which appears as heat. To decrease the loss, the core is made of annealed iron, as it has high permeability but *low* hysteresis loop.

Q. 202. Obtain the relation between current and voltage in an inductive circuit without capacity when connected to an harmonically alternating current supply. Also obtain an expression for the power developed in such a circuit. (*Punjab, 1937*)

Ans. **Relation between Current and Applied E.M.F.** Let a current C given by

$$C = C_0 \sin 2\pi nt, \qquad \qquad (1)$$

where C_0 is its maximum value and n its frequency, pass through a circuit containing resistance R and inductance L. As the current is changing, at any time induced *e.m.f.* $L\dfrac{dC}{dt}$ is produced and is *opposite* to the applied electromotive force E at that moment, so that the effective *e.m.f.* for overcoming resistance R is equal to $E - L\dfrac{dC}{dt}$.

$$\therefore\ E - L\frac{dC}{dt} = CR$$

or

$$E = CR + L\frac{dC}{dt} \qquad \qquad (2)$$

But $$C = C_0 \sin 2\pi nt$$

and $$\frac{dC}{dt} = 2\pi n C_0 \cos 2\pi nt$$

Putting the values of C and $\frac{dC}{dt}$ in (2), we get

$$\begin{aligned}
E &= RC_0 \sin 2\pi nt + L 2\pi n C_0 \cos 2\pi nt \\
&= C_0(R \sin 2\pi nt + 2\pi n L \cos 2\pi nt) \\
&= C_0 \sqrt{R^2+4\pi^2n^2L^2}\left(\frac{R.\sin 2\pi nt}{\sqrt{R^2+4\pi^2n^2L^2}} + \frac{2\pi nL \cos 2\pi nt}{\sqrt{R^2+4\pi^2n^2L^2}}\right) \\
&= C_0 \sqrt{R^2+4\pi^2n^2L^2}\,(\cos\phi.\sin 2\pi nt + \sin\phi \cos 2\pi nt), \\
&= C_0 \sqrt{R^2+4\pi^2n^2L^2}.\sin(2\pi nt+\phi)
\end{aligned}$$

where $\tan\phi = \frac{2\pi nL}{R}$, $\sin\phi = \frac{2\pi nL}{\sqrt{R^2+4\pi^2n^2L^2}}$, and

$$\cos\phi = \frac{R}{\sqrt{R^2+4\pi^2n^2L^2}}.$$

As the greatest value of $\sin(2\pi nt+\phi)$ is 1, maximum value E_0 of the applied *e.m.f.* is equal to $C_0\sqrt{R^2+4\pi^2n^2L^2}$, so that

$$C_0 = \frac{E_0}{\sqrt{R^2+4\pi^2n^2L^2}} \quad \ldots\ldots\ldots \quad (3)$$

and $$E = E_0 \sin(2\pi nt+\phi) \quad \ldots\ldots\ldots \quad (4)$$

Comparing this with (1), we find that the phase of the applied *e.m.f.* is *ahead* of that of the current by angle ϕ, whose tangent is equal to $\frac{2\pi nL}{R}$. This lag of current behind the applied *e.m.f.* is due to the production of the induced *e.m.f.*, which opposes the change of current and retards both its rise and fall. When the applied *e.m.f.* is maximum, current is still increasing and reaches its maximum value *after* some time.

Putting in (1) the value of C_0 obtained in (3), we get

$$C = \frac{E_0 \sin 2\pi nt}{\sqrt{R^2+4\pi^2n^2L^2}}. \quad \ldots\ldots\ldots \quad (5)$$

Power. As both current and electromotive force vary in an alternating circuit, their product, that is, power, also changes, and at any time

Instantaneous power

$$= C_0 \sin 2\pi nt \times E_0 \sin(2\pi nt + \phi)$$

$$= \frac{C_0 E_0}{2} \{\cos(2\pi nt - 2\pi nt - \phi) - \cos(2\pi nt + 2\pi nt + \phi)\}$$

$$= \frac{C_0 E_0}{2} \{\cos\phi - \cos(4\pi nt + \phi\} \quad \ldots \ldots \quad (6)$$

The first factor $\cos\phi$ remains the same throughout, but the second factor $\cos(4\pi nt + \phi)$ changes with time, and in half a period, equal to $\frac{1}{2n}$, $(4\pi nt + \phi)$ changes by 2π, so that the *average* value of its cosine is zero. Thus for any number of half periods, the average value of the second factor is zero, and

$$\therefore \text{Mean power} = \frac{C_0 E_0}{2} \cos\phi \quad \ldots \ldots \quad (7)$$

This shows that the mean power developed in an alternating current circuit depends not only on the maximum strength of current and applied *e.m.f.* but also on the phase angle ϕ by which current lags behind the applied *e.m.f.* For this reason $\cos\phi \left\{= \frac{R}{(R^2 + 4\pi^2 n^2 L^2)^{\frac{1}{2}}}\right\}$ is called *power factor*.

It is shown in Q. 203 that the R.M.S. (root mean square) values of current and *e.m.f.* are equal to $\frac{C_0}{\sqrt{2}}$ and $\frac{E_0}{\sqrt{2}}$ respectively. If they are denoted by C and E,

$$\text{Mean Power} = \frac{C_0 E_0}{2} \cos\phi$$

$$= \frac{C_0}{\sqrt{2}} \cdot \frac{E_0}{\sqrt{2}} \cdot \cos\phi$$

$$= CE \cos\phi \quad \ldots \ldots \quad (8)$$

Q. 203. Distinguish between the mean value and the root mean square value of an alternating current, and find the relation between them. (*Punjab, 1932*)

Ans. **Mean Value.** The magnitude and direction of alternating current change according to the sine law, and its value C at any time t is given by

$$C = C_0 \sin 2\pi nt \quad . \quad . \quad (1)$$

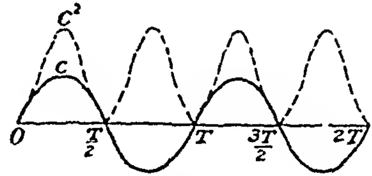


Fig. 149.

where C_0 is its maximum value and n is its frequency. This variation of current with time is shown in Fig. 149, where T is the time period of the current and is equal to $1/n$. For *half* a period, from 0 to T/2 its

$$\text{Mean value} = \frac{1}{\frac{T}{2}} \int_0^{T/2} C.dt$$

$$= \frac{2}{T} \int_0^{T/2} C_0 \sin 2\pi nt.dt$$

$$= \frac{2C_0}{T} \int_0^{T/2} \sin\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{2C_0}{T}\left[-\frac{T}{2\pi}\cos\left(\frac{2\pi t}{T}\right)\right]_0^{\frac{T}{2}}$$

$$= \frac{C_0}{\pi}(\cos 0 - \cos\pi)$$

$$= \frac{2C_0}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Root Mean Square Value. For the next or last half period its mean value is the same but in the *opposite* direction, so that for an integral number of full periods its mean value is *zero*. Therefore a measuring instrument, whose deflection is proportional to the current, does not indicate any deflection when alternating current is passed through it, as the period of the current is very small. Still its passage is accompanied by the consumption of electrical energy, which is proportional to the product of current and potential difference, or resistance and square of current, or square of potential difference and the reciprocal of resistance, and the square root of the

mean value of the *square* of current is called its **root mean square** (R.M.S.), **effective,** or **virtual** value. It is equal to the value of direct current of constant magnitude which passing through a *non-inductive resistance* develops the *same* power as the given alternating current.

In Fig. 149, the dotted curve shows the relation between the square of current C and time t. The mean value of the ordinates of this curve is the same for any integral number of half periods, and is given by

$$\text{Mean square value} = \frac{2}{T}\int_0^{T/2} C_0^2 \sin^2\left(\frac{2\pi t}{T}\right)dt$$

$$= \frac{2C_0^2}{T}\int_0^{T/2}\left\{\frac{1-\cos(4\pi t/T)}{2}\right\}dt$$

$$= \frac{C_0^2}{T}\left[t - \frac{T}{4\pi}\sin\left(\frac{4\pi t}{T}\right)\right]_0^{\frac{T}{2}}$$

$$= \frac{C_0^2}{T}\cdot\frac{T}{2}$$

$$= \frac{C_0^2}{2} \quad \ldots\ldots\ldots \quad (3)$$

$$\therefore \text{ Root mean square value} = \frac{C_0}{\sqrt{2}} \quad \ldots\ldots\ldots \quad (4)$$

Dividing (2) by (4), we get

$$\frac{\text{Mean value}}{\text{Root mean square value}} = \frac{2C_0}{\pi}\times\frac{\sqrt{2}}{C_0} = 0{\cdot}9001.$$

Q. 204. Explain what is meant by (*a*) impedance, (*b*) reactance of a circuit, and (*c*) virtual voltage? Why should alternating currents be used instead of direct currents for long-distance transmission?

(*Punjab, 1933*)

Ans. (*a*) **Reactance.** In an alternating current circuit; due to change of current, induced *e.m.f.* is produced and it opposes the applied *e.m.f.* The effective *e.m.f.* is smaller than the applied *e.m.f.*, and the current produced is weaker than if the circuit has no inductance, or the resistance of the

circuit is greater than its value for direct current of fixed strength. The additional resistance is proportional to the inductance of the circuit and frequency n of the alternating current, and is equal to $2\pi n L$. It is called the **reactance** of the circuit.

(*b*) **Impedance.** Due to the induced *e m f.* produced, current in an alternating current circuit is not in phase with, but lags behind, the applied *e.m.f.*, and the total resistance of the the circuit, called **impedance**, is equal to $\sqrt{R^2+4\pi^2n^2L^2}$, where R is its ohmic resistance. If the maximum value of the applied *e.m.f.* is E_0, the maximum strength of current is equal to $\dfrac{E_0}{\sqrt{R^2+4\pi^2n^2L^2}}$.

(*c*) **Virtual Voltage.** The magnitude and direction of an alternating *e.m.f.* change according to the sine law. Its average value for one period is zero, but some electrical energy is converted into some other form of energy. If the *e.m.f.* is applied to a resistance, electrical energy converted into heat is proportional to the square of current or *e.m.f.*, and the square root of the mean value of the square of the voltage for any integral number of half periods is called **virtual** or **root mean square** value of the voltage. It is equal to the value of a fixed voltage, applied to a non-inductive resistance, in which the transformation of energy is at the *same* rate as with the alternating voltage. If E_0 is the maximum value of the alternating *e m.f*, its virtual value is equal to $\dfrac{E_0}{\sqrt{2}}$.

Transmission of Electrical Energy. When electrical energy has to be transmitted to a distant place, the resistance of the line wires is appreciable, and a good deal of energy is dissipated in the form of heat, the rate of loss of energy being given by the product of the resistance and the *square* of the current. Moreover, there is sufficient fall of potential along the line—product of resistance and current—and, therefore, the voltage of the current at the other station is much smaller than at the transmitting station. In order to decrease this loss, the line resistance may be decreased by using *thick* wires, but in that case the initial cost becomes high, due to the greater mass of the line wires used and the stronger poles required.

The electric power transmitted depends not only on the current but also on the potential difference, and is equal to their product. Thus for transmitting electrical energy at the rate of 11 kilowatts, 50 amperes current may be sent at 220 volts, or 5 amperes current may be sent at 2200 volts. As the current in the first case is 10 times that in the second case, the line loss in the second case is *reduced to one-hundredth*. Moreover, much thinner wires are required in the second case. Therefore *it is economical to transmit electrical energy at high potential and low current.*

For this purpose, transformers are employed, both at the transmitting station and the receiving station, but alternating current *alone* can work them. As the strength of a direct current does not change, it cannot work a transformer, and its voltage cannot be changed. At the transmitting station the primary coil of a '*step up*' transformer is connected with th A. C. generator, while its secondary coil is joined with the line wires. As high voltage is not safe for domestic use, the voltage is brought down to about 220 at the receiving station; the line wires are connected with the primary of a 'step down' transformer, while energy is taken from its secondary. For example, the A. C. generator may supply current at 440 volts; by using a 'step up' transformer of turns ratio 1 : 10, this voltage may be raised to 4400. At the receiving station this may be brought down to 220 volts by a "step down" transformer of turns ratio 20 : 1. For very long distance transmission, this transformation may be carried out in many stages, and the line voltage may be very high. For example, electrical energy is being transmitted from Jogindar Nagar to Lahore at the tremendous voltage of 132000.

Q. 205. Show that in an A. C. circuit $C_v = E_v \cos\phi/R$, where C_v and E_v are the virtual current and e. m. f. respectively and ϕ is the 'angle of lag.'

An alternating e.m.f. of 200 volts with frequency of 60 cycles per second is applied to a circuit having 100 ohms resistance and 0·283 henries inductance. Find C_v and ϕ. (*Bombay, 1933*)

Ans. See Q. 202 for showing that the average power in an alternating current circuit is equal to $C_vE_v\cos\phi$. But

according to the definition of virtual current, it is equal to C_v^2R, where R is the resistance of the circuit.

$$\therefore \quad C_v^2R = C_vE_v \cos\phi$$

or

$$C_v = \frac{E_v \cos\phi}{R}$$

Problem. In an alternating current circuit of resistance R, inductance L, and frequency n,

$$\text{Impedance} = \sqrt{R^2+4\pi^2n^2L^2}$$

$$\text{Resistance} = 100 \text{ ohms.}$$

$$\text{Inductance} = 0{\cdot}283 \text{ henries}$$

$$\text{Frequency} = 60 \text{ per sec.}$$

$$\text{Impedance} = \sqrt{100^2+4\times 3{\cdot}142^2\times 60^2\times 0{\cdot}283^2} \text{ ohms.}$$

$$\therefore \quad \cos\phi = \frac{\text{Resistance}}{\text{Impedance}}$$

$$= \frac{100}{\sqrt{100^2+4\times 3{\cdot}142^2\times 60^2\times 0{\cdot}283^2}}$$

$$= 0{\cdot}6837$$

or

$$\phi = 45{\cdot}89^\circ$$

and

$$C_v = \frac{200\times 0{\cdot}6837}{100}$$

$$= 1{\cdot}367 \text{ ampere.}$$

Q. 206. Explain the action and use of choke coils. Discuss the question of energy losses in them and compare these with the corresponding losses in rheostats or control resistances. (*Punjab, 1928*)

Ans. **Choke Coils.** See Q. 202 for showing that when an alternating potential difference of virtual value E of frequency n is applied to a coil of resistance R and inductance L to produce current of virtual value C, the average power consumed in it is equal to CE $\cos\phi$, where ϕ is the angle of lag and its cosine is equal to $\dfrac{R}{\sqrt{R^2+4\pi^2n^2L}}$. If its resistance R is very small as compared with its reactance $2\pi n$L, angle ϕ is very large, and then, $\cos\phi$ being very small, power

consumed in it is *very low*. Due to large self-inductance L or high frequency n, the induced *e.m.f.* is nearly equal and opposite to the impressed *e.m.f.*, and only a very small part of the latter is effective. In the *extreme* case where R is zero, ϕ is equal to $\pi/2$ and $\cos\phi$ is equal to zero, so that *no* power is consumed in the coil. In this case current in the coil is said to be *idle* or *wattless* as *no* electrical energy is spent in it. Such a coil is called a **choke coil** and is used for regulating alternating current and voltage in a circuit, or to prevent high frequency currents entering it.

A soft iron ring is broken into two semi-circles and a coil of *thick* wire and *many* turns is wound over one of them. The two halves are mounted close to each other, leaving air gaps between their ends. By changing the width of the air gaps, the inductance of the coil is changed, and thereby its reactance $2\pi n\mathrm{L}$ is adjusted.

Choke Coils and Rheostats. When direct current is to be passed through an appliance and the available potential difference is too much for it, a variable resistance, called rheostat or control resistance, is placed in series with it, and its value is adjusted so that the required current is obtained. In this case a good deal of electrical energy is wasted in overcoming the resistance of the rheostat. There is no other way of regulating direct current.

If the same appliance is to be worked on an alternating current supply, a choke coil is used for regulating current in it. By changing its inductance the induced *e.m.f.* produced in the circuit is changed to adjust the effective *e.m.f.* and thereby current of required strength is passed through the appliance. If its resistance is made exceedingly low, its power factor $\cos\phi\left(=\dfrac{\mathrm{R}}{\sqrt{\mathrm{R}^2+4\pi^2 n^2\mathrm{L}^2}}\right)$ approaches zero, and electrical energy wasted in it is negligible. If instead of a choke coil a resistance is used to regulate the current, energy is wasted in it as in the case of direct current.

Suppose an arc lamp which requires 10 amps. at 40 volts is put on an alternating current supply of 50 volts and 50 cycles per sec.

(1) *Choke coil used.*

$$\text{Resistance of arc lamp} = \frac{40}{10} = 4 \text{ ohms.}$$

$$\text{Impedance} = \frac{50}{10} = 5 \text{ ohms.}$$

$$\therefore \quad \text{Reactance} = \sqrt{5^2 - 4^2} = 3 \text{ ohms.}$$

and

$$\cos\phi = \frac{4}{5}$$

$\therefore$ Power consumed by the circuit $= 50 \times 10 \times \frac{4}{5}$

$= 400$ watts

and " " " arc lamp $= 40 \times 10 = 400$ watts.

Thus in the extreme case where the resistance of the choke coil is zero and, therefore, its introduction does not add to the resistance of the circuit, power used in the whole circuit is the *same* as that developed in the lamp, that is, no energy is wasted in the choke. In actual practice the choke has some little resistance and some energy is wasted in it but it is very low as compared with the case of a control resistance.

(2) *Rheostat used.*

$$\text{Total resistance} = \frac{50}{10} = 5 \text{ ohms.}$$

$$\therefore \quad \text{Rheostat resistance} = 5 - 4 = 1 \text{ ohm.}$$

and power wasted in the rheostat $= 10^2 \times 1 = 100$ watts.

Power developed in the arc lamp is equal to 400 watts, so that $20\ \% \left(= \frac{100 \times 100}{400 + 100}\right)$ of the total power consumed is wasted.

Q. 207. Explain the action of a choke coil.

A choke having a resistance of 5 ohms and inductance 2 henry is connected to a 1,000 volt 60 cycle A. C. supply. Calculate the current in the circuit.

(*Punjab, 1935*)

Ans. Choke coil. See Q. 206.

Problem.

$$\text{Resistance} = 5 \text{ ohms}$$
$$\text{Inductance} = 2 \text{ henry}$$
$$\text{Frequency} = 60 \text{ cycles/sec.}$$
$$\therefore \quad \text{Reactance} = 2 \times 3{\cdot}142 \times 60 \times 2 \text{ ohms.}$$
and
$$\text{Impedance} = \sqrt{5^2 + (3{\cdot}142 \times 240)^2} \text{ ohms.}$$
$$\text{Voltage} = 1000$$
$$\therefore \quad \text{Current} = \frac{1000}{\sqrt{5^2 + (3{\cdot}142 \times 40)^2}} = 1{\cdot}326 \text{ ampere.}$$

Q. 208. Describe the construction and working of an alternator. Explain the meaning of two-phase and three-phase currents and the advantage of the last type.

Ans. **Alternator.** See Q. 198. The ends of the armature instead of being connected to the two commutator segments are joined to two separate rings, and the brushes of the external circuit press on these rings, so that the current in the external circuit also is alternating.

Usually commercial generators have a stationary armature whose ends are *permanently* connected to the line wires, and the field magnets rotate inside it. This enables very high voltage to be generated, as there is no danger of sparking and the coils can be well insulated from each other. Many conductors are joined in series, and the resultant *e.m.f.* is the vector sum of the *e.m.fs.* in the individual conductors.

The field poles consist of coils with iron cores and they are excited by *direct* current generated by a small separate *d.c.* dynamo. As an alternating current of about 50 cycles per second is suitable for commercial purposes, an alternator with two poles only has to be rotated very rapidly. To overcome this difficulty, *many* pairs of poles, alternately north and south, are used. The *e.m.f.* induced in conductors under north poles is opposite to that induced in conductors under south poles, and, therefore, the adjacent coils are wound in *opposite* directions so that the direction of the induced current is the same. The period of a cycle is the time in which one north pole takes the position of the *next* north pole, and the direction of current in a conductor is reversed when it is midway between two consecutive poles.

Two-Phase Currents. If two coils are mounted at *right angles* to each other on the same core, the rate of change of flux through one is maximum when it is minimum through the other, and *vice versa*, so that the phase difference between the two induced currents is 90°. In a two-phase alternator another series of coils is used *midway* between the coils of the first set, and two more terminals are added.

Three-Phase Currents. Here *three* exactly similar but independent coils are mounted on the same core making *equal* angles with one another. The three currents induced have the *same* virtual value, but differ in phase by 120° from each other (Fig. 150). When one current has its maximum positive value, the second current is negative and its magnitude is decreasing, while the negative value of the third current is increasing. At any instant the algebraic sum of the three currents is *zero*, that is, the algebaric sum of any two is *equal* and *opposite* to the third.

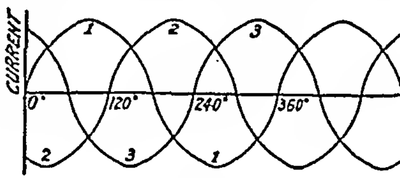


Fig. 150.

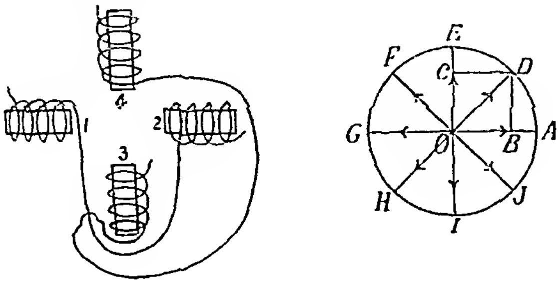
In transmitting electrical energy current is sent by a generator along one line wire and returns to it along a second line wire, and at any instant these currents are equal in magnitude but are flowing in opposite directions. As in a three-phase alternator each current is equal and opposite to the algebraic sum of the other two, the lead wire of one can serve as a return for the other two. Thus instead of using six line wires for the three currents, only *three* line wires are employed. One end of each armature coil is connected to one of the line wires while their other ends are connected together. For this economy of line wires three-phase alternators are generally used.

Q. 209. Explain how a rotating magnetic field can be produced. Mention an important application.

(*Punjab, 1936*)

Ans. **Rotating Magnetic Field.** In Fig. 151 (*a*), 1 and 2 are the poles of one electromagnet and 3 and 4 are the poles of another electromagnet. Both are excited by alternating currents of the same virtual strength but they have a phase difference of 90°. When one current is increasing the other

is decreasing, and when one has its maximum value the other has its minimum value. The strength of the magnetic field produced by each current changes with it, and the resultant of the two perpendicular magnetic fields has a *constant* value,



(*a*) (*b*)

Fig. 151.

but its direction changes at a uniform rate and it completes one rotation in one time period of the alternating currents used.

When the magnetic field due to the first electromagnet has its maximum value from pole 1 towards pole 2, the magnetic field of the second electromagnet has zero intensity, and the resultant of the two is represented by OA [Fig. 151(*b*)]. Then the strength of the first magnetic field decreases, while that of the second increases, from pole 3 towards pole 4. After one-eighth of the period of the alternating currents used, the strength of the first magnetic field is reduced to $\frac{1}{\sqrt{2}}$ of its maximum value and is represented by OB, and the strength of the second increases to $\frac{1}{\sqrt{2}}$ of its maximum value OE and is represented by OC. Their resultant is represented by OD, which is *equal* to the maximum strength of either magnetic field.

After a quarter period from the start, the intensity of the first magnetic field is reduced to zero, while that of the second attains its maximum value, and their resultant is represented by OE. Then the strength of the first magnetic field increases, in the opposite direction, that is, from pole 2 towards pole 1 and that of the second decreases, but the magnitude of their resultant remains constant and its direction changes anticlockwise. At the end of three-eighth of the period, the intensity

of the resultant magnetic field is represented by OF, and after half a period by OG, which is equal and opposite to OA. After five-eighth, six-eighth, seven-eighth, and one period the resultant magnetic field is represented by OH, OI, OJ, and OA respectively, so that the period of the rotating magnetic field is equal to that of the alternating currents used.

Application. This is used in the construction of **induction motors.** Three-phase currents are also used to produce a rotating magnetic field. The two electromagnets may be excited by two-phase currents, or a single-phase current, one part of which is passed through a non-inductive resistance and the second through an inductance of negligible resistance to get a phase difference of 90°. If a cage formed of copper bars is placed in the space between the poles of the two electromagnets, the rotating magnetic field induces current in it. According to Lenz's law the direction of the induced current is such that it opposes the cause which produces it. As the cause of the induced current is the relative motion between the magnetic field and the cage, it begins to rotate in the direction of rotation of the magnetic field to oppose he relative motion.

Q. 210. Explain the working of a transformer, and find a relation between the potential difference between the ends of the secondary coil and the current in it.

Ans. A **transformer** consists of a *closed laminated soft iron* ring C (Fig. 152) over which are wound a primary coil P, P of n_1 turns and resistance R_1 and a secondary coil S, S of n_2 turns and resistance R_2. When the primary coil is connected to a source of *alternating current,* the strength of the current varies from its maximum value in one direction to its maximum value in the opposite direction; the iron ring is magnetised, first in one direction and then in the opposite direction, and the magnetic flux in it varies in accordance with these changes. As the number of magnetic lines of force passing through the secondary coil changes, *e.m.f.* is *induced* in it. When the current in the

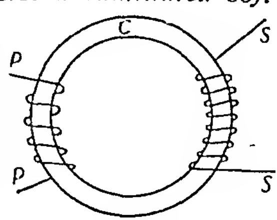


Fig. 152.

primary coil is increasing, the induced *e.m.f.* in the secondary is in the *opposite* direction, and when the primary current decreases, the secondary *e.m.f.* is in the *same* direction. Therefore the induced *e.m.f.* in the secondary is also *alternating*, and its period is the same as that of the primary current.

Due to change of flux, *e.m.f.* is induced in the primary also and is opposite to the applied *e.m.f.* at every stage. The ratio of the induced *e.m.f.* in the secondary coil to the induced *e.m.f.* in the primary is the same as the ratio of their respective *number of turns*, as there is no leakage of the lines of force between the two coils and, therefore, electromotive force induced in each turn of the two coils is the same.

$$\frac{e.m.f. \text{ in the secondary}}{e.m.f. \text{ in the primary}} = \frac{\text{no. of turns in the secondary}}{\text{no. of turns in the primary}} = \frac{n_2}{n_1}$$

As the energy of the induced current in the secondary is *derived from that of the primary current*, through the changing magnetic field, the product of current and *e.m.f.* is the same for both the coils, and, therefore, the secondary current is *inversely* proportional to the secondary *e.m.f.* The transformer is called "step up" or "step down" according as the number of the secondary turns is greater or smaller than the primary turns. The coil of greater number of turns is of thinner wire than the other.

If C_1 and C_2 are the currents in the primary and secondary coils respectively and E_1 and E_2 are the corresponding induced *e.m.fs.*,

$$E_1C_1 = E_2C_2$$

or

$$\frac{C_1}{C_2} = \frac{E_2}{E_1} = \frac{n_2}{n_1}$$

If the secondary is open, no current passes through it and there is a *very small* current in the primary for magnetising the core. As the resistance of the primary is very small, the back *e.m.f.* induced in it is very nearly equal to the applied potential difference V_1.

When the secondary is closed, current passes in it in a direction *opposite* to that of the primary current. The magnetic flux due to the secondary current is *opposite* to that due to the

primary current and tends to demagnetise the core (*Lenz's law*). Owing to decrease in flux, back electromotive force E_1 in the primary decreases and current in it increases until the resultant flux almost regains its original value. Thus the transformer works *automatically*. If the secondary is closed, energy of the primary circuit is used according to the load in the secondary, but no energy is withdrawn from the primary, except that used for magnetizing the core, when the secondary is open.

The fall of potential C_1R_1 in the primary circuit is equal to the difference between the applied potential difference V_1 and the induced back electromotive force E_1, so that

$$V_1 = E_1 + C_1R_1$$

or

$$E_1 = V_1 - C_1R_1$$

In the secondary the induced electromotive force E_2 produces fall of potential C_2R_2 in it and the available potential difference V_2 between its ends is given by

$$V_2 = E_2 - C_2R_2$$

or

$$E_2 = V_2 + C_2R_2$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2 + C_2R_2}{V_1 - C_1R_1} = \frac{n_2}{n_1}$$

or

$$V_2 + C_2R_2 = \frac{n_2}{n_1} V_1 - \frac{n_2}{n_1} C_1R_1$$

$$= \frac{n_2}{n_1} V_1 - \frac{n_2^2}{n_1^2} C_2R_1 \qquad \left[\because C_1 = \frac{n_2C_2}{n_1}\right]$$

or

$$V_2 = \frac{n_2}{n_1} V_1 - C_2\left(\frac{n_2^2}{n_1^2} R_1 + R_2\right)$$

Q. 211. Derive the dimensions of the units of (*a*) current, (*b*) potential difference in the two systems of units.

Discuss the question of the difference of the dimensions in the two systems indicating any important result to which it leads. (*Bombay, 1931*)

Ans. **Electrostatic Units.** The force F between two charges Q_1 and Q_2 placed at a distance d from each other

in a medium of specific inductive capacity K is given by

$$F=\frac{Q_1Q_2}{Kd^2}$$

Taking the fundamental units of mass M, length L, and time T and putting the dimensions of F and d, we get

$$M^1L^1T^{-2}=\frac{[Q^2]}{[K]L^2}$$

or $$[Q^2]=[K]M^1L^3T^{-2}$$

and $$[Q]=[\sqrt{K}]M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1} \quad . \quad . \quad (1)$$

(a) As current is equal to the rate of flow of electricity,

$$\therefore \ [\text{Current}]=\frac{[Q]}{T^1}$$

$$=[\sqrt{K}]M^{\frac{1}{2}}T^{\frac{3}{2}}T^{-2} \quad . \quad . \quad (2)$$

(b) The amount of work done in carrying a charge from one point to another is equal to the product of the charge and the potential difference between the two points.

$$[\text{Potential difference}]=\frac{[\text{Work}]}{[\text{Charge}]}$$

$$=\frac{M^1L^2T^{-2}}{[\sqrt{K}]M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}$$

$$=[K^{-\frac{1}{2}}]M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1} \quad . \quad . \quad (3)$$

Electromagnetic Units. The force F between two magnetic poles m_1 and m_2 placed at a distance d from each other in a medium of permeability μ is equal to $\frac{m_1m_2}{\mu d^2}$.

$$\therefore \quad [m^2]=[F][\mu][d^2]$$

$$=M^1L^1T^{-2}[\mu]L^2$$

$$=[\mu]M^1L^3T^{-2}$$

and $$[m]=[\sqrt{\mu}]M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$$

As the force experienced by a magnetic pole in a magnetic field is equal to product of the pole strength and the intensity

of the magnetic field,

$$[\text{Intensity}] = \frac{[\text{Force}]}{[\text{Pole strength}]}$$
$$= \frac{M^1L^1T^{-2}}{[\sqrt{\mu}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}$$
$$= [\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$$

The magnetic intensity at the centre of a circular wire of radius R when a current of C electromagnetic units passes through it is equal to $\frac{2\pi C}{R}$.

(a) $$\therefore \quad [\text{Current}] = [\text{Intensity}] \times [R]$$
$$= [\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} \times L^1$$
$$= [\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1} \quad . \quad . \quad . \quad (4)$$

and $$[\text{Charge}] = [\text{Current}] \times [\text{Time}]$$
$$= [\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (5)$$

(b) $$[\text{Potential difference}] = \frac{[\text{Work}]}{[\text{Charge}]}$$
$$= \frac{M^1L^2T^{-2}}{[\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{\frac{1}{2}}}$$
$$= [\mu^{\frac{1}{2}}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} \quad . \quad . \quad . \quad (6)$$

Relation between Units on the two Systems. The above results show that the dimensions of the same quantity on the two systems of units are not the same in the units of mass, length, and time. But the dimensions of a physical quantity must be the same whatever the system of units adopted. The apparent discrepancy is due to the fact that the dimensions of K and μ have not been expressed in terms of the fundamental units. Equating the dimensions of charge on the two systems we get

$$[\mu^{-\frac{1}{2}}]\,M^{\frac{1}{2}}L^{\frac{1}{2}} = [\sqrt{K}]\,M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$$

or $$\left[\frac{1}{\sqrt{\mu K}}\right] = L^1T^{-1} = [\text{Velocity}]$$

This shows that though the dimensions of μ or K are not

known separately, the dimensions of $\frac{1}{\sqrt{\mu K}}$ are the same as those of velocity. The same result is obtained by equating the dimensions of potential difference or any other quantity on the two systems.

If a and b represent numerically the value of a given charge on the electrostatic and electromagnetic systems respectively, then

$$a[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}K^{\frac{1}{2}}]=b[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

or
$$\frac{a}{b}[L^1T^{-1}]=\left[\frac{1}{\sqrt{\mu K}}\right]$$

Here L^1T^{-1} is a unit of velocity of 1 cm. per sec.

$$\therefore \quad \frac{a}{b}=\frac{1}{\sqrt{\mu K}}=v \text{ cms. per sec.}$$

and
$$\frac{\text{Electromagnetic unit of charge}}{\text{Electrostatic unit of charge}}=v,$$

as the numerical value of a quantity is inversely proportional to the magnitude of its unit. Similarly, it is found that the electromagnetic unit of capacity divided by the electrostatic unit is equal to v^2.

By comparing experimentally the magnitudes of any one electrical quantity on the two systems, the value of v is found to be equal to 3×10^{10} cm. per sec., and this is the velocity of light in vacuum.

The action of electric and magnetic fields in rotating the plane of polarisation of light shows that the medium for the electric and magnetic phenomena is the same as that in which light waves are propagated. Maxwell found from theoretical considerations that the velocity of an electromagnetic wave in a medium should be equal to $\frac{1}{\sqrt{\mu K}}$, or the value of the electromagnetic unit of charge divided by the electrostatic unit. From these facts he suggested the **electromagnetic theory of light** according to which light is a form of electromagnetic disturbance in the ether.

Q. 212. Give a method by which the ratio of the electrostatic to the electromagnetic unit of current can be obtained. What is the value of the ratio, and what part did it play in the electromagnetic theory of light? (*Punjab, 1937*)

Ans. **Ratio of Electromagnetic and Electrostatic Units of Current.** *See* Q. 211. The ratio of the electromagnetic and electrostatic units of current is the same as that of the corresponding units of charge. As shown in Q. 211, this is equal to v, and can be determined from the ratio of the units of capacity on the two systems of units.

$$\text{Capacity} = \frac{\text{Charge}}{\text{Potential difference}}$$

Electrostatic Units

$$[\text{Capacity}] = \frac{[\sqrt{K}]M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}{[K^{-\frac{1}{2}}]M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}} = [K]L^1$$

Electromagnetic Units.

$$[\text{Capacity}] = \frac{[\mu^{-\frac{1}{2}}]M^{\frac{1}{2}}L^{\frac{1}{2}}}{[\mu^{\frac{1}{2}}]M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}} = [\mu^{-1}]L^{-1}T^2$$

If c and m be the values of the capacity of a condenser on the electrostatic and electromagnetic systems of units respectively, then

$$c[KL^1] = m[\mu^{-1}L^{-1}T^2]$$

or
$$\frac{c}{m}[L^2T^{-2}] = \left[\frac{1}{\mu K}\right]$$

But $[L^2T^{-2}]$ is the square of the unit of velocity of 1 cm. per second.

$$\therefore \quad \sqrt{\frac{c}{m}} = \left[\frac{1}{\sqrt{\mu K}}\right] = v \text{ cms. per second}$$

Determination of 'v'. The value c of the capacity of a condenser in electrostatic units is calculated from the dimensions of its coatings and the distance between them, while its value m in electromagnetic units is found by the following method :—

In one arm of a Wheatstone bridge a *small* condenser C is connected, while the other three arms contain resistances (Fig. 153). One coating of the condenser is connected to the arm of the bridge, while its other coating is connected to a vibrator which vibrates n times per second in the gap between the points K and M. The resistances are adjusted until the galvanometer shows no deflection.

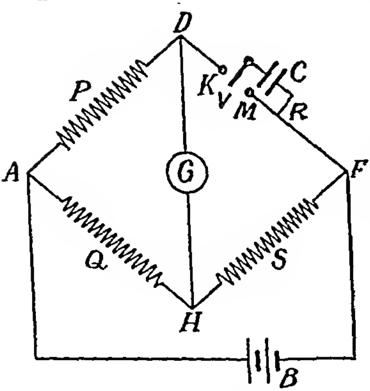


Fig. 153.

Every time V comes in contact with K, the condenser is charged and when it goes to the other extreme M it is discharged, so that the condenser is charged and discharged n times per second. If E electromagnetic units be the potential difference between F and D, the charge on the condenser is Em electromagnetic units. The amount of charge that pssses through the arm DF in one second is equal to $Em \times n$ electromagnetic units and this is the strength of current in it. If this arm contained a resistance only, its value R electromagnetic units for the same value of current would be given by

$$\frac{E}{R}=Emn$$

or
$$R=\frac{1}{mn} \text{ } e.m. \text{ units}$$

Thus the condenser behaves like a resistance R, and

$$\frac{S}{R}=\frac{Q}{P}$$

or
$$Smn=\frac{Q}{P}$$

or
$$m=\frac{Q}{n.S.P}$$

$$\therefore \quad v=\sqrt{\frac{c}{m}}=\sqrt{\frac{cnSP}{Q}}$$

The value of v is equal to 3×10^{10}, and is the same as the velocity of light in vacuum in c.g.s. units. See Q. 211 for the electromagnetic theory of light.

Q. 213. Explain the action of a triode valve as a detector and an amplifier.

Draw the diagram of a typical receiving circuit.

(*Punjab, 1934*)

Ans. **Triode Valve.** It consists of an exhausted tube containing a filament F, a wire gauze gird G, at a very short distance from the filament, and a cylindrical plate P surrounding both F and G, [*Fig.* 154 (*a*)]. When the filament is heated

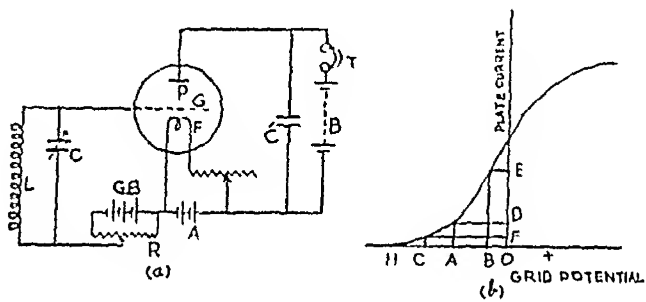


Fig. 154.

by connecting it to the low tension battery A through a variable resistance, it gives out electrons, and they are attracted by the plate P which is connected to the *positive* pole of the high tension battery B. Thus current passes in the plate-filament circuit, and its strength can be increased by increasing the plate potential or by heating the filament more by sending stronger current through it.

The plate current is more effectively changed by varying the potential of the grid, which is *very near* to the filament, and it increases or decreases according as the grid potential is raised or lowered by moving its point of contact to the right or left of the resistance R, connected across the poles of the grid bias battery G.B. Fig. 154 (*b*) is the *characteristic curve* of the valve, and it shows the relation between grid potential and

plate current. It has one *bend* near the bottom and one near the top, and the part between the two bends is *straight*.

Detector. When the negative potential of the grid is equal to OH, the plate current is stopped altogether. On raising its potential (decreasing the negative value), plate current increases, but the curve being bent upward, the plate current is increased *more* by any rise of the grid potential than it is decreased by the same fall of its potential. The grid potential is made equal to OA, so that the normal plate current is equal to OD, and the inductance of the coil L and capacity of the condenser C are adjusted to tune the circuit to the incoming waves, whose alternate positive and negative halves raise and lower the potential of the grid. If in any wave the grid potential is raised and lowered by *equal* amounts to OB and OC respectively, the corresponding values of the plate current are OE and OF, so that this current is increased by a *greater* amount than it is decreased, that is, its average value is *increased.*

Thus while the waves are being received, [the average plate current is *greater* than its normal value OD. Its value changes with the amplitude of the radio waves, and its variation is according to that of the modulated wave received, that is, its frequency of variation is equal to that of the *audio-frequency* wave, which modulated the carrier wave at the transmitting station, and *not* that of the carrier wave. In this way the radio waves which are of too high a frequency to affect the telephone T produce in it audible sound of the frequency at which the carrier wave is modulated. A small fixed condenser C' is connected across the plate load circuit to by-pass any radio-frequency currents not rectified by the valve.

[**Second Method.** The above arrangement is called *plate detection* as the rectification takes place in the plate circuit after radio frequency amplification in the valve. Another arrangement for the same purpose is called *grid leak detection*, where rectification takes place in the grid circuit and then there is audio-frequency amplification in the plate circuit. Here a condenser G.C. and a very high resistance G.L., called grid leak, are connected with the grid, as is shown with the second valve in Fig. 155, and the plate current is adjusted below its saturation valve.

When the valve is switched on, due to the presence of the grid condenser, there is no current in the grid filament circuit. As the electrons come to the grid from the filament, they can not escape and accumulate on it until due to the lowered potential further electrons are repelled. When the waves arrive the potential of the grid is alternately made positive and negative. The positive half of a wave raises the grid potential and immediately electrons begin to flow to it; its potential begins to fall, and at the end of the positive half it becomes *lower* than it was at the beginning. The next negative half of the wave still further lowers the grid potential. This fall of the grid potential continues with the subsequent waves and the plate current decreases.

The valve would be blocked but for the grid leak resistance. Its value is so adjusted that current begins to pass through it when the waves corresponding to half of the *audio cycle* have arrived, and the excess of electrons leak off just before the arrival of the next audio cycle of waves. The grid potential, instead of varying according to the signal potential, goes on decreasing during the first half of the *wave train* and then, owing to leakage, increases with the next half. The plate current decreases during the first half and increases during the second.]

Amplifier. To use the valve as an amplifier, potential of the plate or the grid is adjusted to work on the *straight* part of the characteristic curve, and with the normal grid potential the position is at the *middle* of this straight line. When the signals are impressed on the grid, its potential increases and decreases alternately. In the first case the plate current increases while in the second case it decreases, and if the grid potential remains over the straight part, equal changes in its value produce *equal* changes in the plate current. The *average* plate current remains the *same*, and the plate current wave is of *exactly* the same shape as that of the signal impressed on the grid.

The variation of the plate current changes the potential of a point in the plate circuit *many times more* than the corresponding change of the grid potential. The potential difference between the ends of a resistance or an inductance placed in

the plate circuit changes and this is impressed inductively or through a capacity, on the grid circuit of the next valve. The ratio of the change of plate potential required to produce a given change in the plate current to the change of grid potential producing the same change in the plate current is called the **amplification factor** of the valve.

Receiving Circuit. A typical receiver has three valve circuits (Fig. 155). The first valve on the left is coupled

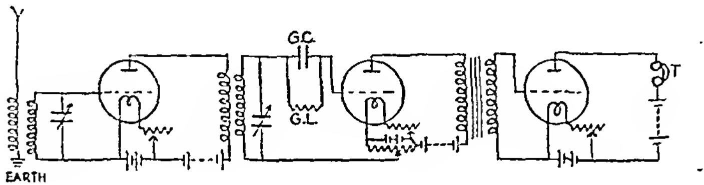


Fig. 155.

inductively with the aerial and is a radio-frequency amplifier. Its circuit is tuned to the incoming waves, and the amplified changes of potential between the ends of the primary of a transformer placed in its plate circuit are impressed through its secondary on the tuned grid circuit of the second valve, which acts as the rectifier. This transformer has *no* iron core, otherwise its impedance for the radio-frequency currents become very great. Then the rectified changes of current of audio-frequency are impressed, through a transformer with iron core, on the grid of the third valve, which is an audio-frequency amplifier, and the telephone T (or loudspeaker) is placed in its plate circuit. In actual practice only one high tension battery is used to supply the plates of the different valves with different potentials and a single low tension battery heats all the filaments. By adjusting the plate potential and the filament current the required position on the characteristic curve and the grid potential respectively are obtained.

Q. 214. Explain the use of a triode as an oscillator. Draw a diagram of a simple transmitter.

(*Punjab, 1935*)

Ans. **Triode Valve as an Oscillator.** In Fig. 156, plate

P, high tension battery B, oscillatory circuit consisting of a variable condenser C and a variable inductance L are connected through a key K with the filament F, which is heated by the low tension battery A. The grid-filament circuit contains an inductance L_1 coupled inductively with L. The potential of P is adjusted to make the plate current much below its saturation value.

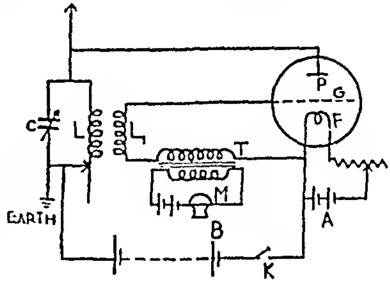


Fig. 156.

On closing the key K there is a sudden rush of electrons and, due to their inertia, the condenser C is overcharged. It then discharges through the inductance L, and, as shown in Q. 215, this discharge of the condenser is *oscillatory*. Due to damping, the amplitude of the oscillations gradually decreases, and *damped* electromagnetic waves, whose frequency is determined by the capacity and inductance of the circuit and can be changed by varying either or both of these quantities, are produced. If the charge of the condenser can be raised to its original value at the right moment after every period of oscillation, the oscillatory current and the electromagnetic waves become undamped.

This is done by the inductance L_1, which charges the grid alternately positively and negatively, because electromotive force is induced in it due to the variation of the magnetic field of coil L. The oscillatory current in the LC circuit acts inductively on coil L_1 and produces oscillatory current in the grid-filament circuit. It is *amplified* by the valve, as the operating point is on the straight part of the characteristic curve of the valve and equal changes of grid potential produce *equal* changes in the plate current, and reappears in the LC circuit. If the coils L and L_1 are correctly coupled, the original oscillations in the LC circuit are *strengthened*. The reinforced oscillations are again further amplified, and the process is repeated. In this way the amplitude of the oscillations goes on increasing, and equilibrium is attained when the plate current reaches its saturation value. The aerial and earth are connected

to the LC circuit to radiate undamped continuous waves, whose energy is derived from the high tension battery. Thus because of its ability to amplify a thermionic valve it can generate undamped oscillations.

Modulation. The microphone M used by the speaker is connected with a battery and the primary of an iron core transformer T whose secondary is in the grid circuit. When the broadcaster speaks in front of the microphone, current in the primary of the transformer changes and induces electromotive force in its secondary. The potential of the grid varies, and corresponding changes are produced in the amplitude of the oscillatory current in the plate circuit. The carrier wave is of a very high frequency, of the order of 10^6 per second, but its amplitude is changed by the microphone current at the frequency of the *speech* broadcast.

Q. 215. Write short notes on any three of the following :—

(*a*) Oscillatory discharge of a condenser.

(*b*) Television. (*c*) X-rays. (*d*) Transformer.

(*Punjab, 1939*)

Ans. (*a*) **Oscillatory Discharge of a Condenser.** In Fig. 157, a condenser C is charged by an induction coil, the right coating having positive charge and the left coating negative charge, and it is connected with an inductance L through a *narrow* spark gap. When it is being charged, electric lines of force run from A to B. The potential difference between the coatings of the condenser becomes too high, and spark passes between A and B. Current passes from A to B and magnetic lines of force encircle the line joining these two points.

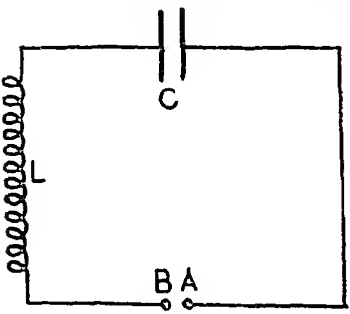


Fig. 157.

Owing to self-induction the current does *not* stop when the potential of the two coatings of the condenser becomes the same. The condenser becomes *over discharged*; the coatings acquire charges *opposite* to those

they first possessed, and the electric lines of force run from B to A. Again the spark passes, but now from B to A, and ... magnetic lines of force encircle the line BA, in the *opposite* direction to that in the last case.

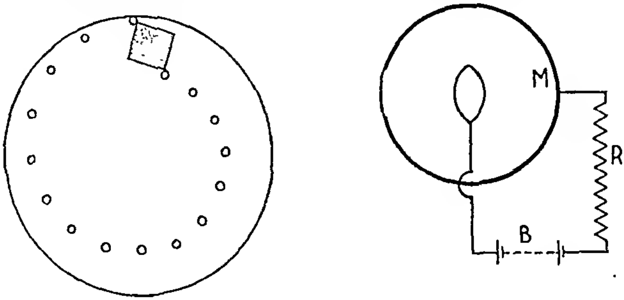
The condenser is again charged, the right coating positively and the left coating negatively. The process is repeated, but, owing to the dissipation of energy, successive surges of current become weaker and weaker. Thus when the resistance of the circuit is *very small* and below a certain value, the discharge of the condenser is *oscillatory*, and the discharge current is alternating and of decreasing amplitude. When the spark is examined in a rotating mirror, it is found to consist of a number of separate lines.

Due to the changes in the magnetic and electric fields in perpendicular directions, electromagnetic waves are given off in a direction perpendicular to the two fields, and their period depends on the capacity and inductance of the circuit.

(*b*) **Television.** The object is strongly illuminated and its image is formed on the scanning disc, which has a *large* number of holes arranged around it in the form of a spiral [Fig. 158 (*a*)]. The height of the image is equal to the distance between the first and last holes and its breadth extends between any two consecutive holes. On rotating the disc, each hole moves in front of a narrow strip of the image and one *different* from those of the last and next holes. When one hole leaves the image on the right, the next lower hole is on its left and scans the next lower strip. In this way the whole of the image is scanned in one rotation of the disc and this takes *less* than $\frac{1}{16}$ sec.

A photoelectric cell [Fig. 158 (*b*)] is placed behind the disc, and it receives successively through the disc minute spots of light, whose intensity is proportional to the intensity of illumination of the point of the object from which it comes. The spots of light fall on the metal film M, which is connected to the *negative* pole of the high tension battery B, and the electrons given out by it are received by a ring connected to the positive pole of this battery. The strength

of the current in the photoelectric cell is proportional to the intensity of illumination of the point of the object from which



(*a*) Fig. 158. (*b*)

the spot of light comes. These changes of the photo-electric current in the resistance R are amplified and then made to modulate the carrier wave of the transmitting station.

At the receiving station the signals are amplified, rectified, and again amplified through many stages, and the final plate current, which *changes according to the current of the photoelectric cell*, is passed through a neon tube, instead of a telephone. This tube has two plates placed a very small distance apart in neon gas. When the potential difference between these plates exceeds a certain limit (about 100 volts), the negative plate gives out a pinkish glow. The intensity of this glow varies *instantaneously* and *directly* as the changes in the potential difference impressed on the plates.

A scanning disc rotating in *exact synchronism* with that at the transmitting station is placed between the observer and the neon tube. Each hole moving in front of the neon tube describes narrow strip, and the observer sees a combination of spots of light whose relative intensity and arrangement are exactly the *same* as at the corresponding strip at the transmitting station. The observer receives these strips of light in quick succession, and as one rotation takes less than $\frac{1}{16}$ sec., due to persistence of vision, the image is seen as a whole.

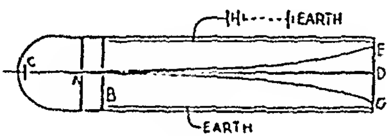
(*c*) X-Ray. *See* Q. 219.

(*d*) Transformer. *See* Q. 210.

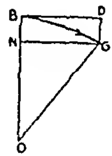
Q. 216. Explain Sir J. J. Thomson's method of ascertaining the values of $\frac{m}{e}$ and v for cathode rays.

(Bombay, 1935)

Ans. **Thomson's Method. (*a*) Electrostatic Field.** The cathode rays coming from the cathode C are passed through a narrow slit in the earthed anode A and a narrower slit in another earthed plate B to get a *narrow* beam, and they fall at D on a fluorescent screen at the other end of the exhausted tube BD [Fig. 159(*a*)]. Tube BD contains two



(*a*)



(*b*)

Fig. 159.

parallel metallic plates, one near its top, and the other near its bottom. The upper plate is connected to the positive pole of a high voltage battery whose negative pole is earthed, while the lower plate is connected with earth, so that an electric field of *uniform* strength F *e m. units* is produced from above downward *in* the plane of paper and *perpendicular* to the path of the cathode rays.

When there is no electric field, the cathode rays move along a straight line CABD. On switching on the battery, the cathode rays, being negatively charged, are deflected upwards and follow a parabolic path BE, and their displacement DE is equal to d cm. If e *electromagnetic* units be the charge of an electron and m gm. its mass, it experiences a force of Fe dynes upward and its acceleration is equal to $\frac{Fe}{m}$ cm. per sec. per sec. perpendicular to its original path. Let v cm. per sec. be the velocity of an election and suppose it takes t sec. to travel the distance BD of length l cm. It moves forward with a uniform velocity of v cm. per

sec., and also upward with a uniform acceleration of $\frac{Fe}{m}$ cm. per sec. per sec.

$$\therefore \quad vt = l \text{ cm.}$$

or
$$t = \frac{l}{v} \text{ sec.}$$

and
$$d = \frac{1}{2}\left(\frac{Fe}{m}\right) t^2 = \frac{1}{2}\,\frac{Fe}{m}\,\frac{l^2}{v^2} \text{ cm.}$$

$$\therefore \quad \frac{m}{e} = \frac{Fl^2}{2dv^2} \text{ gm./e.m. unit} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(*b*) **Magnetic Field.** A uniform magnetic field of strength H gauss is applied to the whole length of the tube BD and *perpendicular* to the plane of paper from above downward. The stream of electrons experience a force in the plane of paper perpendicular to the magnetic field and their direction of motion at *each* point on their path, and not their original path. They describe a circular arc of radius R cms., and their deflection DG is *opposite* to that produced by the electrostatic field.

If N is the number of electrons per c.c., Nav electrons pass through a cross-section area a sq. cm. of the beam in one second. Therefore a charge of $eNav$ electromagnetic units passes in one sec., and this is the strength of current i. The force exerted by the magnetic field on a length L of the beam of electrons is equal to HLi, or $HLeNav$ dynes, so that the force experienced by one electron is equal to Hev dynes, as the number of electrons in a length L is equal to NLa. This is the centripetal force $\left(\frac{mv^2}{R}\right)$ which makes the electron move in a circular path of radius R cm. From Fig. 159 (*b*),

$$DG(2R - DG) = NG^2 = BD^2,$$

where NG is drawn perpendicular on the radius OB. As DG is very small as compared with 2R, $2R - DG$ is practically equal to 2R,

$$\therefore \quad d.2R = l^2$$

cr
$$R = \frac{l^2}{2d}$$

But
$$Hev = \frac{mv^2}{R} = \frac{mv^2 2d}{l^2}$$

$$\therefore \quad \frac{m}{e} = \frac{Hl^2}{2dv} \text{ gm./e.m. units} \quad . \quad . \quad . \quad . \quad (2)$$

(c) **Combined action of Electrostatic and Magnetic Fields.** The intensities of the two fields are so adjusted that their deflections are *equal* and *opposite*, that is, under their combind action the path of the beam remains unchanged.

$$\therefore \quad Hev = Fe$$

or
$$v = \frac{F}{H} \text{ cm. sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Putting this value of v in (1) or (2), the value of $\frac{m}{e}$ is found.

Q. 217. Give an account of some recent determination of e the electronic charge, and give the theory of the method.

Taking e as $4{\cdot}7 \times 10^{-10}$ electrostatic units, the electrochemical equivalent of hydrogen as 0·0000104 gm. per coulomb, and the standard density of hydrogen as 0·00009 gm. per c.c., estimate the number of molecules per c.c. of hydrogen under standard temperature and pressure. (*London University*)

Ans. **Determination of Electronic Charge. Millikan's Method.** Very fine drops of oil or mercury are produced in a chamber with an atomiser A, and some of them pass into the lower chamber through a hole in plate C, which is placed at a small distance from, and parallel to, another horizontal plate D (Fig. 160). The lower chamber is lighted with a strong beam of light, so that the fine drops appear as bright points of light when looked at through a microscope, whose eyepiece is fitted with a scale. These drops do

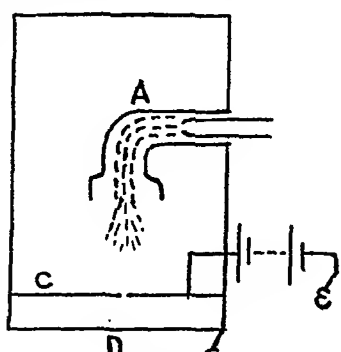


Fig. 160.

not evaporate appreciably, and *individual* drops can be kept under observation for a long time.

The two plates, C and D, are connected together by a wire so that there is no potential difference between them. A drop of mass M gm., radius a cm., and density ρ gm. per c.c. has weight of $Mg = \frac{4}{3}\pi a^3 \rho g$ dynes, but it experiences upward thrust of air of $\frac{4}{3}\pi a^3 \sigma g$ dynes, where σ gm. per c.c. is the density of air, so that it falls under a force of $\frac{4}{3}\pi a^3(\rho - \sigma)g$ dynes. Its downward motion should be uniformly accelerated, but the viscosity of air opposes its motion. The retarding force due to viscosity increases with the velocity of the drop and it soon becomes equal to its effective weight. Then the drop moves with a *steady* velocity of v_1 cms. per sec, which according to Stokes' law is given by

$$v_1 = \frac{2a^2(\rho - \sigma)g}{9\mu},$$

$$\therefore\ a = \left\{\frac{9\mu v_1}{2(\rho - \sigma)g}\right\}^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where μ is the co-efficient of viscosity of air. By noting the time that the drop takes to fall through a known distance, its steady velocity is found.

Plate C is connected with the positive pole of a high voltage battery B, while the lower plate D and the negative pole of the battery are connected with earth. If the distance between the two plates is d cm. and V electromagnetic units is the electromotive force of the battery, the intensity F of the uniform electric field produced between the two plates is equal to V/d electromagnetic units.

Then the air between the two plates is ionized by exposing it to X-rays. The drop picks up one or more ions, and it experiences an upward or downward electric force according as the ion is negative or positive. If the drop takes up n ions, each of charge e electromagnetic units, the force of the electric field on it is equal to Fne dynes, and the resultant force on it is equal to $\frac{4}{3}\pi a^3(\rho - \sigma)g \pm Fne$ dynes. The drop acquires a new steady velocity of v_2 cms. per sec. which is proportional to the new force on it.

$$\therefore\ \frac{v_2}{v_1} = \frac{\frac{4}{3}\pi a^3(\rho - \sigma)g \pm Fne}{\frac{4}{3}\pi a^3(\rho - \sigma)g}$$

or $$nc = \pm \frac{(v_2 - v_1)4\pi a^3(\rho - \sigma)g}{3v_1 F}$$

Putting the value of a from (1) and the value of F equal to V/d, we get

$$nc = \pm \frac{(v_2 - v_1)d4\pi(\rho - \sigma)g}{3v_1 V}\left\{\frac{9\mu v_1}{2(\rho - \sigma)g}\right\}^{\frac{3}{2}}$$

$$= \pm\frac{4\pi d}{3V}\left(\frac{9\mu}{2}\right)^{\frac{3}{2}}\left\{\frac{v_1}{(\rho - \sigma)g}\right\}^{\frac{1}{2}}(v_2 - v_1) \qquad \ldots (2)$$

The value of $(v_2 - v_1)$ changes with the number of ionic changes taken up by the drops, but it is always an *integral* multiple of a certain least value which corresponds to n equal to one. Putting this minimum value of $(v_2 - v_1)$ and the other known quantities in (2), the value of e is obtained.

An alternative method is to adjust the electric force to balance the effective weight of the drop when it becomes *stationary*.

$$Fnc = \tfrac{4}{3}\pi a^3(\rho - \sigma)g$$

or $$\frac{V}{d}nc = \tfrac{4}{3}\pi(\rho - \sigma)g\left\{\frac{9\mu v_1}{2(\rho - \sigma)g}\right\}^{\frac{3}{2}}$$

and $$nc = \frac{2\pi d}{3V}\;\frac{(9\mu v_1)^{\frac{3}{2}}}{2(\rho - g)^{\frac{1}{2}}} \qquad \ldots \qquad \ldots \qquad \ldots (3)$$

Problem. In an electrolyte an hydrogen ion consists of one atom of it and it carries a positive charge which is numerically equal to the charge of an electron.

Mass of 1 c.c. of hydrogen = 0·00009 gm.

Electrochemical equivalent of hydrogen = 0·000104 gm./coulomb.

$$\therefore \text{ Charge on ions in one c.c.} = \frac{0{\cdot}00009}{0{\cdot}0000104} = \frac{9}{1{\cdot}04} \text{ coulomb.}$$

But Charge on one hydrogen ion $= 4{\cdot}7 \times 10^{-10}$ e.s. units.

$$= \frac{4{\cdot}7 \times 10^{-10}}{3 \times 10^9} = \frac{4{\cdot}7}{3} \times 10^{-19} \text{ coulomb.}$$

$$\therefore \text{ No. of ions in one c.c.} = \frac{9 \times 3}{1{\cdot}04 \times 4{\cdot}7 \times 10^{-19}} = 5{\cdot}526 \times 10^{19}$$

and No. of molecules in one c.c. $= 2{\cdot}763 \times 10^{19}$, as one molecule of hydrogen consists of two atoms.

Q. 218. Describe and explain Thomson's method for analysing positive rays, and discuss the meaning of isotopes.

Ans. Positive or Canal Rays Thomson's Method. The positive ions in the kathode dark space move towards the kathode, and the faint glow on the kathode is due to them. Thomson's apparatus consists of two parts: the discharge tube T where the rays are produced, and the camera tube on the right where they are examined and their presence detected by their action on the photographic plate P [Fig. 161 (*a*)]. The

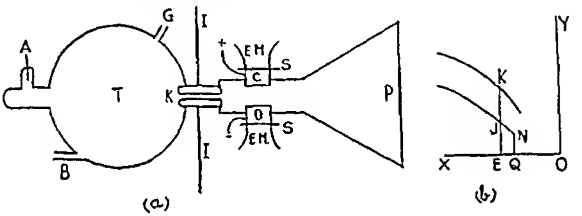


Fig. 161.

gas to be examined is supplied to tube T through a side tube B, and is removed by a pump through a tube G to keep the gas in T at a suitable low pressure. The camera tube is connected to a tube containing charcoal immersed in liquid air for absorbing the gas. The pressure in the camera tube is made much below that in T, and can be easily maintained as the diffusion through the kathode tube is very slow.

The kathode K is pierced with a *very narrow* long slit, through which a very fine pencil of positive rays passes into the camera tube. The kathode tube is surrounded by a thick tube of ion to protect the rays in it from the magnetic field, while the tube T is protected by the iron sheets I. Two blocks of soft iron, C and D, are connected to the poles of a battery which produces an electric field between them, and at the same time they form the pole pieces of a very strong electromagnet E.M. from which they are insulated by mica sheets S.

Thus the rays pass through a magnetic field of intensity H gauss and an electric field of intensity F electromagnetic units, both in the plane of paper and perpendicular to the path of

the rays. The deflection x produced by the electric field is in the plane of paper, perpendicular to the path of the rays, and is proportional to $\frac{eF}{mv^2}$ while the displacement y due to the magnetic field is proportional to $\frac{eH}{mv}$ and is perpendicular to both the magnetic field and the path of rays, that is, perpendicular to the plan of paper. *See* equations 1 and 2 of Q. 216.

$$\therefore \frac{y^2}{x} = \frac{e^2H^2}{m^2v^2} \cdot \frac{mv^2}{eF} = \frac{H^2}{F} \cdot \frac{e}{m}$$

or

$$y^2 = \frac{H^2}{F} \cdot \frac{e}{m} \cdot x \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here $\frac{H^2}{F}$ is a constant and if $\frac{e}{m}$ is also constant, y^2 is proportional to x, and a parabola is obtained on the photographic plate with its axis parallel to the axis of x [Fig 161(b)]. Different points on the same parabola correspond to particles of the *same* value of $\frac{e}{m}$ moving with different velocities, while different parabolas indicate particles of different values of $\frac{e}{m}$, for which both e and m may be different.

$$\frac{y}{x} = \frac{eH}{mv} \cdot \frac{mv^2}{eF} = \frac{H}{F}v \quad . \quad . \quad . \quad . \quad (2)$$

Thus $\frac{y^2}{x}$ measures the ratio of the charge to the mass of the particles, while $\frac{y}{x}$ is a measure of their velocities. If KJE and NQ are drawn perpendicular to the X-axis, the ratio $\frac{e}{m}$ of the particles deflected to K to that of the particles deflected to F is equal to $\left(\frac{KE}{JE}\right)^2$, while the ratio of the velocities of particles deflected to N and J on the same parabola is given by

$$\frac{NQ}{OQ} \cdot \frac{OE}{JE}.$$

Isotopes. The ratio $\frac{e}{m}$ of any element to that of a single charged hydrogen atom is called the *electric atomic weight* of the former. Taking the mass of hydrogen atom as the unit, the masses of the *individual atoms* of other elements can be measured. With instruments of greater sensitive power it is found that a single parabola of an element, which corresponds. to the same charge, consists of a number of parabolas. This shows that atoms of the *same* element differ in mass and that any particular atom belongs to one of the several groups. Such substances which have identical chemical properties, but different atomic weights, are called *isotopes*. All the isotopes of the same element have the same atomic number and cannot be separated by chemical methods. Taking the at. wt. of oxygen as 16, their atomic weights are *whole* numbers, and experiment shows that the fractional atomic weight of an element is due to the mixture of its two or more isotopes. Thus the fractional atomic weight 35·46 of chlorine is due to its two isotopes of at. wts. 35 and 37 mixed in a proportion which is the same no matter how chlorine is obtained.

Q. 219. Describe the production, properties, and uses of X-rays.

Ans. **Rontgen Rays or X-Rays. Production.** When the cathode rays fall on an obstacle of high atomic weight, X-rays are given off. The cathode K is made concave, and at its centre of curvature is placed a tungsten plate B, inclined at angle of 45° to the axis of the cathode. Plate B is called *anticathode* and is connected to the anode A (Fig. 162). The tube is exhausted until the Crookes' dark space occupies the whole of it and there is no cathode glow. The cathode rays are focussed on B, and X-rays are given off from it. They pass out from one side of the tube, as shown by arrows.

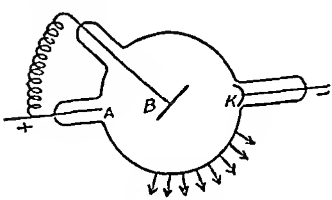


Fig. 162.

The velocity with which the cathode rays strike the target depends on the potential difference between the two electrodes of the tube. Owing to their very high velocity, they penetrate

into the inner energy levels of the orbital electrons of the atoms of the anticathode, and by repulsion expel some orbital electron out of the atom. To take the place of this ejected electron, another electron from an outer arbit falls into the inner orbit, and X-rays are radiated.

Properties. These rays are not deflected by an electric or a magnetic field, and, therefore, are not charged particles. They are a form of electromagnetic radiation, and their velocity is the same as that of light, but their wave-length is short (10^{-7} to 10^{-9} cm.) as compared with that of the visible part of the spectrum (10^{-5} cm.). With high potential difference *hard* X-rays of high frequency are produced, while *soft* rays of low frequency are generated when a low potential difference is applied to the tube.

These rays affect a photographic plate and produce fluorescence in a barium platinocyanide plate placed several metres away. When they are passed through a gas, it is ionized and becomes a conductor of electricity. They are very penetrating and their penetrating power depends on the atomic weight of the anti-cathode and the substance through which they pass. The greater the density of the substance, the smaller is the penetrating power of the rays for it. A 0·5 mm. thick plate of lead is more opaque than a 5 cm. thick plate of aluminium. They pass more easily through wood than iron, and more easily through iron than through gold.

When they fall on a metal plate, some pass out without any change and some are scattered. In addition to this, electrons are ejected with a velocity equal to that of the cathode rays in the X-ray tube in which the primary X-ray were produced, and X-rays of *smaller* frequency are given off. These secondary X-rays depend on the penetrating power of the primary rays and the atomic weight of the element struck, and are *characteristic* of the element. Most substances give out one or more kinds of characteristic homogeneous X-rays, but for their production the primary X-rays must be of *greater* frequency. The characteristic rays form different series, according to their penetrating power, and some substances give out lines of two series simultaneously. These series are called K, L, and M in decreasing order of penetrating power and frequency.

The spectra of a given series of different elements resemble each other. Not only the number of lines is the *same* in each, but in each case they are *similarly* placed. The wave-lengths of corresponding lines *decrease* with atomic weight but there is no fixed ratio. Mosley has found that the square root of the frequency of a given line for different elements is *proportional* to their *atomic numbers*, which are equal to the resultant positive charges on the nuclei of their atoms or the number of the orbital electrons. The high frequency waves are given out by the electrons of an inner orbit, while the outer electrons are responsible for the emision of the low frequency waves.

Uses. These rays pass more readily through flesh than through bones. When they are passed through a part of the human body and the emergent rays received on a fluorescent screen, a shadow picture of the bones and imbedded bullet or any metal piece is seen on it. For rays of different hardness the transparency of flesh is different. Therefore thick muscles can be distinguished from others, and even the actual beating of the heart can be seen. For taking a photographic record, the active face of the fluorescent screen is placed against the photographic plate.

In addition to finding fracture or dislocation of bones, any imbedded foreign matter, and the diseases of the internal organs, these rays are also used for the treatment of certain diseases, specially of the skin. Due to their ionizing power they produce chemical changes which are responsible for their physiological effects.

They may be used for testing the proper nailing or fitting of costly shoes. Imitation pearls are less transparent than the real pearls owing to these rays, and thus they can be distinguished from one another.

These rays are used for studying the structure of the crystals, that is, the arrangement of atoms in them. By studying the different effects produced when they fall on a substance, the structure of its atoms, the charge on the nucleus, and the arrangement of electrons in them are found.

Q. 220. Explain the photoelectric effect. Describe a photoelectric cell and its working, and mention some of its uses.

Ans. **Photoelectric Effect.** When light of certain frequencies falls on certain metal plates, particularly the amalgams of the metals of the alkali group, they emit electrons and acquire a slight positive potential. As the positive potential of the metal plate increases it resists the escape of electrons, and after a certain stage their emission stops. This phenomenon is called **photoelectric effect,** and the strength of the photoelectric current is increased by increasing the intensity of the incident light, or connecting the metal plate with the *negative* pole of a high voltage battery whose positive pole is connected with another plate placed at a small distance from the first plate.

The kinetic energy of the emitted electrons is measured by the positive potential which when applied to the plate just prevents their emission, and from this their velocity is calculated. It is found to be different for different electrons, but it has a certain maximum value. Those electrons which come from the surface have maximum velocity, while those coming from below the surface lose energy in coming to the surface and emerge from it with a smaller velocity. Experiments conducted in vacuum show that the maximum velocity of the emitted electrons depends on the wave-length of the incident light. It is *independent* of its intensity, but increases with its frequency. With greater intensity of the incident light, the rate of emission of the electrons increases, but their velocity remains unchanged. For every metal there is a certain *minimum* frequency of the incident light, called *threshold frequency*, below which no electrons are emitted.

The wave theory cannot account for these results, but the quantum theory explains them satisfactorily. According to this theory, a radiation of frequency n consists of discontinuous bundles of energy each of value hn, where h is a universal constant and is called Planck's constant. These bundles of energy are called **photons,** and they are absorbed or emitted in *whole* numbers. When light falls on the plate, an electron absorbs a photon of energy hn. In detaching itself from the metal plate a certain amount of work w has to be done by the electron of mass m, and it emerges with a velocity v, so that

$$\tfrac{1}{2}mv^2 = hn - w.$$

If the frequency of the incident light is equal to the threshold frequency n_o for the given metal, the energy of the photon is just sufficient to detach the electron, and

$$hn_o = w$$

Combining these two equations, we get

$$\tfrac{1}{2}mv^2 = h(n - n_o)$$

If a positive potential V given to the metal plate just prevents the emission of electrons of charge e, Ve is equal to $\frac{1}{2}\,mv^2$,

and $$Ve = h(n - n_o),$$

so that the graph between V and n is a straight line.

When hn is smaller than w, no electrons are emitted however great the intensity of the incident radiation may be, because an increase in intensity simply increases the number of photons falling on a unit area of the surface in a unit time, but the energy of each photon remains unchanged.

Photoelectric Cell. A very fine film of a metal is more sensitive than a thick plate of it, and a film of its hydride or oxide is still more sensitive. On the inside of a glass bulb a thin layer of silver is deposited, except for a small window on the left through which light is to enter [Fig. 158(b)], and a part of the silver coating is covered with a fine layer of the active material. The silver coating is connected with a cap on the glass bulb, and thus the active material can be connected with the *negative* pole of a high voltage battery. The anode is placed in the middle of the bulb and is in the form of a ring, so that it may not obstruct the light falling on the active material.

The cell is either completely exhausted or after complete evacuation it is filled with a *small* amount of one of the rare gases, such as helium, argon, or neon. In the vacuum type of the cells, the photoelectric current is solely due to the electrons emitted by the active material, while in the gaseous type the ejected electrons ionize the gas and expel electrons which further increase the current and make the cell more sensitive. As the photoelectric current is *very low*, of the order of *few microamperes*, it is passed through a resistance and magnified many times with a number of thermionic valves.

Uses. In order to use a photoelectric cell for different purposes, a suitable source of light is used and light is passed to the cell in a suitable manner. It is the *interruption* of the light falling on it, and the consequent change in the photoelectric current, that makes its applications so varied. It is used in television, sound pictures, sorting and sampling devices, fire and burglar alarms, controls, colour analyzers, and light intensity meters.

Q. 221. Describe the chief characteristics of α, β, and γ rays. Is there any relationship between any of these rays and X-rays? (*Calcutta, 1936*)

Ans. When the nuclei of the atoms of radio-active elements explode, helium nuclei (alpha particles), or electrons (beta particles), or both are ejected, and at the same time electromagnetic waves (gamma rays) are radiated.

α-Particles. These particles cause fluorescence and produce *scintillations* on a *fluorescent* screen, which are visible through a microscope. On passing through a gas they ionize it by knocking out electrons from its molecules. When their velocity falls below a certain critical value, they are unable to ionize the gas any further. This range is measured by receding the ionization chamber away from the source of these particles until the ionization current just stops. The ionizing activity of a radio-active substance is mainly due to these particles, owing to the large amount emitted, but their photographic action is very weak.

They are deflected by an electric or a magnetic field applied perpendicular to their path, and their direction of deflection shows that they are *positively* charged. Their deflection is very small as compared with that of the β-particles, showing thereby that their mass is comparatively *large*. The value of $\frac{e}{m}$ is the *same* for all α-particles, and its value is equal to 4823 *e.m.* units per gram, while the corresponding value of hydrogen ions in electrolysis is 9650 in the same units. This shows that either the mass of an α-particle is twice that of an hydrogen ion and it carries a single charge or it carries two units of charge and its mass is four times that of the hydrogen ion. By collecting these particles in a tube and allowing them

to remain there for some days, when their velocity is decreased on passing through mass and they pick up electrons to become neutral atoms, spectrum of helium is obtained. Therefore these particles are atoms of *helium* carrying two units of positive charge.

The velocity of expulsion of these particles is characteristic of the parent radio-active substance and is of the order of 10^9 cms. per sec. They penetrate through matter, but their penetrating power or range is smaller than that of the β-particles and γ-rays. The stopping power of a substance for α-particles increases with its atomic weight and is proportional to its square root, so that their range in hydrogen is four times that in oxygen. Further, their range in a gas varies inversely as its pressure.

When an α-particle is ejected, the nucleus loses two units of positive charge, and its atomic weight decreases by four units. The number of orbital electrons is decreased by two, and a new element of smaller atomic number and atomic weight is formed.

β-Particles. They produce brilliant fluorescence, particularly on a barium platinocyanide plate, the colour depending on the nature of the plate. These particles produce ionization, and are about 100 times *more* penetrating than α-particles, so that they are easily detected by the photographic method. They do not exhibit any definite range, because when they collide with atoms, due to their small mass, they are deflected and, therefore, their path is *zigzag*.

Unlike α-particles, the β-particles from a substance are not homogeneous but possess *different* velocities, ranging from 0·3 to 0·99 of that of light. They are deflected by a magnetic or an electric field much more than the α-particles and in the *opposite* direction, so that they are *negatively* charged particles of a very small mass as compared with that of the α-particles. The value of $\frac{e}{m}$ is not constant, but *decreases* with velocity, showing that their mass *varies* with velocity and *increases* as their velocity approaches that of light. For low velocities, the value of $\frac{e}{m}$ is equal to $1{\cdot}77 \times 10^7$ *e.m.* units per gram, which is the same as that of the kathode rays.

When a β-particle is ejected by the nucleus of an atom of a radioactive substance, its positive charge increases by one unit but no appreciable change is produced in its mass, so that its atomic number increases by one though its atomic weight remains the same. It acquires one more orbital electron and a new element of different chemical properties is produced.

γ-Rays. These rays differ greatly from the α and β particles. They are not deflected by a magnetic or an electric field so that they carry no electric charge. They produce fluorescence and photographic action and ionize the gas through which they pass. Their penetrating power is very great, being about 100 times that of the β-particles.

Their velocity is *equal* to that of light, and they consist of electro-magnetic waves of very small wave-length, ranging from $1{\cdot}7 \times 10^{-10}$ cm. to $4{\cdot}1 \times 10^{-8}$ cm. and depending on the kind of transformation in which they are produced. The nucleus after emitting a β-particle is left in an excited condition, and settles down to the new kind of atom after giving out γ-rays.

Thus γ-rays resemble X-rays, both being electromagnetic waves in the ether, except in their great penetrating power and smaller wave-length. The X-rays in passing through matter produce electrons of low velocity while γ rays liberate electrons of high velocity and produce ionization.

Q. 222. Write a short note about what you know about the structure of the atom. (*Punjab, 1934*)

Ans. **Structure of the Atom.** According to the electron theory, all kinds of matter is made up of positively charged particles called protons, and negative charges known as electrons. Every atom, like the solar system, consists of a massive and very minute nucleus, made up of protons and electrons, at the centre, around which one or more electrons revolve in regular orbits. The nucleus and the electrons occupy a *very small* part of the volume of the atom, so that there is a relatively very large amount of *empty* space. Each proton has a unit positive charge while an electron is a unit of negative electricity, and the positive charges of the nucleus is *greater* than the number of electrons in it, that is its resultant charge is *positive*. Under normal conditions every atom is electrically neutral, so that the number of electrons revolving

around the nucleus is equal to the resultant positive charge on it. The mass of the atom is almost solely due to the nucleus, as the mass of an electron is about $\frac{1}{1840}$ of that of the proton.

In going from lighter to heavier atoms, the mass of the nucleus increases by one or more units at each step but its resultant positive charge or the number of orbital electrons rises by *one* unit only. The simplest of all atoms is that of hydrogen, and consists of a single proton as its nucleus around which revolves a single electron. The next simplest atom is that of helium, where there are four protons and two electrons in the nucleus with two orbital electrons, while the most complex atom is that of uranium consisting of a nucleus of 238 protons and 146 electrons, and around it revolve 92 electrons.

For stability the maximum number of electrons in any orbit is fixed, and is 2, 8, 8, 18, 18, 32, 32 in successive outer orbits. No electron can exist in any orbit unless the next inner orbit contains its maximum number. Those elements in which the number of electrons in the outermost orbit is maximum do not enter into chemical combination, while those in which the quota of this orbit is not complete tend to enter into chemical combination with other elements in a similar dissatisfied condition by the interchange of these electrons. The electrons in the outermost orbit are not so strongly bound to the nucleus as the electrons of the inner orbits, and their arrangement is *disturbed* by the proximity of other atoms. For this reason the compound formed does not exhibit those properties of its components which depend on the electrons in the outermost orbit. These properties, such as chemical affinity, viscosity, surface tension, atomic volume, melting and boiling points, co-efficient of expansion, *optical* spectra, etc., depend on the number of the outermost layer electrons and are *periodic*.

The number of orbital electrons in a neutral atom or the resultant positive charge on its nucleus determines its *atomic number*. All the isotopes of an element have the same atomic number though their atomic weights are different, and have the same chemical properties. Thus the chemical properties of an element are determined by its atomic number and not its atomic weight. Elements with similar properties are found at regular intervals when arranged according to their atomic numbers.

The electrons in the nucleus are very strongly held captive, and their arrangement is not easily disturbed by other atoms. Threrefore properties depending on these electrons are exhibited even when the element enters into chemical combination with another substance. These propertics, such as X-ray spectra, are not periodic but increase with the atomic number of the element.

An electron when revolving in its orbit does not radiate energy continuously. According to Bohr, only certain orbits, called *stationary states*, are possible, and in such a state no radiation of energy occurs. When an electron is displaced from an inner orbit to an outer orbit, work is done *on* it and its potential energy *increases*, while its potential energy *decreases* when it falls from an outer orbit into an inner orbit. It is only when an electron, disturbed and displaced to an outer orbit, passes from it to an inner orbit that energy is radiated in the form of electromagnetic waves. The different lines of the spectrum of an element correspond to different initial and final orbits, and all the lines in which the *final* orbit is the same belong to the same series.

Q. 223. Describe Zeeman effect and give its explanation on the simple electronic theory.

Ans. **Zeeman Effect.** When a source of monochromatic light is placed between the poles of a *very strong* electromagnet and examined with a spectrometer of very high resolving power, a single line of its spectrom is found to be split up into *two* or *three* lines according as the source is observed *along* the magnetic field, though a hole bored in the pole pieces, or *perpendicular* to it. Fig. 163 (*a*) gives the normal position of the

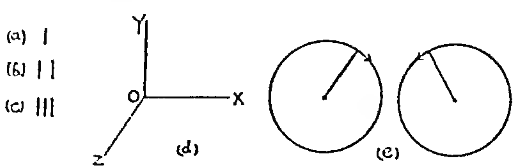


Fig. 163.

line in the absence of the magnetic field. The two lines observed in the direction of the field are both displaced but on the opposite sides of the original line, as shown in Fig. 163 (*b*),

and are *circularly* polarized in *opposite* directions. Fig. 163 (c) shows the three lines seen in a direction perpendicular to the magnetic field. The middle line occupies the position of the original line and is plane polarized, while the outer two lines are also plane polarized, but in a direction perpendicular to that of the middle line.

Light is a kind of electromagnetic radiation, and its emission is due to the vibrations of the electrons, the electric displacement in the wave being in the direction of vibration. These vibrations are very complicated, but they may be considered to consist of three sets of circular vibrations in the three mutually perpendicular planes XOY, YOZ, and ZOX (Fig. 163*d*), each set containing vibrations in opposite directions. (Fig. 163*e*). If the magnetic field is along OZ, the vibrations in the plane XOY have no component in its direction, while the other two sets of vibrations may be resolved into *linear* vibrations along OZ and circular vibration in the plane XOY.

The magnetic field has *no* effect on the linear vibrations, because they are *in* its own direction, and their period of vibration remains unchanged ; but it exerts equal and *opposite* forces on the electrons vibrating in opposite directions in circular orbits in the plane XOY, the direction of the force being perpendicular to the direction of motion of an electron and that of the magnetic field. The force on one set of electrons is towards the centres of their orbits, and their period of vibration is decreased and the frequency of the emitted light increased. The second set of electrons experience the force *away* from the centres of their orbits ; the centripetal force on them decreases, and their period of vibration is increased and the frequency of the emitted light is decreased.

When the observer looks along the direction of the magnetic field, no light is received due to the linear vibrations along OZ, as no light is given out in the direction of vibration of the electrons, and only the two circularly polarized lines, polarized in opposite direction, are observed. The light waves due to the linear vibrations along OZ are propagated in the XOY plane, and as these vibrations are not affected by the magnetic field, a plane polarized line is seen in its unchanged position when the source of light is observed perpendicular to the

magnetic field. The circular vibrations in the plane XOY give out plane polarized waves and not circularly polarized waves in *this* plane, because in sending waves, say, along OX the component vibrations along OY *alone* are effective and no waves due to the component vibrations along OX can be received in this direction. Therefore the two outer lines are also plane polarized but in a direction perpendicular to that of the middle line, as in this case the vibrations for the outer lines are along OY and those for the middle line are along OZ.

APPENDIX

PUNJAB UNIVERSITY PAPERS

B.A. & B.Sc.

1931

PHYSICS—Paper (*a*)

1. What is a dimensional equation ?

An apprentice engineer found that the volume (V) of water which passes any point of a canal during t seconds is connected with the cross-section (a) of the canal and velocity (u) of water by the relation—

$$V = kau^2t.$$

Test by the Method of Dimensions if the relation is correct.

2. What is the moment of inertia of a body ? Calculate the moment of inertia of a uniform rod about one end.

Two spheres of the *same* mass are exactly similar from outside, but one is hollow, while the other is solid. Explain how they can be distinguished.

3. How will you determine the angle of contact for mercury and glass ?

Prove that the excess of pressure inside a soap bubble to the outside pressure $= \frac{4T}{r}$, where T stands for the surface tension of the soap solution and r for the radius of the sphere.

4. How will you determine the specific heat of a gas at constant volume ? Will it be equal to the specific heat of the gas at constant pressure ? If not, why not ?

5. Deduce Boyle's law from the kinetic theory of gases, and show how the theory explains the deviations from the law in the case of actual gases.

6. Show that no heat engine can be more efficient than a reversible heat engine working between the same limits of temperature.

7. Prove the formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ for refraction through a lens.

The rays of a vertical sun are brought to focus by a lens at a distance of 30 cm. from the lens in air. If the same lens is held *just* over a smooth pond of water, at what depth in the water will the rays come to focus? (The index of refraction of water is 1·33.)

8. Explain Foucault's method of measuring the velocity of light. How did it help the wave theory of light to get firmly established?

9. Explain the formation of colours in a thin film. Show why the colour of a film when viewed by reflected and transmitted light are complementary.

10. The waves of light are said to be transverse. What is the evidence for this?

A thick tourmaline crystal allows only a single polarized beam to pass through it, and so does a Nicol prism. Is the action in the two cases similar?

1931

PHYSICS—Paper (*b*)

1. What are organ pipes? What are their modes of vibration when (*a*) both ends are open, (*b*) one end is closed?

2. How would you determine the frequency of vibration of a tuning-fork? Give details of your experimental arrangement.

3. Work out completely the laws of transverse vibrations of a string, and show how the diameters of two wires of the same material can be compared with the help of a sonometer.

4. Define electrostatic potential at a point.

What potential would enable the tension on an insulated sphere of 3 cms. radius to balance 5×10^{-6} of an atmosphere?

5. How would you determine the specific inductive capacity of a solid substance? Give a sketch of the experimental arrangement you would employ.

6. Define the terms Declination, and Dip.

Describe a method of accurately determining the dip at a place.

7. Describe the construction of a galvanometer of the moving coil type, and explain how, with the addition of a shunt, it can be used as an ammeter for large currents.

8. A normal Daniel's cell has an e.m.f. of 1·07 Volts and resistance 2 ohms. Its terminals are connected by two wires in parallel of resistances 3 and 4 ohms. Assuming that the electro-chemical equivalent of copper is 0·000328 gram per coulomb, calculate the weight deposited in the cell, and also the heat developed (*a*) in the cell, (*b*) in each of the wires, during an hour of the working of the cell.

9. Explain what is meant by a simple magnetic shell, and find the potential of such a shell at any point.

Calculate the magnetic force at a point inside a long solenoid of N turns carrying a current of A amperes.

10. Describe the construction and use of a thermo-couple for the measurement of temperatures.

A themo-couple is made of iron and constantan wires. Find the e.m.f. developed per °C. difference of temperature between the two junctions, given that the themro-e.m.f.'s of iron and constantan against platinum are respectively $+1600$ and -3440 micro-volts per 100°C. difference of temperature.

What do the opposite signs in the above indicate?

11. Write short notes on any *three* of the following:—

(*a*) Cathode rays.

(*b*) Properties of α and β rays.

(*c*) Thermionic valves and their uses.

(*d*) Constitution of the atom.

1932

PHYSICS—Paper (*a*)

1. Explain the use of the compound pendulum in an accurate determination of the acceleration produced by gravity, and give the theory of the method.

2. Define Young's modulus, and indicate the methods of determining it.

A wire of length 250 cm. and radius 0·3 mm. is stretched by hanging on a weight of 12 kilos and the elongation produced is 8 mm. Calculate the value of Young's modulus for this wire.

3. What is meant by the viscosity of a fluid ?

How will you find the co-efficient of viscosity of a liquid ?

4. Describe an accurate method of determining the mechanical equivalent of heat.

5. Define the co-efficient of heat conductivity, and describe a method of investigating the heat conductivity of a solid material.

6. State the equation of an 'isothermal', and obtain that of the 'adiabatic' for a perfect gas.

Show that the adiabatic elasticity of air is γP, where γ is the ratio of the two specific heats and P is the pressure.

7. A compound achromatic lens is to be constructed having a focal length of 50 cm., the surface of contact of the crown and flint glass lenses having a common radius of 30 cm. The dispersive powers of crown and flint glass being taken as 0·22 and 0·46 and refractive indices for the middle of the spectrum assumed as 1·52 and 1·63 respectively. Calculate the radii of curvature of the second faces of the two lenses.

8. Explain clearly how interference fringes are formed, and give a formula connecting their width with the wave-length of the light used.

9. Give the theory of the diffraction grating, and explain the effect of the number of lines per cm. on its action.

What is a normal spectrum, and how is it produced ?

10. Explain the following :—

(*a*) Double refraction.

(*b*) Brewster's Law.

(*c*) Rotation of plane of polarization.

1932

PHYSICS—Paper (*b*)

1. Show that the velocity of waves in a stretched string is given by the expression $\sqrt{\frac{T}{\rho}}$, where T is the tension and ρ the mass of unit length of the string. Hence, deduce an expression for the frequency of a vibrating string.

2. Distinguish between the Diatonic scale and the scale of Equal Temperament, and show how each scale has been built up.

3. If a vibrating fork is rapidly moved towards a wall, beats may be heard between the direct and reflected sounds. Account for these, and calculate their frequency, if the fork makes 512 vibrations per sec. and approaches the wall with a velocity of 300 cms. per sec. The velocity of sound may be taken as 330 metres per sec.

4. What is meant by the Magnetic Permeability of a substance? How can it be measured? Describe how the permeability of a piece of soft iron varies with the magnetizing force.

5. (*a*) Find an expression for the force per square cm. of surface on a conductor due to its charge.

(*b*) Find the mechanical stress per sq. cm. on the glass plates of a condenser, charged to a potential of 30,000 volts. S.I.C. of glass = 4 and thickness = 4 mms.

6. What is meant by the S.I.C. of a medium? How would you explain physically that the force between two charges is diminished when they are placed in a medium of greater S.I.C.?

How would you experimentally determine the S.I.C. of either a gas or a liquid?

7. State the laws governing the distribution of current in a network of wires. A battery of 6 volts e.m.f. and 0·5 ohm internal resistance is by oversight joined in parallel with another of 10 volts e.m.f. and 1 ohm internal resistance, and the combination is used to send current through an external resistance of 12 ohms. Calculate the current through each battery and the external resistance.

Comment on this method of connecting cells.

8. The heat of combustion of hydrogen and oxygen to form water is 34.200 calories for each gramme of hydrogen burnt. A C.G.S. unit current decomposes in one second 0·000945 gm. of water. The mechanical equivalent of heat being $4{\cdot}2 \times 10^7$ ergs, find in volts the smallest e.m.f. which can decompose water.

9. What is the Thomson effect in thermo-electricity? What led to its discovery, and how can it be demonstrated? Why is it sometimes called the specific heat of electricity? Heat is being generated at a certain part of an electric circuit

through which a current is flowing. How can you determine whether the heating is due to thermo-electricity or whether it is merely Joule heating?

10. Distinguish between the mean value and the root mean square value of an alternating current, and find the relation between them

11. Write short notes on any *three* of the following :—

(*i*) Difference between

(*a*) electrons and β particles,

(*b*) X rays and γ rays,

(*c*) α rays and positive rays.

(*ii*) Isotopes.

(*iii*) Photo-electricity.

(*iv*) Spontaneous disintegration of atoms.

(*v*) Nature and properties of kathode rays.

1933

PHYSICS—Paper (*a*)

1. Define co-efficient of rigidity, and give its dimensions.

Show that the couple required to twist a wire is $\frac{\pi n r^4}{2l}\theta$, where the letters have their usual significance.

2. How does the value of g change at different places on the surface of the earth due to its rotation?

The mass of a railway train is 100 tons. What will be its weight when (*a*) stationary, (*b*) travelling due east, (*c*) travelling due west, along the equator at 60 miles per hour? Radius of the earth is 4,000 miles.

3. What are the requisites of a balance? Obtain the general expression used for determining the conditions for these requisites, and show that the conditions for two of these are mutually contradictory.

4. What is the thermodynamic scale of temperatures? How is it defined, and what are its advantages over other scales? Can it be used practically? Give reasons.

5. Describe briefly how the specific heat of a gas at constant pressure may be determined. Obtain an expression

for the difference between the two specific heats of a gas in terms of other measurable quantities.

6. Explain Prevost's theory of exchanges, and show that the emissive power of a body is equal to its absorptive power. Describe some instrument by which radiant heat can be measured.

7. What is meant by an achromatic combination of lenses, and what are the principles underlying its construction? Prove the formulæ involved.

8. Explain the formation of colours in thin plates.

A parallel beam of sodium light ($\lambda=5890\times10^{-8}$ cm) is incident on a thin glass plate ($\mu=1·5$), such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.

9. In what respects does plane polarized light differ from unpolarized light, and how will you experimentally distinguish one from the other?

Define polarizing angle. How is it related to the index of refraction?

10. What is meant by the resolving power of a diffraction grating? Derive the expression for it.

1933

PHYSICS—Paper (*b*)

1. Give a short account of the various methods for the determination of the frequency of sound vibrations and compare their accuracy.

2. Describe an experiment by which you can determine the velocity of sound in hydrogen. Work out any formula which you may need for the purpose.

3. Describe and explain Melde's experiment. In an experiment it was found that the string vibrated in 5 loops when 10 grammes were placed in the scale pan. What mass must be placed in the pan to make the string vibrate in 7 loops? (Neglect the weight of the scale pan.)

4. How will you determine experimentally the value of the vertical component of the earth's magnetic field?

The correct value of the dip at a certain place is 69°. What is the apparent dip if the circle be turned 45° out of the magnetic meridian? (Trigonometric tables will be provided.)

5. How will you determine the S.I.C. of a glass plate?

6. A disc 16 cm. in diameter is surrounded by a guard ring and is situated at a distance of 1 mm. from a large parallel metal plate connected to earth: the potential of the disc is 1·70 E.S. Units. Calculate in grams weight the attraction on the disc.

Describe an electrical measuring instrument based on this principle.

7. Explain clearly how you would use a potentiometer for measuring currents. How would you adapt it for use with large and small currents respectively?

8. Discuss the equivalence of a magnetic shell and a circuit carrying a current. Calculate the field at a point on the axis of a plane circular shell or coil of wire in which a current flows.

9. Explain what is meant by (*a*) impedance, (*b*) reactance of a circuit, and (*c*) virtual voltage? Why should alternating currents be used instead of direct currents for long-distance power transmission?

10. Explain as clearly as you can the action of a thermionic valve as a detector of radio signals.

11. How has the ratio e/m for an electron been determined?

1934

PHYSICS—Paper (*a*)

1. Find the periodic time of a compound pendulum; and explain the meaning of "the centre of suspension," and "the centre of oscillation." Show that they are interchangeable.

2. What do you understand by the viscosity of a liquid? Define viscosity, and give an experimental method for its determination. What is the effect of temperature on it?

3. Deduce the formula for determining the moment of inertia of a sphere about a diameter.

A fly-wheel weighs 10 tons, and the whole of the weight may be considered as concentrated at a distance 3 feet from

the axis. What is the amount of energy stored in the fly-wheel when rotating at a speed of 100 revolutions per minute ?

4. What is meant by J, the mechanical equivalent of heat ? Explain the principles underlying the various methods of determining the same.

5. Derive the *reduced* equation of a gas, starting from Van der Waal's equation of state; and show that if two gases have the same *reduced* pressure and volume, they also have the same *reduced* temperature.

6. Define thermal conductivity. How is this constant determined in the case of a gas?

Describe an experiment to show that hydrogen has a greater conductivity than air.

7. Obtain the relation between the distances of an object and its image formed by a lens in terms of the radii of curvature of the lens.

Show that the minimum distance between an object and its real image formed by a convex lens is four times the focal length of the lens.

8. Describe and explain the colours seen when a thin film of oil spreads over the surface of water.

9. Describe the phenomena of double refraction, and discuss its connection with the polarization of light.

Explain the ordinary and the extraordinary rays in a crystal, and the construction of a Nicol's prism.

10. Write brief notes on the following :—

(*a*) Fraunhofer lines.

(*b*) Infra-red spectroscopy.

(*c*) Absorption and Emission spectra.

1934

PHYSICS—Paper (*b*)

1. What are Lissajous' Figures ? Calculate the resultant of two rectangular simple harmonic vibrations, whose amplitudes as well as periods are in the ratio of 1 : 2, and the phase-difference is 90°.

2. Describe an experimental arrangement by which you would determine the absolute frequency of a tuning-fork.

3. Describe the various parts of a harmonium, and explain their use. Explain the existence of 12 notes to an octave in this instrument.

4. Prove that the intensity of the magnetic field due to a small magnet of magnetic moment M, at a point situated d cm. from its middle point on a line making an angle θ with the axis of the magnet, is

$$\frac{M}{d^3}\sqrt{3\cos^2\theta+1}.$$

5. Define the terms intensity of magnetization (I), magnetic induction (B), and the magnetic force (H): and obtain the relation $B=H+4\pi I$.

Explain the difference between the lines of force and the lines of induction inside a piece of soft iron placed in a magnetic field.

6. Describe the construction of the Dolezalek Quadrant Electrometer. How is it used to measure (*a*) small, and (*b*) large potential differences?

7. Describe fully and explain the action of a moving coil galvanometer. How can it be made (1) very sensitive, (2) ballistic, and (3) dead-beat?

8. Give an account of the chemical changes which occur in a storage cell during charge and discharge.

An accumulator has a capacity of 28 ampere hours. What is theoretically the least weight of PbO_2 on its positive plates, given that the PbO_2 is reduced to PbO, and that (*a*) the electro-chemical equivalent of hydrogen is 0·00001038 gm./coulumb, and (*b*) the atomic weight of lead is 207, and of oxygen 16?

9. Explain why there is a phase-difference between E.M.F. and current in an A.C. circuit. What is power-factor, and when is the current Watt-less?

10. Explain the action of a Triode valve as a detector and and amplifier.

Draw the diagram of a typical receiving circuit.

11. Write a short note on what you know about the structure of the atom.

1935

PHYSICS—Paper (*a*)

1. Find the resultant of two mutually perpendicular S.H. motions which agree in period, but differ in phase. Consider the important cases for phase differences varying from 0 to 2π.

2. What is meant by the co-efficient of rigidity of a substance? Explain how it can be determined experimentally, deducing the formula used.

3. What is moment of inertia of a body? State the units in which it is generally measured.

Find the moment of inertia of a thin uniform circular plate of mass M and radius R_1, with a concentric hole of radius R_2 about an axis passing normally through the centre.

4. Describe Joly's method of determining the specific heat of a gas at constant volume. Will the specific heat at constant volume be equal to the specific heat at constant pressure? If not, why not?

5. Describe Lindes' method for liquefying gases, and discuss the principle on which it is based.

6. Explain how it is possible to define temperature in terms of energy, and thus arrive at an absolute scale of temperature from thermo-dynamical considerations.

7. Explain the theory of a direct-vision spectroscope.

8. Explain the action of a diffraction grating. Deduce the formula which connects the wave-length of the diffracted light, its deviation, and the grating constant.

9. Describe the phenomenon of rotatory polarization. Explain in full the main parts of some form of instrument for measuring the strength of sugar solution by means of this property.

10. Explain the formation of colours in a thin oil-film.

1935

PHYSICS—Paper (*b*)

1. Obtain an expression for the velocity of sound in a gas. How does the velocity depend on humidity, temperature, and pressure?

2. Compare the notes given by a vibrating string and an open organ pipe.

3. Explain Doppler effect in sound.

The frequency of the whistle of a stationary engine is 600. What is the apparent frequency of the whistle to passengers in a train travelling at 60 miles an hour before and after passing the engine?

4. Give a short account of the molecular theory of magnetism, and explain in a general way—diamagnetism, paramagnetism, and ferromagnetism.

How are diamagnetic and paramagnetic substances distinguished experimentally?

5. Define normal electric induction and tubes of force. State and prove Gauss's theorem. Deduce from the theorem that the intensity of the field near a charged surface of density σ is $4\pi\sigma$.

6. Describe and explain the use of an attracted disc electrometer. How do you make use of this to measure the dielectric constant of a solid?

7. Describe a method of determining the specific resistance of an electrolyte. Why can we not measure it like the resistance of a metallic conductor?

8. Define self-inductance. What are its dimensions? How can it be demonstrated to a large audience?

9. Explain the use of a triode as an oscillator. Draw a diagram of a simple transmitter.

10. Draw diagrams of series wound and shunt wound motors. To what use are they put? Explain the construction and use of a starter.

How is the efficiency of a series motor determined?

11. Write short notes on *any three* of the following :—

(1) Thunderstorms, (2) Choke coil, (3) Zeeman effect, (4) Photoelectric cell, (5) Positive rays.

1936

PHYSICS—Paper (*a*)

1. Deduce the dimensions of (*a*) the co-efficient of Viscosity, and (*b*) the constant of Gravitation, G.

Obtain a formula for the time of swing of a simple pendulum, from a knowledge of the dimensions of the physical quatities involved.

2. What is meant by Young's modulus, the modulus of rigidity, and the bulk modulus ?

A straight cylindrical rod is fixed at one end and twisted by a couple applied at the other. Deduce the relation between the twisting couple and the co-efficient of rigidity.

3. What do you understand by the surface tension of a liquid ? Describe one experimental method of determining it. How do temperature and pressure affect surface tension ?

4. Give an account of the measurements of low temperatures.

5. Deduce Boyle's law from the kinetic theory of gases. How are deviations from the law explained on this theory ?

6 Show that the efficiency of a reversible heat engine working between two specified temperatures is a maximum for those temperatures, and deduce an expression for the efficiency in terms of the temperature scale you adopt.

7. What is meant by an achromatic combination of lenses ? How will you correct the chromatic abberation of a plano-convex, crown glass lens of 30 cm. mean focal length, the following data being given ? Refractive indices for red and blue light respectively are 1·520 and 1·540 for the crown glass and 1·630 and 1·660 for a flint glass, various lenses of which material are available and with any desired focal length, and radii of curvature.

8. How is the rectilinear propagation of light explained on the wave theory ? Calculate the velocity of light in diamond, the refractive index from air to diamond being 2·5.

9. Describe and explain fully one method involving the phenomenon of interference for determining the wavelength of a monochromatic radiation.

10. What is elliptically polarized light, and how can it be produced ? How would you distinguish between such light on the one hand and a mixture of plane polarized light and unpolarized light on the other ?

1936

PHYSICS—Paper (B)

1. Obtain an expression for the resultant of two wave-motions travelling along the same direction, with the same velocity and amplitude, but with slightly different frequencies, and explain its meaning.

Distinguish between beats and combination tones.

2. What is Newton's expression for the velocity of sound in a gas? How did Laplace modify this expression, and why?

Describe Kundt's experimental arrangement for the determination of the velocity of sound in gases.

What other measurements are possible with this arrangement?

3. Explain:—Free vibrations, forced vibrations, and resonance, giving an example for each.

How is the principle of resonance used to analyse a complex note?

4. Define magnetic permeability and susceptibility. How are the two related?

Describe an experimental method of getting the hysteresis curve for iron or steel.

Point out briefly the importance of these curves in the construction of dynamos.

5. How was the inverse square law in electrostatics established theoretically and experimentally by Cavendish?

6. Find the electric intensity at a point between two large parallel plates, one of which is earthed, the surface density of the charge on the insulated plate being σ.

Two large metal plates are fixed horizontally at a distance of $\frac{1}{2}$ cm. from each other. What potential in volts should be applied between the plates, if a droplet of oil of mass $1{\cdot}5 \times 10^{-11}$ gm., and carrying a charge of $4{\cdot}9 \times 10^{-10}$ E.S.U., is to be held at rest between the plates? [$g=980$; one E.S.U. of potential $=300$ volts.]

Mention any application of this arrangement.

7. Find the intensity of the magnetic field due to a circular coil carrying a current, at a point on its axis. How can it be experimentally determined?

How can a local uniform field be obtained with two such coils ?

8. Describe instruments to measure 10 amperes, 10^{-3} ampere and 10^{-8} ampere.

9. Explain how a rotating magnetic field can be produced. Mention an important application.

10. Describe a triode. How can it be used as an amplifier ?

Draw a neat diagram of a circuit used for amplifying audio-frequency currents.

11. Write short notes on *any two* of the following :—

(*a*) Transformer.

(*b*) Production and application of X-rays.

(*c*) Structure of the atom.

(*d*) Isotropes.

(*e*) Photo electric cell.

1937

PHYSICS—Paper (*a*)

1. Define the term "rigidity." What is the relation between the modulus of torsion and the modulus of rigidity for a given wire ?

If you are given a wire 100 cm. long and 0·04 cm. in diameter, what apparatus and observations will you require for the determination of its modulus of rigidity ?

2. What are the conditions which determine the sensitiveness of a common beam-balance ? What are the disadvantages of an extremely sensitive balance ?

3. Why is the upper surface of mercury in a glass capillary tube convex upward, while for water it is concave ?

Assuming the surface tension of rain-water to be 72 dynes per cm., find the difference of pressure inside and outside a rain-drop of diameter 0·02 cm. What would this difference of pressure amount to if the drop were to be decreased by evaporation to a diameter of 0·00002 cm. ?

4. Derive for a perfect gas the relation connecting pressure and volume during an adiabatic change.

Calculate the rise in temperature when a gas, for which γ (gamma)$=1{\cdot}5$, is compressed to eight times its original pressure, assuming the initial temperature to be 27° Centigrade.

5. What is the "Joule-Thomson effect," and how has it been accurately measured? Describe the method used for liquefying hydrogen.

6. Give an account of Forbes' method for determining the thermal conductivity of a metal. What is a "steady state"?

7. Explain clearly the distinction between "resolving power" and "magnifying power" of optical instruments. Derive an expression for the magnifying power of a microscope.

8. Derive a formula expressing the relation between μ, u, v and r in the case of refraction at a single spherical surface separating two transparent media. Using the result, show that $1/f=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ for a convex lens. What approximations have you made, and what limitations do they impose upon the use of this formula?

9. Give a brief account of the use of the spectrometer in the study of the sun and stars. What kind of information does it give, and how?

10. Write short notes on any *three* of the followings :—

(*a*) Raman Effect; (*b*) Light Quanta;
(*c*) Half-Wave Plate; (*d*) Echelon Grating:
(*e*) Michelson-Morley Experiment.

1937

PHYSICS—Paper (B)

1. Obtain an expression for the velocity of transverse vibrations along a stretched string, and thence deduce the frequency of a string vibrating in p segments.

2. What is Doppler effect?

The whistle of an engine moving at 30 m.p.h, is heard by a motorist driving at 15 m.p.h. and estimated to have a pitch of 500. What must be the actual pitch of the whistle, to the nearest whole number, when—

(a) the two are moving in opposite directions but approaching each other ;

(b) the two are moving in opposite directions but away from each other ;

(c) the two are moving in the same direction, the motorist being behind the engine ;

(d) the two are moving in the same direction, the motorist being in front of the engine ?

The velocity of sound may be taken to be 1,200 ft. per sec.

3. Describe the action of a mouth-piece of an organ flute pipe, and show that an open end pipe will produce sounds richer in overtones than a closed end one.

4. Describe the action of a magnet in (a) uniform, (b) a non-uniform, field.

5. Give the theory and use of a quadrant electrometer. How has an increase of sensitivity been obtained in the modern forms of the instrument ?

6. The energy in a charged parallel plate condenser is supposed to reside in the dielectric between the plates. Show that this is $\frac{2\pi D^2}{k}$ or $\frac{kE^2}{8\pi}$ per c.c. of the dielectric where D and E are respectively the electric density and intensity of the field k the S.I.C. of the medium.

7. A pointer galvanometer of 10 ohm resistance has a 50 division scale and indicates one micro-ampere per division. Draw a complete plan of connections, showing how it could also be used at will as a milliammeter of 50 milliamperes range and a voltmeter of 5 volts range. Calculate the necessary resistances, etc.

8. Obtain the relation between current and voltage in an inductive circuit without capacity when connected to an harmonically alternating current supply. Also obtain an expression for the power developed in such a circuit ?

9. Give a method by which the ratio of the electrostatic to the electromagnetic unit of current can be obtained. What is the value of the ratio, and what part did it play in the electromagnetic theory of light ?

10. Write short notes on *any two* of the following :—
(1) Seebeck and Peltier effects.
(2) Nucleus of the atom.
(3) Radioactivity.
(4) Atomic number and its significance.

1938

PHYSICS—Paper (a)

1. Derive the formula for the period of a compound pendulum, and prove that the centres of oscillation and suspension are reversible. Indicate how this principle is utilized in an accurate determination of gravity.

2. Define Poisson's ratio, and show that the rigidity n and Young's modulus Y are connected by the relation

$$n=\frac{y}{2(1+\sigma)},$$

where σ is the Poisson's ratio.

3. Define Viscosity, and describe any method for determining it in the case of liquids.

What is *critical velocity* ?

4. Show that for a perfect gas $C_p - C_v = R$, where C_p is the specific heat of a gram-molecule of a gas at constant pressure and C_v, the specific heat at constant volume and R is the gas constant.

Calculate C_v for hydrogen, given that

$C_p = 6{\cdot}85$ Cal.

Density of hydrogen at N.T.P. $= 0{\cdot}0899$ gm./litre, and $J = 4{\cdot}19 \times 10^7$ erg./cal.

5. Distinguish clearly between adiabatic change and Joule-Thomson effect.

Describe briefly how these processes have been used in the liquefaction of gases.

6. What is a reversible process ? Prove that the efficiency of a reversible heat engine is maximum.

7. Calculate the focal length of a Ramsden's and a Huyghens' eyepiece. Which one of them is more achromatic ?

8. Mention the different methods for determining the wave-length of light, and give an account of the one you ...sider the most accurate.

9 Describe a method for obtaining elliptically polarized ...ht. How will you distinguish experimentally between ...iptically polarized light and a mixture of plane polarized ...d unpolarized light ?

10. Write short notes on any *three* of the following :—

(*a*) Spherical aberration.

(*b*) The theory of the rainbow.

(*c*) Half-shade device in Polarimeter.

(*d*) The bigger the diameter of a telescope, the smaller is the image of a star.

(*e*) Aplanatic points.

1938

PHYSICS—Paper (*b*)

1. Derive an expression for the velocity of sound in a gas, and hence show that the change in the velocity of sound in air is about 0·6 metre per degree (Centigrade) change in temperature. [Velocity in air at 0°C.= 330 metres/sec.]

2. The frequency of a tuning fork is nearly 50. Describe a method for its accurate determination.

What is the effect of (i) loading the prongs, and (ii) change of temperature on the frequency of a fork ?

3. Describe how you will measure the velocity of sound in carbon dioxide.

4. Derive an expression for the intensity of the field ...lue to a small magnet at a large distance from it. Hence ...educe that in case of the earth

$$2 \tan \lambda = \tan \delta,$$

where δ is the angle of dip at a place whose magnetic ...atitude is λ.

5. Describe the phenomenon of magnetic hysteresis and explain how it is measured. What is the significance of such measurements ?

6. State Gauss's theorm. Prove that every eleme a charged conductor experiences an outward force equ $2\pi\sigma^2$ per unit area, where σ is the charge per unit a the point considered.

A sphere of radius 100 cms. is charged to a potenti 1,500 E.S. units. Calculate the force per unit area o sphere.

7. Describe the construction of a moving coil galv meter and derive an expression for the relation between current and deflection.

8. What is the real meaning of the statement "ele current flows from the positive to the negative pole"?

Twenty lamps of 30 watts each are run from the s mains (230 volts) for five hours. Calculate :—

(i) The units consumed ;
(ii) The current through the mains ;
(iii) The resistance of one lamp ; and
(iv) The heat (in calories) given out by all the l in one hour.

9. Draw the circuit diagram of a two valve receive explain the action of the various parts.

10. Write short notes on any *three* of the following :—

(a) Self-inductance.
(b) Rotating magnetic field.
(c) Positive rays.
(d) Zeeman effect.
(e) Transmutation of elements.

1939

PHYSICS.—Paper (a)

1. Define moment of inertia.

Calculate the moment of inertia of a disc of mass grammes and radius R cms. about an axis at right angles the plane of the disc and passing through its centre.

2. Find the acceleration of a body of mass M, moving w a velocity V in a circle of radius r and show how the va

6α,7α-DIFLUOROMETHYLENE-11β,17α-DIHYDROXY-21-p-BROMOBENZOYLOXY-16α-METHYL-2,4-PREGNADIEN-20-ONE (2,3-D)-1'-PHENYLPYRAZOLE